Letter

Signal Processing

Robustness of the decorrelating discrete time Rake receiver (DDRR) in the presence of dynamic propagation conditions

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SUMMARY

Recently, an optimum structure of the discrete time Rake receiver according to the maximum a posteriori criterion has been proposed. The obtained receiver, known as the decorrelating discrete time Rake receiver (DDRR), uses estimates of the channel statistics in order to compute its decision variable. In this letter, we study the DDRR robustness in the presence of dynamic statistics due to birth-death and moving paths.

1. INTRODUCTION

The Rake receiver is the conventional matched filter in direct sequence spread spectrum systems. It consists in combining the output of correlators locked on the most significant paths. We will call this implementation continuous time Rake receiver (CRR) since it is based on an estimation of path delays. These estimates are obtained from acquisition and tracking modules. One of the main drawbacks of the CRR comes from the unrealistic assumption of multipath propagation channels with discernable paths. A second drawback results from the limited time resolution of the acquisition system and the incapacity to distinguish paths separated by less than the chip period [1]. A final drawback comes from the inability of the delay locked loop to follow paths separated by less than the chip period [2].

To circumvent the problem of path delays estimation, a discrete time implementation of the Rake receiver (DRR) has been proposed [1, 3]. The DRR is a matched filter to an estimate of the sampled version of the global channel resulting from the convolution of the impulse responses of the shaping filter and the multipath channel. While avoiding the need for complex acquisition and tracking modules, this receiver faces several drawbacks arising from its extreme sensitivity to discrete time channel estimation quality [1, 3]. In fact, the global channel is estimated over an arbitrary delay window so that many estimated channel samples are due only to noise. Besides, the contribution of the side lobes of the global channel is very sensitive to noise.

In order to improve the DRR performance, an optimum structure of this receiver, according to the maximum a posteriori (MAP) criterion, has been proposed in Reference [4]. The obtained receiver, called the decorrelating DRR (DDRR), uses estimates of the discrete time global channel covariance matrix in the generation of a decorrelated version of the discrete time channel by means of the Karhunen-Loève (KL) orthogonal expansion. DDRR performance analysis in the presence of estimated channel

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statistics has been evaluated in Reference [4] for a diffuse multipath channel. In this letter, we evaluate its robustness in the presence of dynamic propagation conditions yielding a modification of the statistics due to birth-death and moving paths. These statistics are estimated using an exponential averaging window.

The letter is organised as follows. The next section gives both correlated and uncorrelated models of the discrete-time despreading filter output of the DRR. Section 3 describes both the conventional and decorrelating discrete time Rake receivers. Section 4 deals with noise and channel statistics estimation. Section 5 gives some simulation results. A discussion about the complexities of the CRR, the DRR and the DDRR is presented in Section 6. Finally, section 7 draws some conclusions.

2. DISCRETE-TIME DESPREADING FILTERS

2.1. Correlated despreading filter outputs

The DRR is composed of \( L = L_+ + L_+ + 1 \) despreading filters, one for each discrete time path \( f_i = f(\frac{IT_c}{2}) \) where \( T_c \) is the chip period and \( f(t) \) is the global channel. The lower and upper bounds \( L_+ \) and \( L_- \) are chosen so that in average most of the energy of the discrete global channel is concentrated between these bounds [4]. For large values of the spreading factor, the spreading sequence has good autocorrelation properties and the despreading filters’ outputs for symbol \( s_k \) can be written as

\[
z_k = f s_k + b_k
\]

where \( z_k = (z_{k,-L}, \ldots, z_{k,L})^T \), \( z_{k,l} \) is the despreading filter output corresponding to delay \( IT_c/2 \) and symbol \( s_k \), \( f = (f_{L-1}, \ldots, f_0)^T \), \( b_k = (b_{k,-L}, \ldots, b_{k,L})^T \) is a zero mean complex Gaussian noise with covariance matrix \( \Sigma^2 I_L \), \( \Sigma^2 = 2N_0/T_c \), \( N_0 \) is the channel noise PSD and \( T \) is the transpose operator.

2.2. Uncorrelated despreading filter outputs

An uncorrelated version of the despreading filter outputs can be obtained by means of the KL expansion of \( f \). More precisely, we have

\[
y_k = U^T z_k = e s_k + v_k
\]

where \( U = (u_0, \ldots, u_{L-1}) \), \( \{u_l\}_{l=0}^{L-1} \) are the normalised eigenvectors of the Hermitian covariance matrix \( \mathbf{F} = E(\mathbf{f f}^H) \), \( \dagger \) denotes the Hermitian transpose operator, \( e = (e_0, \ldots, e_{L-1})^T \), \( e_l = u_l^T f \) and \( v_k = U^T b_k \) is a zero mean complex Gaussian noise with the same statistical properties as \( b_k \).

3. DISCRETE-TIME RAKE RECEIVER

3.1. Conventional DRR

The conventional DRR (CDRR) soft output for symbol \( s_k \) is given by [1, 2]

\[
\Lambda_k = \frac{\hat{f}^T z_k}{\Sigma^2} \tag{3}
\]

where \( \hat{f} \) is an estimate of \( f \) obtained from the set of pilot symbols by

\[
\hat{f} = \frac{\sum_{k \in A_p} z_k s_k^T}{N_p E_p} \tag{4}
\]

where \( A_p \) is the set of pilot symbol indices in the current time slot, \( N_p \) is the number of pilot symbols and \( E_p \) is the transmitted energy per pilot symbol.

Therefore, the CDRR is based on a maximum ratio combining (MRC) of the discrete time despreading filter outputs. This rule is optimum when the channel is known. However, this rule is no longer optimum when the channel is estimated and its components are correlated. The optimum implementation of the receiver according to the MAP criterion is detailed in the next subsection.

3.2. Decorrelating DRR

For Rayleigh fading channels, the optimum implementation of the DRR according to the MAP criterion is given by [4]

\[
\Pi_k = \frac{\hat{e}^T W y_k}{\Sigma^2} \tag{5}
\]

where

\[
\hat{e} = \frac{\sum_{k \in A_p} y_k s_k^T}{N_p E_p} \tag{6}
\]

\( W \) is a diagonal matrix with \( l \)-th entry

\[
w_l = \frac{1}{1 + \sigma^2 \left( \frac{e_l^2}{\Sigma^2} + \Gamma_l \right)} \tag{7}
\]

\( \sigma^2 = \Sigma^2/(N_p E_p) \) and \( \{\Gamma_l\}_{l=0}^{L-1} \) are the eigenvalues of \( \mathbf{F} \).
If we examine the expression of the optimal decision variable (5), we learn two things. First of all if the channel is perfectly estimated by increasing towards infinity, the number \( N_p \) and/or the energy \( E_p \) of the pilot symbols, all weighting coefficients \( w_l \) converge to unity. Hence, we verify that the CDRR and DDRR are equivalent when the channel is perfectly estimated. Second, if the components of \( f \) are decorrelated, all weighting coefficients \( w_l \) are equal so that the CDRR and the DDRR become equivalent.

Since the global channel results from the convolution of the impulse responses of the shaping filter and the multipath channel, the components of \( f \) are always correlated.

### 4. NOISE AND CHANNEL STATISTICS ESTIMATION

In a practical implementation of the DDRR, the noise PSD \( N_0 \) and the discrete time channel covariance matrix \( F \) must be estimated. In order to reduce the effect of noise on the quality of the statistics estimates, successive time slot estimates must be averaged. In this letter, we propose to use an exponential averaging window, which allows a simple reduced-noise tracking of these statistics. According to this technique, the estimate of the discrete time channel covariance matrix at time slot \( m \) is given by

\[
\hat{F}^m = (1 - \varepsilon) \hat{F}^{m-1} + \varepsilon \left[ \hat{F}^m \hat{F}^m \right] - \frac{2 \hat{N}_0^m}{T_c N_p E_p} I_c
\]

where \( 0 < \varepsilon \leq 1 \) is the forgetting factor, \( \hat{F}^m \) is the discrete-time channel estimate at time slot \( m \) and the noise PSD estimate at time slot \( m \) is expressed as

\[
\hat{N}_0^m = (1 - \varepsilon) \hat{N}_0^{m-1} + \varepsilon \frac{T_c}{2(N_p - 1) L} \sum_{k \in A_p} \tilde{z}_k^m \tilde{z}_k^m - N_p E_p \hat{F}^m \hat{F}^m
\]

where \( \tilde{z}_k^m \) is the despreading filter outputs for symbol \( \tilde{s}_k \) at time slot \( m \).

Note that the terms multiplied by \( \varepsilon \) in Equations (8) and (9) are respectively the ML estimates of \( F \) and \( N_0 \) based on the received signal over the \( m \)-th time slot [4]. The optimal choice of \( \varepsilon \) depends on the rapidity of the channel statistics variations. If the statistics vary slowly, they can be tracked using a small value of \( \varepsilon \) which allows a large averaging window. If the statistics vary rapidly, a large value of \( \varepsilon \) must be used in order to track the statistics evolution.

### 5. SIMULATION RESULTS

In this section, the optimal choice of \( \varepsilon \) is investigated in the presence of dynamic propagation conditions. The simulated channel is Rayleigh fading with classical Doppler power spectrum and unit average energy. Except in Figure 3, the maximum Doppler frequency is set to 18.5 Hz corresponding to a mobile speed of \( v = 10 \text{ km/h} \) and a carrier frequency of 2 GHz. According to the UMTS specifications [5], the chip period is fixed to 1/3.84 \( \mu \text{s} \) and the transmitted symbols are dropped from a QPSK constellation. The spreading factor is equal to 256 and \( N_p = 4 \) pilot symbols having the same energy as the data symbols are used.

The conventional acquisition module [6] and the coherent delay locked loop (DLL) [7] were used to estimate path delays for the CRR. The acquisition module was applied during the first 500 slots. Path delays were searched with a resolution equal to the chip period in a 10 \( \mu \text{s} \) delay window. A threshold of \(-10 \text{ dB}\) relative to the most significant peak is used for secondary paths selection [6]. The obtained estimates are then refined by a coherent DLL using a first order loop filter [8] of forgetting coefficient 0.7. Note that a despreading filter is used by the acquisition module to detect new appearing paths in a 10 \( \mu \text{s} \) delay window. At each time slot, the despreading filter tests a given

![Figure 1. Robustness of the DDRR in the presence of birth-death propagation conditions.](image-url)
path delay. In order to reduce the effect of noise and interference, the obtained correlations are then averaged three times.

5.1 Birth-death propagation conditions

The birth-death propagation conditions consist in two Rayleigh fading paths with equal average powers [9]. The two paths alternate between ‘birth’ and ‘death’ every 191 ms. Relative path delays are randomly selected from the group \([-5,-4,-3,-2,-1,0,1,2,3,4,5]\) ms.

Figure 1 shows the DDRR robustness with respect to \(\varepsilon\) for \(E_s/N_0 = 10\) dB and 16 dB, where \(E_s\) is the transmitted energy per data symbol. We notice that DDRR performance degrades for \(\varepsilon \leq 10^{-6}\) because the evolution of the statistics is not perfectly tracked. The optimal region for \(\varepsilon\) is \(10^{-5}-10^{-3}\), which allows better performance than the CDRR due to a good compromise between noise averaging and statistics tracking. The DDRR performance degrades for \(\varepsilon > 5 \times 10^{-3}\) since the statistics are not sufficiently averaged. Finally, note that \(\mathbf{F}\) has a single non-null eigenvalue for \(\varepsilon = 1\) so that the DDRR and the CDRR become equivalent.

In Figure 2, simulation results of the DDRR with estimated channel statistics for \(\varepsilon = 0.001\) and \(\varepsilon = 0.007\) are compared to those of the DDRR with perfect channel statistics (PCS), the CDRR, the CRR with known path delays, the CRR with estimated path delays and the matched filter bound (MFB) [1] in the presence of birth-death propagation conditions. We notice that the DDRR offers 0.9 dB gain compared to the CDRR for \(\varepsilon = 0.001\). However, DDRR performance for \(\varepsilon = 0.007\) is worse than that of the CDRR because the estimated statistics are not sufficiently averaged. DDRR performance for \(\varepsilon = 0.001\) are 1 dB worse (resp. 2.5 dB better) than that of the CRR with known (resp. estimated) path delays for a bit error rate (BER) equal to \(10^{-2}\). Performance degradation of the CRR with estimated path delays is due to the fact that new emerging paths are not immediately detected by the acquisition module.

5.2. Moving propagation conditions

The moving propagation channel model has two Rayleigh fading paths with equal average powers. The first path has a static delay and the other path has a moving delay. The delay difference between the two paths is given by [9]

\[
\Delta \tau = B + \frac{A}{2} \left[1 + \sin(\Delta \omega t)\right]
\]

where \(A = 5\) ms, \(B = 1\) ms and \(\Delta \omega = 0.04\) Hz.
Figure 3 shows the robustness of the DDRR in the presence of moving propagation conditions for ($\Delta w = 0.04$ Hz, $v = 10$ km/h) and ($\Delta w = 0.2$ Hz, $v = 50$ km/h). The value of $E_s/N_0$ was set to 10 dB. The optimal region for $\varepsilon$ is $10^{-5}$–5 $10^{-4}$ (resp. $10^{-5} – 10^{-3}$) for $\Delta w = 0.2$ Hz (resp. $\Delta w = 0.04$ Hz). Therefore, the value of $\varepsilon$ should be adapted using an estimate of the mobile speed. Performance degradation of both receivers for $v = 50$ km/h compared to $v = 10$ km/h is due to the fact that the channel changes during the slot duration so that channel estimates obtained from pilot symbols are no longer valid for data symbols.

Figure 4 compares the BER performance of the different receivers for $A = 5$ $\mu$s, $B = 1$ $\mu$s and $\Delta w = 0.04$ Hz. We notice that the CRR with estimated path delays offers very close performance to that of the CRR with known path delays. This is due to the fact that path delays are far apart so that the coherent DLL can track them.

Figure 5 compares the performance of the different receivers in a close path environment characterised by $A = B = T_c/2$ and $\Delta w = 0.04$ Hz. We notice that the DRR for $\varepsilon = 0.001$ offers 5 dB gain compared to the CRR with estimated path delays for a BER equal to $10^{-2}$. Performance degradation of the CRR with estimated path delays is due to the fact that the coherent DLL confuses the two paths when their delays are separated by less than the chip period.

6. COMPLEXITY CONSIDERATIONS

The complexity of the CRR originates essentially from the despreading filters needed in the maximum ratio combining of the most significant paths, in the detection of emerging paths (acquisition) or the tracking of the continuously time varying delays of paths. In general, three despreading filters are needed for tracking and MRC combining for each path and at least one despreading filter is needed for acquisition. For the CDRR, the complexity comes essentially from the despreading filters which are systematically used to span and combine the half-chip period regularly spaced paths of the discrete time global channel. A comparison of the complexities of the CRR and CDRR is not immediate. On the one hand, the CDRR requires only one despreading filter for each combined discrete path while the CRR requires three. On the other hand, the number of systematically combined paths in the CDRR is always larger than that of the CRR because of shaping filtering at the transmitter side. In summary, the complexity of both CRR and CDRR is tributary of the multipath
intensity profile of the propagation channel. As a consequence, there is no immediate and unique answer to the comparison of the complexities of both receivers.

Obviously, the extra complexity of the DDRR with respect to the CDRR is the price to pay for the enhancement of the receiver performance. It is mainly due to the computation of the uncorrelated version of the despreading filter outputs, which should be carried for each received symbol. It is also due, but to a lesser extend, to the regular processing of the KL orthogonal expansion and the estimation of noise and channel statistics. Complexity reduction techniques for the DDRR are provided in Reference [4].

7. CONCLUSION

In this letter, the DDRR robustness in the presence of dynamic propagation conditions is investigated. By setting the value of the forgetting factor of the exponential averaging window, $\varepsilon$, between $10^{-5}$ and $10^{-4}$, we have verified that the DDRR is able to offer better performance than the CDRR even in the presence of dynamic statistics.

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