High Performance Quasi-Continous HOSM Controller for Sensorless IPMSM Based on Adaptive Interconnected Observer

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Abstract—This paper proposes a robust control strategy for interior permanent magnet synchronous motor (IPMSM) without rotor position sensor. The proposed strategy is based on quasi-continuous higher order sliding mode controller, which allows to attenuate the effects of the chattering, improve the robustness with respect to external disturbances and parameters uncertainties. Additionally, the maximum-torque per ampere (MTPA) strategy is applied to increase the efficiency of the controlled drive. Furthermore, an adaptive interconnected observer is used to estimate the speed, the position, the load torque and the stator resistance. The closed-loop system stability is studied and conditions are obtained to ensure the practical stability. Performances of the controller-observer are illustrated in a simulation study under parameter uncertainties and external disturbances.

NOMENCLATURE

HOSM High Order Sliding Mode
IPMSM Interior Permanent Magnet
$R_s$ stator resistance
$L_{d}, L_{q}$ dq-axis inductances
$\phi_f$ permanent-magnet flux linkage
$i_d, i_q$ stator currents
$u_d, u_q$ stator voltages
$v_d, v_\beta$ stator voltages
$\Omega$ rotor mechanical speed
$\theta$ rotor angular position $\theta = p\theta$
$\theta_e$ Rotor electrical position
$J$ moment of inertia
$f_v$ viscous friction coefficient
$p$ number of pole pairs
$T_l$ load torque.

I. INTRODUCTION

Interior Permanent Magnet Synchronous Motors (IPMSMs) are more and more used for drive applications. To control IPMSM, position and speed sensors are usually implement in the machine. However, the use of sensors introduce several drawbacks such as increased cost and reduced system reliability. Hence, elimination of these sensors is desirable. Several sensorless control schemes have been proposed in the recent literature to achieve sensorless operation. The back electromotive force is often used for the sensorless control of IPMSM [1]. But this method does not perform well at both standstill and low speed. Then, the extended electromotive force is used, see for instance [2]. In addition, adaptive observers, and Extended Kalman Filter [3] have been used to estimate both the speed and the position. With the Extended Kalman Filter, it is difficult to guarantee convergence of the estimated speed and position. A Quasi-continuous HOSM control is proposed in [4] to reduce the effects of the chattering phenomena, this controller combines a high order sliding mode control and the backstepping approach. This method is applied for sensorless surface-mount permanent magnet synchronous motor [5] by using sliding mode observer. This controller does not take into account the unmatched perturbations due to the parameter uncertainties. In the other hand, the major disadvantage of this observer (used in [5]) is the chattering phenomena due to discontinuous functions. This motivated us to use a new algorithm for this controller, for the case of the sensorless interior permanent magnet synchronous motor with account the unmatched perturbations. Moreover, by designing this adaptive interconnected observer that, additionally, estimates online the stator resistance.

This paper proposes a nonlinear sensorless speed control for IPMSM based on Quasi-continuous High Order Sliding Mode control and an adaptive interconnected observer. This controller ensures the finite time exact tracking of the desired output in spite of the presence of unmatched perturbations. The proposed observer is used to estimate the rotor speed, the rotor position, the load torque and the stator resistance on the complete trajectory with different speed range. Furthermore, the maximum-torque-per-ampere (MTPA) [6] strategy is applied to increase the efficiency of the control.

This paper is organized as follows: in section II, the nonlinear mathematical model of the IPMSM system is briefly introduced. In section III, the Quasi-continuous high order sliding mode MTPA controller is derived in order to improve the robustness under parametric uncertainties. Furthermore, in order to estimate the rotor position, the velocity, the rotor resistance and the load torque (required for the controller), an adaptive interconnected observer design is presented in section IV. In section V, the convergence of this observer is studied under parameter uncertainties. Next, the stability of the closed-loop system with the observer-controller scheme is proven in section VI. Simulation results are presented in section VII showing...
the performance of the proposed observer-control scheme on the framework of an industrial Benchmark [7]. Finally, some conclusions are given.

II. PROBLEM STATEMENT

In a synchronous rotating frame the mathematical model is described in [8]:

\[
\begin{align*}
\dot{x} &= f(x) + g(x)u \\
y &= h(x)
\end{align*}
\]

(1)

where \( x = [i_d, i_q, \Omega, \theta] \), \( u = [u_d, u_q] \), \( y = [h_1, h_2] = [i_d, i_q] \)

\[
f(x) = \begin{bmatrix}
\frac{p}{L_d} i_d + p L_q \Omega i_q \\
\frac{p}{L_q} i_q - p \frac{L_q}{L_d} \Omega i_d - p \frac{1}{L_d} \Phi_f \Omega \\
\frac{\Omega}{L_d} (L_d - L_q) i_d i_q - \frac{\Omega}{L_q} \Omega i_d - \frac{\Phi_f}{L_d} i_q - \frac{1}{L_d} T_i
\end{bmatrix},
\]

\[
g_1(x) = \begin{bmatrix}
\frac{1}{L_d} & 0 & 0 & 0
\end{bmatrix}, \quad g_2(x) = \begin{bmatrix}
0 & \frac{1}{L_q} & 0 & 0
\end{bmatrix}
\]

Control objective. To design a controller such that the rotor speed tracks a desired reference (\( \Omega^* \)) and the current \( i_d \) is forced to \( i_d^* \) (defined later) despite the parametric uncertainties and the load torque.

Observation objective. By using only the measurement of the currents and voltages, the observation objective is to reconstruct the speed, the position, the load torque and the stator resistance of the IPMSM.

III. MTPA QUASI-CONTINUOUS HIGHER ORDER SLIDING MODE CONTROL

Technical aspect of the proposed solution are given in the following of this section. Firstly, the MTPA strategy is considered in III-A. After preliminaries and the quasi-continuous sliding mode control algorithm are given in III-B, and this algorithm is applied to the case of the IPMSM in III-C.

A. MTPA Strategy

The electromagnetic torque of IPMSM can be expressed as follows:

\[
T_e = p [\Phi_f i_q + (L_d - L_q) i_d i_q],
\]

(2)

Many researchers have attempted to control the d-axis current by forcing it to zero. i.e., \( i_d^* = 0 \) [9]. Nevertheless, this approach does not efficiently utilize the electromagnetic torque of an IPMSM. The Maximum-Torque-Per-Ampere (MTPA) control method provides a maximum torque/current ratio, hence increasing the efficiency of the control ([6]). To ensure the full use of the reluctance torque and to operate the motor with optimum efficiency, \( i_d^* \) is determined based on MTPA control strategy below the rated speed. This is obtained by differentiating equation (2) with respect to \( i_d \), keeping the absolute value of stator current \( i_d \) constant at its maximum value \( I_m \) as in [10] with.

The relationship between the stator currents, \( i_d \) and \( i_q \), \( i_d = f(i_q) \) is:

\[
i_d^* = -\frac{\Phi_f}{2(L_d - L_q)} \sqrt{\frac{\Phi_f^2}{4(L_d - L_q)^2} + i_q^*}.
\]

(3)

B. Quasi-continuous higher order sliding mode controller

Now, we introduce a quasi-continuous high order sliding mode control proposed by [11]. Consider the following nonlinear system

\[
\Sigma: \begin{cases}
\dot{x}_i &= f_i(x_i, t) + g_i(x_i, t)s_i + \omega_i(x_i, t) \\
x_{i,n-1} &= f_{i,n-1} + g_{i,n-1} x_i + \omega_{i,n-1}
\end{cases}
\]

(4)

for \( i = 2, \ldots, n - 1 \); where \( x_i \in R^n \) is the state, \( x_i = [x_{i,1}, \ldots, x_{i,n}]^T \); \( u_i \in R \) is the control input. Furthermore, \( f_i(x_i, t) \) and \( g_i(x_i, t) \) are smooth function, \( \omega_i(x_i, t) \) is a bounded unknown perturbation term due to parameter variations and external disturbances, with at least \( n - i \) bounded derivatives. \( g_i(x_i, t) \neq 0 \) for \( x \in X \in R^n \), \( t \in [0, \infty) \). To simplify the exposition, we assume that the relative degree of system (4) is equal to \( n \). Now, the control design algorithm for the class of nonlinear system with unmatched perturbations given by (4) is introduced.

Step 1.

Defining \( x_2 = \phi_1 \) where \( \phi_1 \) is \( (n-1) \) times differentiable function defined by

\[
\phi_1 = g_1(x_1, t)^{-1} \{f_1(x_1, t) + u_1,1\} \\
u_{1,1} = u_{1,2} \\
\vdots \\
u_{n,n-1} = \alpha \Psi_{n-1,1}(\sigma_1, \sigma_2, \ldots, \sigma_{i-1}^{-1})
\]

(5)

where \( \sigma_1 = y - y_{ref} \). The first virtual control is constituted of two parts, \( \phi_1 \) compensates the nominal part of the system and \( u_{1,1} \) compensates the perturbation introduced through \( n-1 \) integrators. The i-th step is given as follows

Step i.

Defining the virtual control as follows \( x_{i+1} = \phi_i \) where \( \phi_i \) is \( n - i \) times differentiable function defined by

\[
\phi_i(x_i, t, u_i) = g_i(x_i, t)^{-1} \{f_i(x_i, t) + u_{i,1}\} \\
u_{i,1} = u_{i,2} \\
\vdots \\
u_{n-i,n-1} = \alpha \Psi_{n-i,n-1}(\sigma_1, \sigma_2, \ldots, \sigma_{i-1}^{-1})
\]

(6)

where \( \sigma_i = x_i - \phi_{i-1} \). The last step allows to calculate the robust control. Step n. Defining \( \sigma_n = x_n - \phi_{n-1} \), then the real control \( u \) is given by

\[
u = g_n(x_n)^{-1} \{f_n(x_n) + u_{n,1}\} \\
u_{n,1} = \alpha \Omega \text{sgn}(\sigma_n)
\]

(7)

C. IPMSM controller design

Now, we apply the Quasi-Continuous HOSM control algorithm [11] to the IPMSM. The IPMSM model (1) can be represented in the following form

\[
\Sigma_i: \begin{cases}
\dot{\Omega} &= \omega_0 - \frac{L_q}{L_d} \Omega^2 - \frac{1}{L_d} T_i + \omega_{1,1} \\
i_q &= -p \frac{L_q}{L_d} \Omega \frac{1}{L_d} \Omega i_d - \frac{1}{L_d} T_i + \frac{1}{L_d} v_q + \omega_{1,2} \\
i_d &= -\frac{L_d}{L_q} i_q + \frac{p}{L_q} \Omega i_d + \frac{1}{L_d} v_d + \omega_{2,1}
\end{cases}
\]

(8)

(9)
where \( X_1 = [x_{1,1}, x_{1,2}]^T = [\Omega, i_q]^T \) and \( X_2 = [x_{2,1}] = [i_d] \), \( m = 2 \). The terms \( \omega_{1,1} \) and \( \omega_{2,1} \) are the unmatched terms due to the parametric uncertainties. Since the outputs to be controlled are \([\Omega, i_q]^T \). Then, it is clear that the relative degree of each subsystem are equal to (2,1). Now, using the above representation, the first uncertain subsystem can be described in the following strict-feedback form.

\[
\Sigma_1 : \begin{cases}
    x_{1,1} = f_1(x_{1,1}) + g_1(x_{1,1})x_{1,2} + \omega_{1,1} \\
    x_{1,2} = f_2(x_{1,1}, x_{1,2}) + g_2(x_{1,2})u_1 + \omega_{2,1}
\end{cases}
\]  

(10) with

\[
f_1(x_{1,1}) = -\frac{L_d}{L_q} \Omega - \frac{1}{J} \tau_i, \quad g_1(x_{1,1}) = \frac{L_d}{L_q} (L_d - L_q) i_d + \frac{R_s}{L_q} i_q,
\]

\[
f_2(x_{1,1}, x_{1,2}) = -\frac{\phi_f}{L_q} \Omega - p \frac{L_d}{L_q} \Omega i_d - \frac{R_s}{L_q} i_q, \quad g_1(x_{1,1}, x_{2,1}) = \frac{1}{L_d} \Omega
\]

and the second subsystem can be represented as follows

\[
\Sigma_2 : \begin{cases}
    x_{2,1} = f_2(x_{1,1}, x_{1,2}, x_{2,1}) + g_2(x_{2,1})u_2 + \omega_{2,1}
\end{cases}
\]  

(11) with

\[
f_2(x_{1,1}, x_{1,2}, x_{2,1}) = -\frac{R_s}{L_d} i_d + p \frac{L_d}{L_q} \Omega i_d \quad \text{and} \quad g_2(x_{2,1}) = \frac{1}{L_d} \Omega
\]

**Step 1.**

Define \( x_{1,2} = \phi_{1,1} \); where

\[
\phi_{1,1} = g_1(x_{1,1})^{-1} (-f_1(x_{1,1}) + u_{(1,1,1)} \}
\]

**Step 2.**

The virtual control \( u_1 \) is calculated as follows

\[
u_{(1,1,1)} = -\alpha \Omega (\sigma_1 - \Omega)
\]

Next, to compute the control action \( u_q = u_1 \), let be

\[\sigma_{1,2} = x_{1,2} - \phi_{1,1} \]

Then, the control for the first subsystem is given by

\[u_1 = g_1^{-1}(x_{1,2}) (-f_1(x_{1,1}, x_{1,2}) + u_{(1,1,2)} \}

where

\[
u_{(1,1,2)} = -\alpha \Omega (\sigma_1 - \Omega)
\]

Finally, we obtain

\[
u_q = L_q \{ p \frac{\Omega}{L_q} \Omega + p \frac{L_d}{L_q} \Omega i_d + \frac{R_s}{L_q} i_q + \alpha \Omega (\sigma_1 - \Omega) \}
\]  

(12)

\[
\phi_{1,1} = \frac{p}{L_d - L_q} i_d + \frac{\phi_f}{L_q} \Omega - \frac{f_c}{J} \Omega + \frac{1}{J} \tau_i + v_{(1,1,1)} \}
\]

**Current i_d loop.**

To apply a maximum torque per ampere strategy, the current \( i_d \) is forced to track the reference value computed in (3). Then, defining the following sliding surface

\[\sigma_q = \sigma_{2,1} = i_d - \bar{i}_d \]

Notice that the relative degree is \( r_2 = 1 \). Then, the control action for the second subsystem is given by

\[
u_2 = g_2^{-1}(x_{2,1}) (-f_2(x_{1,1}, x_{1,2}, x_{2,1}) + u_{(2,1,1)} \}
\]  

(13)

where

\[u_{(2,1,1)} = -\alpha \Omega (\sigma_1 - \Omega)
\]

Finally, the controller is given by

\[
u_q = L_d \{ \frac{R_s}{L_d} i_d - \frac{L_d}{L_d} \Omega i_q + \alpha \Omega (\sigma_1 - \Omega) \}
\]

(14)

**IV. ADAPTIVE INTERCONNECTED OBSERVERS DESIGN FOR IPMSM**

Now, an adaptive interconnected observer (see [12]) for the IPMSM sensorless control, is designed to estimate the position, the rotor speed, the load torque and the stator resistance. The dynamics of these two last variables are assumed to be piecewise function such that

\[
T_i = 0 \quad \bar{R}_s = 0.
\]

(15)

Thus, the extended IPMSM model (1)-(15) can be seen as the interconnection between subsystems (16) and (17). Suppose that each subsystem satisfies some required properties defined later in order to build an observer and assume that, for each separate observer, the state of the other is available. Then one has

\[
\Sigma_1 : \begin{cases}
    x_1 = A_1(y) x_1 + g_1(x_2, u) \\
    y_1 = C_1 x_1
\end{cases}
\]  

(16)

\[
\Sigma_2 : \begin{cases}
    x_2 = A_2(y) x_2 + g_2(x_1, x_2, u) + \phi T_i \\
    y_2 = C_2 x_2
\end{cases}
\]  

(17)

where

\[
A_1(\cdot) = \begin{bmatrix} 0 & -\frac{L_d}{L_q} \\ 0 & 0 \end{bmatrix}, \quad A_2(\cdot) = \begin{bmatrix} 0 & \frac{L_d}{L_q} i_q \\ 0 & -\frac{1}{J} \end{bmatrix}, \quad \phi = \begin{bmatrix} 0 \\ -\frac{1}{J} \end{bmatrix}
\]

\[
g_1(\cdot) = \begin{bmatrix} -\frac{L_d}{L_q} \Omega i_d + \frac{\phi_f}{L_q} \Omega + \frac{1}{L_d} u \\ 0 \end{bmatrix}, \quad g_2(\cdot) = \begin{bmatrix} -\frac{R_s}{L_d} i_d + \frac{1}{L_d} u \\ \frac{\phi_f}{L_q} i_d + \frac{L_d}{L_q} (L_d - L_q) i_q \end{bmatrix}, \quad C_1 = C_2 \begin{bmatrix} 1 & 0 \end{bmatrix}
\]

\[
X_1 = [i_d R_s]^T, \quad X_2 = [i_q \Omega]^T \quad \text{are the states, } u = [v_d v_q]^T \quad \text{is the input, and } y = [i_d i_q]^T \quad \text{is the output of the IPMSM model.} \]

\(T_i\) is considered as an unknown parameter to be identified by the adaptive part of the observer. \(g_1\) and \(g_2\) are the interconnection terms.

The adaptive interconnected observer developed in the sequel for the sensorless IPMSM, is based on the interconnection between several observers satisfying some required properties, in particular the property of input persistence (see [12] for more details). As defined in this latter reference, the input persistence is related to the observability properties of system (16)-(17).

**Remark 1:** \(X_2\) and \(X_1\) are respectively considered as inputs for subsystems \((\Sigma_1)\) and \((\Sigma_2)\). When the IPMSM remains in the observable area [13], \(X_1\) and \(X_2\) satisfy the regularly persistence condition: then, asymptotic convergence of the observer is proved.
Remark 2: It is clear that
1 $A_1(y)$ is globally Lipschitz w.r.t. $X_1$.
2 $A_2(y)$ is globally Lipschitz w.r.t. $X_1$.
3 $g_1(X_2)$ is globally Lipschitz w.r.t. $X_1$ uniformly w.r.t. $(u,y)$.
4 $g_2(X_1,X_2)$ is globally Lipschitz w.r.t. $X_1,X_2$ uniformly w.r.t. $(u,y)$.

Then, adaptive interconnected observers for (16) and (17) are given by

$$\begin{align*}
O_1 : & \begin{cases}
Z_1 = A_1(y) Z_1 + g_1(y, Z_2, u) + S_1^1 C_1^T (y_1 - \hat{y}_1) \\
\dot{S}_1 = -\rho_1 S_1 - A_1^T (y) S_1 - S_1 A_1(y) + C_1^T C_1
\end{cases} \\
\hat{y}_1 = C_1 Z_1
\end{align*}$$

$$\begin{align*}
O_2 : & \begin{cases}
Z_2 = A_2(y) Z_2 + g_2(Z_2, Z_2, u) + \Phi T_i \\
\dot{S}_2 = -\rho_2 S_2 - A_2^T (y, y_2) S_2 - S_2 A_2(Z_1, y_2) + C_2^T C_2 \\
\hat{y}_2 = C_2 Z_2
\end{cases}
\end{align*}$$

with $Z_1 = \begin{bmatrix} \hat{y}_1 & \hat{\theta}_1 \end{bmatrix}^T$ and $Z_2 = \begin{bmatrix} \hat{y}_2 & \hat{\Omega} \end{bmatrix}^T$ are the estimated state variables respectively of $X_1$ and $X_2$. For $i = 1, 2, 3$, the $\rho_i$ are positive constants. $S_1$ and $S_2$ are symmetric positive definite matrices, with $S_3(0) > 0$. $B(Z_1) = k_L^B \hat{\theta}_1 i_q$, $B_1(Z_1) = k_L^B (L_d - L_q) \hat{\theta}_1$

$$K_e = \begin{bmatrix} -k_{c1} & -k_{c2} \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 1 & 0 \\ 0 & \alpha \end{bmatrix}$$

The convergence of the observer is stated by the following lemma, (see proof in [12].)

Lemma 1: Assume that the input $v$ is regularly persistent for a given state affine system and consider the following Lyapunov differential equation

$$S(t) = -\Theta S(t) - A^T (v(t)) S(t) - S(t) A (v(t)) + C^T C,$$

with $S(0) > 0$, then $\exists \theta_0, \forall \theta > \theta_0, \exists \alpha > 0, \beta > 0, t_0 > 0$

$$\forall t_0, \alpha I \leq S(t) \leq \beta I$$

were $I$ is the identity matrix.

For the proposed observer, it is clear that $v = (u, X_3)$ and $S(t) = S_1$ for subsystem (16), and for subsystem (17) one has $v = (u, X_1)$ and $S(t) = S_2$.

It is worth mentioning that the conditions of observability loss have been stated in Remark 2, in the unobservable region of the IPMSM, such inputs are "bad input" and the observer convergence is not guaranteed. The use of practical stability properties introduced in [14] can solve this problem.

Furthermore, to estimate the rotor position, we consider the following observer

$$\frac{d\hat{\theta}}{dt} = \hat{\Omega} + K_\theta (i_q - \hat{i}_q)$$

where $K_\theta$ is the gain of the observer.

V. Stability Analysis of Observer under Uncertain Parameters

Assumes that all parameters of IPMSM are uncertain but with well-known bound values. The robustness of the observer under parametric uncertainties is analyzed and the system is rewritten in the following form

$$\begin{align*}
\Sigma_1 : & \begin{cases}
X_1 = A_1(y) X_1 + g_1(X_2, u) + \Delta A_1(y) + \Delta g_1(X_2) \\
y_1 = C_1 X_1
\end{cases} \\
\Sigma_2 : & \begin{cases}
X_2 = A_2(y) X_2 + g_2(X_1, X_2, u) + \Phi T_i + \Delta A_2(y) + \Delta g_2(X_1, X_2, u)
\end{cases}
\end{align*}$$

where $\Delta A_1(X_2, y)$, $\Delta A_2(X_1)$, $\Delta g_1(u, y, X_2)$, $\Delta g_2(X_1, X_2)$ are respectively the uncertain terms of $A_1(X_2, y)$, $A_2(X_1)$, $g_1(X_2)$, $g_2(X_1, X_2)$.

In this way, the uncertain terms are given by

$$\begin{align*}
\Delta A_1(y) &= \begin{bmatrix} 0 & -i_q \frac{\Delta \theta}{\Delta \theta} \\ 0 & 0 \end{bmatrix}, \Delta A_2(y) &= \begin{bmatrix} 0 & p \frac{\Delta \theta}{\Delta \theta} i_q \\ 0 & 0 \end{bmatrix}, \\
\Delta g_1(y) &= \begin{bmatrix} -p \frac{\Delta \theta}{\Delta \theta} \Omega d - p \frac{\Delta \theta}{\Delta \theta} \Omega + \frac{1}{\Delta \theta} u \\ 0 \end{bmatrix}, \\
\Delta g_2(y) &= \begin{bmatrix} -\frac{\Delta \theta}{\Delta \theta} \Omega d - \frac{\Delta \theta}{\Delta \theta} \Omega + \frac{1}{\Delta \theta} u \\ -p \frac{\Delta \theta}{\Delta \theta} \Omega d - \frac{\Delta \theta}{\Delta \theta} \Omega + \frac{1}{\Delta \theta} u \end{bmatrix}.
\end{align*}$$

Assumption 1. Considering the IPMSM physical operation domain $\mathcal{D}$, assume that the uncertain terms are bounded such that there exist positive constants $\rho_i > 0$, for $i = 1, \ldots, 4$, such that

$$\|\Delta A_1\| \leq \rho_1, \quad \|\Delta A_2\| \leq \rho_2, \quad \|\Delta g_1\| \leq \rho_3, \quad \|\Delta g_2\| \leq \rho_4.$$}

The parameters $\rho_i$, $i = 1, \ldots, 4$, are positive constants determined from the maximal values of $\Delta A_1(\cdot)$, $\Delta A_2(\cdot)$, $\Delta g_1(\cdot)$ and $\Delta g_2(\cdot)$ in the physical domain $\mathcal{D}$. Let us define the estimation errors as

$$\epsilon_1 = X_1 - Z_1, \quad \epsilon_2 = X_2 - Z_2, \quad \epsilon_3 = T_i - \hat{T}_i.$$

Applying the transformation $\epsilon_2 = \epsilon_2 - \Lambda \epsilon_3$, the dynamics of the estimation errors are given by

$$\begin{align*}
\dot{\epsilon}_1 &= \begin{bmatrix} A_1(y) - S_1^{-1} C_1^T C_1 \epsilon_1 + \Delta A_1(y) X_1 \\ + g_1(X_2) + \Delta g_1(X_2) - g_1(Z_2) \end{bmatrix} \\
\dot{\epsilon}_2 &= \begin{bmatrix} A_2(y) - \Gamma S_2^{-1} C_2^T C_2 \epsilon_2 - B_{12} \epsilon_2 + \Delta A_2(y) X_2 - \Delta g_2(X_1, X_2) \\
- B_{22} \epsilon_2 + \Delta A_2(y) X_2 + \Delta g_2(X_1, X_2) - g_2(Z_1, X_2) \\
- g_2(Z_1, X_2) + \Delta A_2(y) X_2 + \Delta g_2(X_1, X_2) - g_2(Z_1, X_2) \end{bmatrix} \\
\dot{\epsilon}_3 &= -B_1 \epsilon_1 - \begin{bmatrix} \sigma S_3^{-1} A^T C_3^T C_2 + B_2^T \epsilon_3 \end{bmatrix}.
\end{align*}$$
Since \((u, X_2)\) and \((u, X_1)\) are the inputs for subsystems (16)-(17) respectively, and from Lemma 1, then there exist \(t_0 \geq 0\) and real numbers \(\eta_{Si}^{\max} > 0\), \(\eta_{Si}^{\min} > 0\), which are independent of \(\rho_i\), such that \(V(t, e_i) = e_i^T S_i e_i\) for \(i = 1, 2, 3\); satisfies

\[
\eta_{Si}^{\min} \|e_i\|^2 \leq V(t, e_i) \leq \eta_{Si}^{\max} \|e_i\|^2, \quad \forall t \geq t_0. \tag{27}
\]

Then, one can establish the following result about the convergence of the observer under parametric uncertainties.

Theorem 1: Consider the IPMSM dynamic model represented by (16)-(17). System (18)-(19) is an adaptive interconnected observer for system (16)-(17) with strongly uniformly practical stability of estimation error dynamics.

For a lack of space, the proof of Theorem 1 is not given here. For details, see the proof in [15].

VI. STABILITY OF THE OBSERVER-CONTROLLER SCHEME

The main goal of this paper is to synthesize a robust sensorless control of IPMSM. The proposed quasi-continuous HOSM control law is implemented using the estimates of the rotor position and speed, the rotor resistance and the load torque. Then, in order to guarantee the correct behavior of the proposed observer-controller scheme a rigorous analysis is necessary. Now the observation error dynamics will be taken into account in order to derive the attractiveness condition of the estimated surfaces \(\sigma_\Omega\) and \(\sigma_i\), given by

\[
\begin{bmatrix}
\dot{\sigma}_\Omega \\
\dot{\sigma}_i
\end{bmatrix}
= \begin{bmatrix}
\Omega - \Omega^* \\
\dot{i}_d - \dot{i}_d^*
\end{bmatrix}.
\tag{28}
\]

However, the estimated sliding surface can be expressed as

\[
\begin{bmatrix}
\dot{\sigma}_\Omega \\
\dot{\sigma}_i
\end{bmatrix}
= \begin{bmatrix}
\Omega - \Omega' - e_{\Omega} \\
\dot{i}_d - \dot{i}_d^* - e_{i_d}
\end{bmatrix},
\tag{29}
\]

where

\[
\begin{bmatrix}
e_{\Omega} \\
e_{i_d}
\end{bmatrix}
= \begin{bmatrix}
\Omega - \hat{\Omega} \\
\dot{i}_d - \dot{i}_d^*
\end{bmatrix},
\tag{30}
\]

with \(e_{\Omega}\) and \(e_{i_d}\) are the speed and \(i_d\) current estimation errors. From theorem 1, the estimation errors are bounded by a positive constant \(\varepsilon^*\). Then, it follows that

\[
|\Omega - \hat{\Omega}| < \varepsilon^*
\]

\[
|\dot{i}_d - \dot{i}_d^*| < \varepsilon^*.
\]

Finally, the tracking error is bounded, which ensures the practical stability (see [15]). This can be summarized in the following lemma.

Lemma 2: Consider system (1) in closed-loop with the HOSM control (12)-(14) designed to track the smooth references \(\Omega^*\) and \(\dot{i}_d^*\), using the estimates variables given by the interconnected observer (16)-(17). Then the tracking error satisfies \(|\Omega - \Omega^*| < \varepsilon^*\) and \(|\dot{i}_d - \dot{i}_d^*| < \varepsilon^*\), where \(\varepsilon^*\) is a positive constant.

VII. SIMULATION RESULTS

The proposed sensorless scheme has been tested by using Matlab/Simulink software following the specification of an industrial Benchmark [7]. The rating and motor parameters used in this simulation are given in Table I. The figures 1 to 4 give the simulation results of sensorless control for IPMSM with nominal parameters. Figure 1.a shows the measured and observed speeds. The speed error due to the perturbation is very small and quickly converges to zero after the transients induced by the load torque application (see figure 1.b). Figure 2 shows the estimated position and the measured one between [0s, 2.7s]. We can observe that the estimated position tracks the actual position very well. By figures 3 and 4, the resistance and load torque estimations are respectively given. It is clear that the observer has good performances for these estimations. The inputs voltages and \(d-q\) currents are shown by figure 5. In order to illustrate the robustness of the sensorless control scheme, the influence of parameter deviations is investigated. Parameter deviations are intentionally introduced in the observer-controller scheme. Figures 6 and 7 show the responses for a +50% increase of the stator resistance. The robustness and the efficiency of the proposed sensorless control under parameter variations and load torque clearly appear.

The gains of the observer are chosen as follows:

\[
\rho_1 = 900, \quad \rho_2 = 800, \quad \rho_3 = 15, \quad \sigma = 80, \quad k_{i_1} = 0.1, \quad k_{i_2} = 0.01, \quad \alpha = 0.1, \quad K_\theta = 15.
\]

Those of the controller are chosen as follows:

\[
\alpha_{i_1}=1500, \quad \alpha_{i_2}=400, \quad \alpha_{i_3}=200.
\]

TABLE I

<table>
<thead>
<tr>
<th>Motor parameters (nominal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
</tr>
<tr>
<td>Speed</td>
</tr>
<tr>
<td>(R_s)</td>
</tr>
<tr>
<td>(L_s)</td>
</tr>
<tr>
<td>(J)</td>
</tr>
</tbody>
</table>

VIII. CONCLUSION

A high order sliding mode controller for sensorless control of interior permanent magnet synchronous motor is given by using an adaptive interconnected observer that estimates the speed, the position, the load torque and the stator resistance. The \(d\) axis reference current is generated on the basis of MTPA control strategy. The practical stability of the observer-controller scheme is guaranteed. Simulation results confirm the effectiveness of the proposed method under significant parameters uncertainties deviations and unknown torque load.

REFERENCES


Fig. 1. Nominal case, observed and measured speeds

Fig. 2. Nominal case, observed and real position

Fig. 3. Nominal case, estimated and real resistance

Fig. 4. Nominal case, estimated and real load torque

Fig. 5. Nominal case, dq voltages and currents

Fig. 6. Robustness w.r.t. +50% Rs, observed and measured speeds

Fig. 7. Robustness w.r.t. +50% Rs, estimated and real resistance


