Efficient Embedded Signaling Through Rotated Modulation Constellations for SLM-Based OFDM Systems

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Abstract—In this paper, we propose a novel optimized blind selected mapping (SLM) technique that avoids the need of the use of any side information (SI) through the joint exploitation of a set of rotated and unrotated QAM constellation of the Orthogonal Frequency Division Multiplexing (OFDM) signal and that consequently performs the role of an inherent embedded signaling for the OFDM system. Furthermore, in order to efficiently recover the transmitted sequence, we propose a set of decision schemes that banks on the Max-Log-Maximum A Posteriori (MAP) algorithm from which we investigate hard and soft decision processes. Our proposed method guarantees a reliable and perfect signal recovery at the receiver side and dramatically reduces the peak-to-average power ratio (PAPR) level at the transmitter side. Simulation results are given to support our claims.

Index Terms—OFDM, PAPR, Selected Mapping, Max-Log-Maximum A Posteriori, rotated set and embedded signalling.

I. INTRODUCTION

The Orthogonal Frequency Division Multiplexing (OFDM) modulation technique is a multi-Carrier transmission scheme that has recently been widely adopted in various wireless communication standards (i.e., WLAN, DVB-T,...), thanks to both high spectral efficiency and the robustness offered especially in a frequency selective channel environment.

However, OFDM systems have few limitations with high Peak-to-Average Power Ratio (PAPR) as the key one [1], [2]. This requires a linear high power amplifier (HPA) [3]. The combination of an HPA having insufficient linear-range and a large PAPR leads to in-band distortion and out-of-band radiation. Various techniques have been developed to reduce PAPR such as clipping [4], some methods including coding techniques as block coding [6], [7] or based on the relationship between some coding properties and OFDM modulator [5]. Other challenging techniques were proposed as Selected Mapping (SLM)[8], Partial Transmit Sequence (PTS) [9],…

Among the cited techniques, the SLM presents a special interest since it can lead to a better PAPR reduction without resulting in any OFDM signal degradation. The PAPR reduction is realized by multiplying independent indexed phase sequences by the original data and then selecting the resulted SLM signal that achieves good PAPR reduction whose index constitutes a Side Information (SI). However, the SI is transmitted to the receiver as a set of bits for the data recovery, which represents an overhead resulting in a data rate decrease. In addition, if this index is wrongly estimated at the receiver side, this will lead to a significant performance deterioration of the OFDM system, in terms of the obtained Bit Error Rate (BER) for example. To solve this problem, several suggestions proposed to implement the SLM method without incorporating the SI index, which is referred as semi-Blind SLM [10], [11]. Some of the authors of such suggestions[10], [11] propose to use pilot tones that are transmitted in the OFDM parquets whereas other authors suggested to perform additional processing in the transmitter side such as the use of a Hadamard Matrix [12], which would increase the OFDM system complexity.

On that account, this paper is meant to suggest a novel optimized blind SLM method in which the need to send a side information is omitted. To do so, we conceive an adequate set of rotated and unrotated constellation at the receiver side. This constellation processing, termed as ‘embedded signaling’ guarantees a reliable and perfect signal recovery at the OFDM receiver that is performed by a hard or a soft decision process using a MAX-Log-MAP estimation technique that optimally takes advantages from this embedded signaling. Contrarily to existing works, both the OFDM transmitter and the receiver sides are exploited to jointly reduce the PAPR and guarantee a perfect signal reconstitution without any use of side information, through the incorporation of our proposed embedded signaling that exploits the OFDM signal phase set.

The rest of this paper is organized as follows: In section III we define the OFDM system which is followed by the description of classical SLM technique. After that, we explain in section V the new blind SLM and our motivation. The approaches used to determine the SI index in the received side are given in section VI. We describe the obtained simulation results to emphasize the performance of our proposed method in section VII. Finally, we dedicate section VIII to summarize
the contributions of this paper and to give the conclusions of its work.

II. NOTATIONS

The boldface lower case letters denote vectors and boldface upper case letters denote matrices. The superscripts $^T$ and $^*$ denote the transpose and the element wise conjugation, respectively. We note by $\mathbb{E}$ the expectation operator, $\| \cdot \|$ denotes the absolute value and $Pr$ refers the probability and $\langle x, y \rangle = x^T y$ denotes the scalar product.

III. OFDM SYSTEM

A. Typical OFDM system

In an OFDM system, a frequency bandwidth $B$ is divided into $N$ non-overlapping orthogonal subcarriers of bandwidth $\Delta f$ where $B = N \Delta f$. For a given OFDM symbol, each subcarrier is modulated with a complex value taken from a known constellation. Let $X = [X_0, \ldots, X_{N-1}]^T$ denotes a block of $N$ frequency domain subcarriers. After performing a $N$-length Inverse Fast Fourier Transform (IFFT), we obtain the following sequences $X =$ such

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{j \frac{2 \pi k n}{N}}, \quad n = 0, \ldots, N-1.$$

B. PAPR definition

The PAPR is a figure of merit that describes the dynamic range of the OFDM time domain signal. The conventional definition of the PAPR for the OFDM symbol in the time domain $x$ may be expressed as $PAPR(x) = \max_{n=0, \ldots, N-1} \frac{|x_n|}{\mathbb{E}[\|x\|^2]}$.

IV. CLASSICAL SLM PAPR REDUCTION TECHNIQUE

Conventional SLM scheme is a specific scheme for PAPR reduction [8]. It is achieved by multiplying the original data $X$ by $D$ independent phase sequences of length $N$ denoted $\{\phi^{(d)}\}$ where $1 \leq d \leq D - 1$ such that $\phi^{(d)} = [\phi^{(d)}_0, \ldots, \phi^{(d)}_{N-1}]$ and determining the PAPR of each phase 'sequence & data' combination. The combination having the lowest PAPR is transmitted according to Algorithm 1:

**Algorithm 1 - SLM PAPR reduction Algorithm**

**Require**: $D$, $N$, $X$ and $\{\phi^{(d)}_0, \ldots, \phi^{(d)}_{N-1}\}$

1. for $d = 0$ to $D - 1$ do
2. Compute $X^{(d)} = X e^{i \phi^{(d)}}$
3. Compute $x^{(d)} = \text{IFFT}(X^{(d)})$
4. Calculate $PAPR(d) = \max_{n=0, \ldots, N-1} \frac{|x^{(d)}_n|}{\mathbb{E}[\|x^{(d)}\|^2]}$
5. end for
6. $\hat{d} = \min_{d=0, \ldots, D-1} \text{PAPR}$
7. Keep on $\phi^{(d)}$.

V. NEW BLIND SLM TECHNIQUE

A. Motivation

In the ordinary SLM technique described above, the receiver should know which phase sequence is used in the transmitter side. Hence, a portion of the bandwidth must be allocated for the transmission of this SI index. A wrong estimation of the SI index at the received side leads a damage on the total signal recovery. One may protect the side information with some forms of coding techniques, but this would result in further bandwidth loss.

For that reason, we suggest a new Blind SLM (BSLM) method, where the phase sequence used in the transmitter to minimize the PAPR is detected in the receiver side without sending any information. Basically, this method uses a predefined set $E$ of possible phases. This set $E$ consists of two sub-systems: $E = E_1 \cup E_2$ where $E_2$ is the set of rotated constellation with a specific $\phi_{opt}$ angle whereas $E_1$ leaves unchanged the constellation. As an example for an OFDM signal using a 4QAM constellation the $\phi_{opt} = \frac{\pi}{4}$. $E_1 = \{0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi\}$ and $E_2 = \{\frac{\pi}{4}, \frac{3\pi}{4}, -\frac{\pi}{4}, -\frac{3\pi}{4}\}$.

The $D$ candidate phase sequences $\{\phi^{(d)}\}$ are randomly generated from the set $E$. Thus, it results $D$ code vectors of length $(N \times 1)$ denoted $C_d = [C_0, \ldots, C_{N-1}]^T$ and is defined as follows:

$$C_d^{(n)} = \begin{cases} 0 & \text{if } \phi^{(d)}_n \in E_1, \quad n = 0, \ldots, N-1 \\ 1 & \text{if } \phi^{(d)}_n \in E_2 \end{cases}$$

This constructs a dictionary or a codebook containing codes that represent an embedded signaling. This will significantly simplify the detection of the selected sequence phase ($d$) at the received side. First of all, we have to define the optimal rotation angle $\phi_{opt}$ and this is the object of the next paragraph.

B. Optimum rotation angle $\phi_{opt}$

Consider for example an MQAM constellation and let $\mathcal{Q} = \{q_0, \ldots, q_{M-1}\}$ be the coordinates for constellation points and let $\mathcal{Q}' = \{q'_0, \ldots, q'_{M-1}\}$ be the coordinates for a rotated constellation points where $Pr(q_m) = Pr(q'_m) = \frac{1}{M}$. If we examine the two types of constellation illustrated in Fig.1, we can notice that the distance between the points of $\mathcal{Q}$ and those of $\mathcal{Q}'$ depends on the rotation angle and the chosen modulation.

![Fig. 1. Rotated and unrotated 4QAM (left) and 16QAM (right).](image-url)

Thus, if constellation points are very close, the rate of errors detection phase will increase. That is why we propose to
calculate the minimum distance, \( d_{\text{min}} \), separating two different types of constellation: the selected one \( \phi_{\text{opt}} \) is the one having the biggest \( d_{\text{min}} \). We illustrate in Table I the corresponding \( \phi_{\text{opt}} \) for 4, 16 and 64 QAM.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>OPTIMAL PHASE rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>4QAM</td>
<td>7</td>
</tr>
<tr>
<td>16QAM</td>
<td>7</td>
</tr>
<tr>
<td>64QAM</td>
<td>7</td>
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</tbody>
</table>

Regarding the receiver side, to guarantee an efficient signal recovery, we proposed a powerful decision process that we detail in the sequel.

VI. DECISION PROCESSES

In this section we present the block diagram for the recovery of the SI index as it is shown in Fig.2. We assumed that both transmitter and receiver consider the same phase sequences dictionary for a specific QAM constellation and share the same set \( E \) (see Section V-A).

Fig. 2. Block diagram of candidate generation, transmission and decoding of the side information (SI)

At the transmitter side, IFFT is usually followed by a Cyclic Prefix (CP) insertion in order to mitigate the ISI. Then \( X \) is transmitted. At the receiver side, the CP is first removed and then Fast Fourier Transform (FFT) operation is performed. Finally, the received signal can be expressed as follows:

\[
R_n = H_n X_n^{(d)} + W_n, \quad H_n X_n^{(d)} + W_n, \quad n = 0, \ldots, N - 1, \quad (3)
\]

where \( W_n \) is the AWGN with zero mean and \( \sigma^2 \) as variance for both real and imaginary components and \( H_n \) denotes the channel frequency response of the multipath channel. In the sequel, we assume that \( R_n \) are independent and the response of the channel \( H_n \) is perfectly known during the recovery process. We note also that the transmitted data \( X_n^{(d)} \) belongs to \( E_1 \) that means the unrotated QAM constellation or to the rotated on \( \mathcal{E}_2 \) where we presume that \( Pr(X_n^{(d)} \in \mathcal{Q}) = \frac{1}{2} \). Our main objective now consists in detecting correctly the selected phase sequence \( \hat{d} \), based on the two following hypotheses:

\[
\mathcal{H}_0 : \quad X_n^{(d)} \in \mathcal{Q} \quad \Rightarrow \quad \phi_n^{(d)} \in \mathcal{E}_1 \quad \Rightarrow \quad C_n^{(d)} = 0
\]

\[
\mathcal{H}_1 : \quad X_n^{(d)} \in \mathcal{Q}' \quad \Rightarrow \quad \phi_n^{(d)} \in \mathcal{E}_2 \quad \Rightarrow \quad C_n^{(d)} = 1 \quad (4)
\]

In the next paragraph, we explain a hard decision process to efficiently detect the index \( d \).

A. Hard Decision Process

In the receiver side, we start by defining vectors code as in (2). We propose here a decision technique based on the MAP decoding algorithm that computes the Likelihood Ratio (LR) corresponding to each hypothesis \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \) based on the received symbol \( R_n = [R_0, \ldots, R_N] \) (see (3)):

\[
LR(n) = \frac{Pr(\mathcal{H}_0|R_n)}{Pr(\mathcal{H}_1|R_n)}, \quad (5)
\]

where \( Pr(\mathcal{H}_0|R_n) \) is the A Posteriori Probability (APP) of the hypothesis \( \mathcal{H}_i, \ i = 0, 1 \) given the received data \( R_n \). Assuming that \( Pr(\mathcal{H}_0) = Pr(\mathcal{H}_1) = 1/2 \), we deduce from the Bayes’ theorem that

\[
LR(n) = \frac{Pr(R_n|\mathcal{H}_0)}{Pr(R_n|\mathcal{H}_1)}, \quad n = 0, \ldots, N - 1. \quad (6)
\]

where \( Pr(R_n|\mathcal{H}_0) = Pr(R_n|X_n^{(d)} \in \mathcal{Q}) = \frac{Pr(R_n|X_n^{(d)} \in \mathcal{Q})}{Pr(X_n^{(d)} \in \mathcal{Q})} = M Pr(R_n, X_n^{(d)} \in \mathcal{Q}) \) and \( Pr(R_n|\mathcal{H}_1) = M Pr(R_n, X_n^{(d)} \in \mathcal{Q}' \). It results that

\[
LR(n) = \frac{\sum_{k=0}^{M-1} e^{-\frac{|R_n - H_n q_k|^2}{2\sigma^2}}}{\sum_{l=0}^{M-1} e^{-\frac{|R_n - H_n q_l|^2}{2\sigma^2}}}, \quad n = 0, \ldots, N - 1. \quad (7)
\]

As it easier to manipulate \( \log \) when our function depends on exponential, this leads to define the \( LLR \) expression as:

\[
LLR(n) = \log \left\{ \frac{\sum_{k=0}^{M-1} e^{-\frac{|R_n - H_n q_k|^2}{2\sigma^2}}}{\sum_{l=0}^{M-1} e^{-\frac{|R_n - H_n q_l|^2}{2\sigma^2}}} \right\}, \quad n = 0, \ldots, N - 1. \quad (8)
\]

However, the involvement of the sums in \( LLR(n) \) can be approximated by the maximum on all possible realizations of \( q_k \) and \( q_l \) [13]. It results that the Max-Log-MAP leads to the following \( LLR \) expression:

\[
LLR(n) = \min_k |R_n - H_n q_k|^2 - \min_k |R_n - H_n q_k|^2. \quad (9)
\]

The \( LLR \) as it is expressed in (9) allows us to estimate the code vector \( C^{(d)} \) (see Section V-A) in the following way:

\[
\hat{C}_n^{(d)} = \begin{cases} 
0 & \text{if } LLR > 1, \ n = 0, \ldots, N - 1. \\
1 & \text{else}
\end{cases} \quad (10)
\]

Therefore, the index of selected phase sequence is deduced as follows:

\[
\hat{d} = \arg \min_{0 \leq d \leq D - 1} |C^{(d)} - \hat{C}_n^{(d)}| \quad (11)
\]

Indeed, the hard decision criteria (given by (9)) ensures good decision performance when the absolute value of the \( LLR \) is
greater than a certain threshold but when it is close to zero, that means for low Signal to Noise Ratio (SNR), making a hard decision on $H_0$ or $H_1$ could wrongly estimate the code vector and therefore introduce a major decision error. In the following section, we propose a powerful decision process that overcomes this issue and that we denote ‘Soft decision’.

B. Soft Decision processes

To resolve the issue of very low $\|LLR\|$ that was be evoked in the last paragraph, we adopt a new decision in which the estimation of vector code $C(d)$ depends on the whole received sequence $R$ (see (3)) where the index $d$ is estimated according to the following expression, where $C_n(d)$ is the complementary of $C_n$. 

$$
\hat{d} = \max_{0 \leq d \leq D-1} Pr(R|C^{(d)}) = \max_{0 \leq d \leq D-1} \prod_{n=0}^{N-1} \frac{Pr(R_n|C_n^{(d)})}{Pr(R_n|C_n^{(d)})}
$$

In order to calculate $\frac{Pr(R_n|C_n^{(d)})}{Pr(R_n|C_n^{(d)})}$, we distinguish the two following cases:

1) If $C_n^{(d)} = 0$, then $\phi_n^{(d)} \in E_1$ and $Pr(R_n|C_n^{(d)}) = Pr(R_n|H_0)$ and similarly $Pr(R_n|C_n^{(d)}) = Pr(R_n|H_1)$

2) If $C_n^{(d)} = 1$, then $\phi_n^{(d)} \in E_2$ and $Pr(R_n|C_n^{(d)}) = Pr(R_n|H_1)$ and similarly $Pr(R_n|C_n^{(d)}) = Pr(R_n|H_0)$

Let $\Lambda = [\Lambda_0, \ldots, \Lambda_{N-1}]^T \in \mathbb{R}^N$, it results that

$$
\log \left\{ \frac{Pr(R_n|C_n^{(d)})}{Pr(R_n|C_n^{(d)})} \right\} = \begin{cases} 
\log \left\{ \frac{Pr(R_n|H_0)}{Pr(R_n|H_1)} \right\} = \Lambda_n & \text{if } C_n^{(d)} = 0 \\
\log \left\{ \frac{Pr(R_n|H_1)}{Pr(R_n|H_0)} \right\} = -\Lambda_n & \text{if } C_n^{(d)} = 1 
\end{cases}
$$

We deduce that the estimated phase index $\hat{d}$ in expression (12) can be determined as follows:

$$
\hat{d} = \arg \max_{0 \leq d \leq D-1} \sum_{n=0}^{N-1} \Lambda_n \xi_n^{(d)} = \arg \max_{0 \leq d \leq D-1} \left\{ \xi_n^{(d)}, \Lambda \right\}.
$$

where $\xi_n^{(d)} = \begin{cases} 
1 & \text{if } C_n^{(d)} = 0 \\
-1 & \text{if } C_n^{(d)} = 1, n = 0, \ldots, N-1.
\end{cases}$

VII. Simulation results

The performances of our proposed contribution are evaluated in terms of complementary cumulative distribution function (CCDF) of the PAPR, the side information error rate (SIER) that indicates the percentage of failure detection and the bit error rate (BER) in both AWGN and Rayleigh fading channel. We assume that the used number of sub-carriers are $N = 128$ or 256 and a QAM modulation is performed for the OFDM symbols. The simulation were investigated for both the transmitter and the receiver sides. For the transmitter side, we illustrate the performance in term of CCDF. In the receiver side, we illustrate the efficiency of the proposed recovery techniques in term of SIER and BER where we take benefits from the embedded signaling (that was be detailed in Section VI) that leads two powerful decision techniques: the Hard and Soft decision techniques.

A. Transmitter side

The Fig.3 presents the CCDF of the PAPR for 256 sub-carriers in the case of a 4 QAM modulation. We deduce that our proposed method leads to considerable improvement in the PAPR level reduction compared to classical OFDM systems. This improvement increases exponentially when $D$ increases.

B. Receiver side

In Fig.4, we present the SIER curves for different QAM constellation. We define the SIER as the probability that the detected phase sequence at the receiver does not match the phase sequence used in the transmitter side. We conclude that both the hard decision and soft decisions processes, lead to a good system performance. Furthermore, the soft decision technique leads to a significative gain compared to the hard decision 5dB in the case of 16 QAM.

In order to evaluate the performances of the Blind-SLM method in terms of BER we compare two cases: the situation where we perform a soft decision process and the one where the phase sequence is perfectly known (SI) at the receiver side (ordinary SLM). We consider a complex AWGN channel in Fig. 5 and rayleigh channel with length $L = 4$ in Fig.6. These two figure show that the curves corresponding to the blind SLM technique and those of the ordinary SLM in the case of
a perfect SI knowledge are approximately subplotted and this proves well the efficiency of our method and confirms well our claims. Finally, we compare in Fig.7 the obtained BER when we perform a soft decision process to the BER that is obtained through its theoretical expression in an OFDM system (the BER expression is elaborated in [14]). The results are spectacular: the obtained BER curves for both situations approximately match, which means that our proposed technique achieves BER performances that are near the theoretical ones.

![Fig. 5. Performance BER comparison: soft decision and SI in the case of AWGN channel, $N = 128$ and $D = 5$](image1)

![Fig. 6. Performance BER comparison: soft decision and SI in the case of rayleigh channel, $N = 128$, $L = 4$ and $D = 5$](image2)

![Fig. 7. Performance BER comparison: Theory expression and soft decision in the case of AWGN channel, $N = 128$ and $D = 5$](image3)

**VIII. Conclusion**

In this paper, we investigated a novel, optimized and efficient PAPR reduction technique, the Blind-SLM, that jointly exploits a set of rotated and unrotated QAM constellation of the OFDM signal in order to omit the need to send a side information to the OFDM receiver. Our proposed method represents an embedded signaling where no rate loss and no increase of the transmitted power could occur. Furthermore, the signal recover is guaranteed through this embedded signalling. Soft and hard decision processes are also proposed to quasiperfectly reconstruct the OFDM transmitted signal by using the Max-Log-MAP decoding algorithm. Our performed simulations lead to the following conclusions: the rate of estimation errors that are obtained through our proposed technique is very low, our proposed soft and hard decision schemes efficiently reconstruct the OFDM signal and our proposed method achieves BER performances that are very close to the theoretical ones. A perspective of our research work will be to extend our proposed technique to a MIMO-OFDM system framework and to evaluate its performance in this context.

**References**


