Comparison of Hybrid Localization Schemes using RSSI, TOA, and TDOA

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Abstract—This paper presents a simulation study of non-hybrid and hybrid localization techniques using RSSI, TOA, and TDOA location dependent parameters. Maximum likelihood and weighted least squares are considered and developed for both non-hybrid and hybrid cases. Monte-Carlo simulations using realistic radio parameters extracted from an ultra wide band measurement campaign are carried out in order to assess the performances of different techniques and to show the importance of hybrid data fusion for localization.

I. INTRODUCTION

Although GPS and other GNSS systems have offered up to now localization and navigation services, they can perform localization only if the GPS receiver is in visibility with at least four satellites. In indoor and dense urban environments, this condition is not usually guaranteed. Even if in some countries GPS repeaters are allowed to overcome this problem, the cost of such an approach is still very high and the used repeaters may cause interferences between GPS receivers. In addition to this drawback, the positioning precision offered by GNSS systems is from few meters to some tens of meters. Such a positioning error may not be adequate for many applications and services which request centimetric precision in order to be executed.

A device’s position is usually estimated by monitoring a location dependent parameter (LDP) such as received signal strength indicator (RSSI), time of arrival (TOA), time difference of arrival (TDOA), etc, from another device whose location is known. The localization is done by computing distances from these LDPs and then applying estimation techniques to find the device’s position. Different techniques of estimation are defined such as least-squares, maximum likelihood, and convex optimization. The localization accuracy is mainly factor of the LDP measurement nature and accuracy, the wireless standard, and the estimation technique itself. Since each location based service, before being proposed to users, require a minimal positioning accuracy, the localization system should choose the best standards, the best LDPs, and the best estimators, able to perform accurately the requested service.

Wireless communications are, by any measure, the fastest growing segment of the communications industry. Today’s wireless applications like cellular phone services or television broadcast are a part of the day to day life of many people. Wireless communication has evolved immensely from the time it was first implemented. The ease of setting up a wireless network, tetherless communication, and low cost of deployment are some of the key reasons for its popularity. Also, the reliability of wireless communication has improved significantly and is reflected in its application to a wide variety of civilian and military fields. The today’s landscape of wireless communications is mainly characterized by the coexistence of different technologies (e.g. Bluetooth, WiFi, Zigbee, UWB, Cellular, WiMax, etc). The widespread implementation of these heterogeneous wireless networks make wireless localization a service that is available “anytime” and “anywhere” [1], [2].

The expansion, the heterogeneity, and the coexistence of wireless networks are the motivations make it possible for localization systems to implement novel techniques of localization. These techniques use more than one LDP type and we call them “Hybrid Localization Techniques”. This paper is a contribution to the study of these hybrid localization techniques using the maximum likelihood estimator (ML) and the weighted least-squares (WLS) technique. Therefore, we organize the paper as follows:

Section II presents the assumed generic heterogeneous scenario. Sections III and IV present the application of respectively the WLS and the ML techniques on the fusion of RSSI, TOA, and TDOA. The mathematical formulation of these estimators are developed for both non-hybrid and hybrid localization techniques. Section V uses the generic scenario and statistical models extracted from a UWB measurement campaign to evaluate the proposed hybrid techniques and to show the effect of such hybrid techniques on the positioning accuracy. Finally, our concluding remarks are given in section VI.

II. ASSUMED SCENARIO

The assumed scenario here is a situation where the targeted mobile is connected to different anchors from which it is able to get different LDPs. Let \( K \) be the total number of all anchors implied in the scenario. Without any loss of generality, assume that the targeted MS can get:

- \( p \) RSSIs \((P_k)\) from anchors with indexes \( k \in (1,\ldots,p)\),
- \( q - p \) TOAs \((\tau_k)\) from anchors with indexes \( k \in (p+1,\ldots,q)\),
- \( K - q - 1 \) TDOAs \((\tau_{k(q+1)})\) from anchors with indexes \( k \in (q+2,\ldots,K)\) obtained with reference to the \((q+1)^{th}\) anchor.

Fig. 1 depicts an example of hybrid scenario where \( p = 4 \), \( q = 8 \), and \( K = 12 \). Here, we suppose that the MS get TDOAs
from base stations, RSSIs from access points, and TOAs from some ranging-capable mobiles with whom it cooperates. This scenario will be used for simulations later in this paper. Let \( \mathbf{X} = (x, y) \) and \( \hat{\mathbf{X}} = (\hat{x}, \hat{y}) \) be respectively the actual and estimated MS position and \( \mathbf{X}_k = (x_k, y_k) \) the position of the \( k^{th} \) anchor.

For measurements, we consider Gaussian models. For both TOA and TDOA, we assume that the measurement is centered on the actual value with a standard deviation \( \sigma_k \) and \( \sigma_{k(q+1)} \) for respectively the \( k^{th} \) TOA and the \( k^{th} \) TDOA:

\[
c_{\mathbf{r}} \sim \mathcal{N}(d_k, \sigma_k^2) \tag{1}
\]
\[
c_{\mathbf{r}} \sim \mathcal{N}(d_{k(q+1)}, \sigma_{k(q+1)}^2) \tag{2}
\]

where \( d_k = \| \mathbf{X} - \mathbf{X}_k \|_2 \) is the actual range between the targeted MS and the \( k^{th} \) anchor, \( d_{k(q+1)} = \| \mathbf{X} - \mathbf{X}_{k(q+1)} \|_2 \) is the actual difference of ranges between the targeted MS and the \( k^{th} \) anchor with reference to the \( (q+1)^{th} \) anchor, and \( c \) is the speed of light. RSSI is modeled using log-normal shadowing model. This model is given in (3) and represents the power \( P_k \), received by the \( k^{th} \) anchor, as a random variable centered on the mean received power with a standard deviation of shadowing \( \sigma_{sk,k} \). \( n_p \) is the propagation exponent, \( d_{0} \) is a reference distance taken equal to 1 meter, and \( P_0 \) is the received power at \( d_{0} \).

\[
P_k \sim \mathcal{N}(P_0 - 10n_p \log_{10} \left( \frac{d_k}{d_{0}} \right), \sigma_{sk,k}^2) \tag{3}
\]

After performing RSSI based ranging [3], the obtained ranges are assumed to be Gaussian centered on the true values with a standard deviation equal to \( \sigma_{r,k} \) for the \( k^{th} \) RSSI:

\[
r_k \sim \mathcal{N}(d_k, \sigma_{r,k}^2) \tag{4}
\]

III. WEIGHTED LEAST-SQUARES FOR NON-HYBRID AND HYBRID LOCALIZATION TECHNIQUES

The use of WLS technique is quite different for RSSI, TOA, and TDOA. In the case of RSSI, the ranges are firstly estimated using RSSI based ranging techniques [3] and then WLS techniques are applied on these ranges. In the case of TOA, the WLS techniques are directly applied on ranges given by the TOAs multiplied by the speed of light \( c \). The case of TDOA is quite different because the WLS is applied on differences of ranges not on ranges. The WLS estimate is given by:

\[
\hat{\mathbf{X}} = (\mathbf{A}^T \mathbf{C}^{-1} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{C}^{-1} \mathbf{b} \tag{5}
\]

where \( \mathbf{A}, \mathbf{b}, \) and \( \mathbf{C} \) are matrices defined differently for each localization scheme. For each of considered LDPs, the correspondent matrices are given in Table I for 2D scenario where \( l_k \) is given by \( l_k = x_k^2 + y_k^2, k \in \{1, ..., K\} \) and \( c = 3.10^8 m/s \) is the speed of light. Notice that in the TDOA case, the vector \( \mathbf{X} \) is of length \( m+1 \) and estimates, in addition to the coordinates of the targeted MS, the range between the MS and the reference anchor.

### Table I

<table>
<thead>
<tr>
<th>LDP</th>
<th>Matrices</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RSSI</strong></td>
<td>( \mathbf{A}_{RRSSI} = \begin{pmatrix} x_2 - x_1 &amp; y_2 - y_1 \ x_3 - x_1 &amp; y_3 - y_1 \ \vdots &amp; \vdots \ x_n - x_1 &amp; y_n - y_1 \end{pmatrix} )</td>
</tr>
<tr>
<td><strong>TOA</strong></td>
<td>( \mathbf{A}<em>{TOA} = \begin{pmatrix} x</em>{p+2} - x_{p+1} &amp; y_{p+2} - y_{p+1} \ x_{p+3} - x_{p+1} &amp; y_{p+3} - y_{p+1} \end{pmatrix} )</td>
</tr>
<tr>
<td><strong>TDOA</strong></td>
<td>( \mathbf{A}<em>{TDOA} = \begin{pmatrix} x</em>{q+2} - x_{q+1} &amp; y_{q+2} - y_{q+1} &amp; cT_{(q+2)(q+1)} \ x_{q+3} - x_{q+1} &amp; y_{q+3} - y_{q+1} &amp; cT_{(q+3)(q+1)} \end{pmatrix} )</td>
</tr>
</tbody>
</table>

Applying WLS techniques to fuse different LDPs can be easily made since the used matrices (respectively \( \mathbf{A}, \mathbf{b}, \) and \( \mathbf{C} \)) for each LDP can be fused together and resulting in three new matrices which are denoted respectively \( \mathbf{A}_{Hybrid}, \mathbf{b}_{Hybrid}, \) and \( \mathbf{C}_{Hybrid} \). Using the definition of assumed hybrid scenario in section II, we construct \( \mathbf{A}_{Hybrid}, \mathbf{b}_{Hybrid}, \) and \( \mathbf{C}_{Hybrid} \) respectively as follows [4]:
When different measurements come from different receivers, the assumption of independence between these measurements can be made. For the hybrid scenario where different LDP are collected from different receivers, we can define the hybrid likelihood function as follows:

\[ \nabla f_{\text{hybrid}} = \nabla f_{\text{RSSI}} + \nabla f_{\text{TOA}} + \nabla f_{\text{TDOA}} \] \hspace{1cm} (14)

where \( \nabla f_{\text{RSSI}}, \nabla f_{\text{TOA}}, \) and \( \nabla f_{\text{TDOA}} \) are defined respectively in (10).

V. SIMULATION AND DISCUSSIONS

A. Simulations parameters

Simulations are carried using the scenario defined in section II. The statistical models used in simulations are extracted from an UWB measurement campaign [7]. A summary of these models is given in Table II:

<table>
<thead>
<tr>
<th></th>
<th>RSSI</th>
<th>TOA</th>
<th>TDOA</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_0 )</td>
<td>-36.03 dBm</td>
<td>( \sigma_s = 2.38 )</td>
<td>( \sigma_{sh} = 3.08 dBm )</td>
</tr>
<tr>
<td>( \tau_0 )</td>
<td>1 ns</td>
<td>( \sigma_{k+1} = 1.88 m )</td>
<td></td>
</tr>
</tbody>
</table>

The results presented below are obtained with 1000 randomly drawn iterations of the MS position in the \( L \)-by-\( L \) squared area defined by the generic scenario in Fig.1. \( L \) is taken equal to 20 meters.

B. ML versus WLS performances for non-hybrid and hybrid localization techniques

The performances of these two estimators (i.e. ML and WLS) applied on different non-hybrid and hybrid localization schemes are plotted respectively in Fig.2 and Fig.3. In these figures, for each scheme the CDFs of absolute positioning error using ML and WLS techniques are plotted. Table III summarizes these performances by giving the values of positioning error at respectively 67\% and 95\%. These two figures and the table obviously reveals that the ML technique outperforms the WLS technique. This is verified for all non-hybrid and hybrid localization schemes. The poor performances of the WLS, compared to the ML technique, are mainly due to the linearization of the localization problem. When doing this linearization, some informations are lost which affects the estimation result. In contrast, these informations are still considered by the ML non-linearized objective function. Nevertheless, the ML technique may suffer from some singularities especially in the case of TDOA (see Fig. 2-(c) where the CDFs tend toward infinity because of the presence of singularities which result in large positioning errors). These singularities are due to the non convexity of the ML objective functions.

C. Importance of hybrid data fusion for localization

Since the ML gives the best performances among the studied estimators, we use it below in order to compare the different localization schemes and to study the effect of hybrid
Since it is interesting for operators to reduce complexity and resources consumption, the number of ranging attempts considered in the generic scenario (i.e. 4 RSSIs, 4 TOAs, and 3 TDOAs)

The fusion of RSSI, TOA, and TDOA.

This figure reveals the following points:

- Comparison between non-hybrid schemes shows that time-based LDPs give better accuracies than the power-based LDP (i.e. RSSI) which gives the worst positioning accuracy. Since localization with RSSI relies on path loss models which give an imperfect statistical representation of radio channel, the offered positioning accuracy cannot be very reliable. Moreover, the variation of shadowing is generally higher and makes the estimation of distance very inaccurate. In order to enhance the RSSI-based positioning accuracy, more sophisticated path loss models are needed. TOA and TDOA are generally measured with a high accuracy in UWB standard via ranging techniques.

- Adding TOA, TDOA, or both of them to RSSI drastically enhances the positioning accuracy. By contrast, adding RSSI to TOA, TDOA, or both results in minor enhancement of positioning accuracy. This is justified by the higher precision of time-based LDPs (especially in UWB networks) and the unreliability of RSSI measurements because of shadowing and radio propagation phenomena. Hence, the use of RSSIs is justified by one or more of the following facts:

  1. The number of TOAs or/and TDOAs is not sufficient to perform localization (i.e. less than 3 TOAs in 2D scenario);
  2. The precision of TOAs or/and TDOAs is not accurate;
  3. RSSIs are usually available without any additional cost.

- Fusing all available LDPs is obviously the most accurate localization scheme. This is in line with the estimation theory stating that more are available informations better is the estimation accuracy.

Since it is interesting for operators to reduce complexity and resources consumption, the number of ranging attempts

<table>
<thead>
<tr>
<th>LDP</th>
<th>Technique</th>
<th>90%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>RSSI</td>
<td>WLS</td>
<td>6.6m</td>
<td>14.2m</td>
</tr>
<tr>
<td></td>
<td>ML</td>
<td>4.15m</td>
<td>8.37m</td>
</tr>
<tr>
<td>TOA</td>
<td>WLS</td>
<td>1.59m</td>
<td>2.76m</td>
</tr>
<tr>
<td></td>
<td>ML</td>
<td>1.25m</td>
<td>2.11m</td>
</tr>
<tr>
<td>TDOA</td>
<td>WLS</td>
<td>6.40m</td>
<td>18.13m</td>
</tr>
<tr>
<td></td>
<td>ML</td>
<td>1.69m</td>
<td>3.60m</td>
</tr>
<tr>
<td>RSSI + TOA</td>
<td>WLS</td>
<td>1.88m</td>
<td>5.08m</td>
</tr>
<tr>
<td></td>
<td>ML</td>
<td>1.18m</td>
<td>1.98m</td>
</tr>
<tr>
<td>RSSI + TDOA</td>
<td>WLS</td>
<td>4.88m</td>
<td>10.52m</td>
</tr>
<tr>
<td></td>
<td>ML</td>
<td>1.42m</td>
<td>2.65m</td>
</tr>
<tr>
<td>TOA + TDOA</td>
<td>WLS</td>
<td>1.37m</td>
<td>2.42m</td>
</tr>
<tr>
<td></td>
<td>ML</td>
<td>1.00m</td>
<td>1.69m</td>
</tr>
<tr>
<td>RSSI + TOA + TDOA</td>
<td>WLS</td>
<td>1.75m</td>
<td>4.26m</td>
</tr>
<tr>
<td></td>
<td>ML</td>
<td>0.90m</td>
<td>1.55m</td>
</tr>
</tbody>
</table>
must be reduced because they consume much resources and cause overhead. Moreover, RSSIs are usually available without any additional costs. For these reasons, reaching the requested accuracy while using all available RSSIs and reducing the number of TOA and TDOA seems to be a very interesting opportunistic localization scenario. In order to show the effect of adding TOA (respectively TDOA) to RSSIs on positioning accuracy, let us consider the ML technique applied on the scheme (RSSI+TOA) (respectively (RSSI+TDOA)). Let us assume all four RSSIs available and gradually increase the number of additional TOAs (respectively TDOAs). Fig. 5 plots first the CDFs of absolute positioning error and second the evolution of average absolute positioning error over the area with respect to the number of added TOAs. Fig. 6 represents the same results in the TDOA case. These figures show a gradual enhancement provided by increasing the number of used TOA (respectively TDOA). Nevertheless, adding some LDPs may deteriorate the positioning accuracy (i.e. in the 4 RSSIs + 1 TOA case in Fig.5). This can be explained by the fact that some LDPs are very imprecise or that they come from a device misplaced with respect to other devices.

![Fig. 5. Effect of additional TOA on hybrid (RSSI+TOA) positioning accuracy using ML technique.](image)

![Fig. 6. Effect of additional TDOA on hybrid (RSSI+TDOA) positioning accuracy using ML technique.](image)

VI. CONCLUSION

This paper considered non-hybrid and hybrid localization techniques using RSSI, TOA, and TDOA. The WLS and the ML estimators are presented and compared for both of them. For both techniques, the we have presented the expressions of estimators for non-hybrid and hybrid localization schemes. Monte-Carlo simulations used a generic scenario and radio parameters extracted from an UWB measurement campaign. These simulations have shown that the ML outperforms the WLS technique. Simulations have revealed also that when TOAs and/or TDOAs have high accuracy, the use of RSSIs is either marginal or not necessary. Nevertheless, RSSIs are very important and may enhance positioning accuracy in cases where no sufficient number of TOAs or TDOAs is available or when their precisions are not accurate.

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REFERENCES