On Multipath Detection in CDMA Systems

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Abstract—This paper addresses the problem of multipath detection in CDMA systems. In conventional CDMA receivers, the detection of multipath components and RAKE finger management is normally based on the received signal energy per path. These energy-based schemes essentially overlook the interference component contaminating the total received power. Consequently, they exhibit poor detection capability especially at low signal-to-interference-plus-noise ratio (SINR). In this paper, we present a new scheme for multipath detection and RAKE finger assignment that takes into consideration the interference level in each resolved path individually. The proposed scheme utilizes information provided by the pseudo random code acquisition circuit to estimate the interference power per path. To account for the hardware limitations of the receiver, a low complexity version of the proposed scheme is designed and incorporated into the receiver structure. Analytical and simulation results show that the proposed scheme provides significant improvements in the detection probability of multipath components over the energy-based schemes. For instance, our results show that the proposed scheme can achieve the same detection probability of all multipath components as that of the energy-based scheme with a saving of at least 2 dB in $E_b/N_0$. In some cases, it is shown that the improvement can be as high as 3 dB.

I. INTRODUCTION

CODE division multiple access (CDMA) technologies are being standardized around the world as the core air interface for the Third Generation (3G) wireless communication networks, e.g., cdma2000, WCDMA. In 3G CDMA systems, the signals transmitted are wideband in nature, which gives rise to multipath propagation. A key issue for dealing with multipath fading is to first identify the potential paths at the front end of the receiver. Then, a RAKE receiver is used to coherently combine the energy from these multipath components. A RAKE receiver consists of several correlators called fingers that are time-aligned with the different paths. Each finger is intended to de-spread the corresponding path and then the outputs of these fingers are properly combined to maximize the signal-to-interference-plus-noise ratio (SINR) at the output of the RAKE receiver. The criterion by which a finger is assigned to a multipath component is very crucial as it significantly impacts the overall performance of the receiver [1] [2].

In conventional CDMA receivers, the multipath components are detected using a search block and an acquisition circuit [3] [4]. The search block correlates the received signal with locally generated shifted versions of the intended user’s pseudo-noise (PN) code, and the correlation results are then presented to an acquisition circuit. In the acquisition circuit, the correlation results are subjected to hypotheses testing using a pre-calculated threshold [1] [5]. The resolvable paths with energy exceeding this threshold are detected and assigned to the available RAKE fingers for combining. In case the number of these paths is greater than the number of available fingers, the paths with maximum energy among the potential paths are selected and combined [2], [6]–[8]. The process by which these paths are detected and assigned to the RAKE fingers is normally referred to as the finger assignment algorithm (FAA).

Although the conventional multipath detection and finger assignment schemes are deemed to be practical from an implementation point of view, they suffer from major drawbacks. Mainly, the correlation energy, based on which the multipath components are detected, is typically comprised of the desired signal energy and the interference coming from other multipath components belonging to the desired user as well as other users. At low signal-to-interference ratio (SIR), the desired user’s multipath components are vulnerable to be ‘masked’ by other paths with large interference power. Consequently, the probability of detection and probability of accurate assignment deteriorate considerably.

In this paper, we propose a new multipath detection scheme that addresses the above mentioned problem through estimating the interference level at each delay offset in the search window using a practical estimator at the acquisition stage. We derive the probability of detection and the probability of false alarm for the proposed scheme and compare that with their counterparts for the conventional energy-based scheme. We show that substantial improvements in the probability of detection can be achieved with the proposed scheme over the conventional one. We also present a simple and realizable receiver structure based on the proposed detection scheme. Finally, we validate our analysis through extensive computer simulations.

The remainder of the paper is organized as follows. Section II gives the signal and system model. In Section III, the conventional multipath detection scheme is presented. The proposed scheme is described in Section IV. In Section V, a practical realization of the proposed scheme is described. Numerical and simulation results are presented and discussed in Section VI. Finally, Section VII concludes the paper.

II. SYSTEM MODEL

The system considered in this paper is similar to that of the reverse link of the proposed third generation cdma2000
system where a pilot channel is code multiplexed with the data channel. More specifically, we consider a BPSK direct-sequence spread-spectrum (DS-SS) system operating over frequency selective Rayleigh fading channel with $L$ time varying paths. These paths are represented by the channel coefficients $\{\alpha_l: l = 1, 2, \ldots, L\}$, modeled as independent and identically distributed (i.i.d.) complex Gaussian random variables, with zero mean and variance $0.5$ per dimension.

The average power delivered by the $l$th path, i.e. the channel power delay profile (PDP), is denoted by $\phi(\tau_l)$. The received baseband signal is given by

$$u(t) = \sum_{m=1}^{M} \sum_{l=1}^{L} \sqrt{\phi_m(\tau_l)} \alpha_{ml} s_m(t - \tau_{ml}) + n(t),$$

where $s_m(t)$ is the spread-spectrum signal of the $m$th user, $\alpha_{ml}$ and $\tau_{ml}$ are the channel gain and path delay for the $l$th path of the $m$th user, respectively, $\phi_m(\cdot)$ is the channel PDP of the $m$th user, and $n(t)$ is an additive white Gaussian noise (AWGN) with zero mean and power spectral density $N_0/2$ per dimension.

During the data transmission (traffic mode), the receiver searches continuously for new potential multipath components to be combined in the de-modulation stage. The search algorithm (henceforth referred to as searcher) is basically an integral part of the acquisition circuit used to perform the correlation between the received signal and different replicas of the desired user’s PN code. The searcher examines a window of $C$ possible delays for a period of time (dwell time), and the correlation results are stored for further processing. The search step size, denoted by $S$, is typically a fraction of a chip, e.g., one-half a chip. Hence, the search results, which we call the search delay profile, will consist of $K = C/S$ values. For each effective multipath component in the desired user’s channel, the search result is given as [9]

$$h_k(n) = \sqrt{E_b\phi_1(k)}\alpha_{1k}(n) + \sqrt{\sigma^2(k)}I(n),$$

where the index $k$ represents the $k$th delay offset within the search window, $n$ is the search time index within received frame, $E_b\phi_1$ is the energy per bit for the first user, $I(n)$ is a normalized complex Gaussian random variable with zero mean and variance 0.5 per dimension, and $\sigma^2(k)$ is the variance of the interference component [9].

To improve the probability of multipath detection, $N_A$ independent search results are obtained through repeating the search process at different time instants, e.g., bits or search blocks, during a data frame. These time instants are usually chosen to be spaced sufficiently far from each other within a frame. This gives the searcher the required time to correlate the received signal with all shifted versions of the PN code. It also gives the fading process some time to de-correlate.

III. ENERGY-BASED MULTIPATH DETECTION SCHEME (EMDS)

In the energy-based multipath detection scheme (EMDS), the acquisition circuit averages the correlation energies over $N_A$ independent search blocks at each delay offset and compares the result to a threshold. If the average energy at a certain delay offset exceeds the threshold, the path with that delay offset is acquired. This process is done for all delay offsets in the search window. The selected delay offsets are then fine-aligned through the tracking process, which is accomplished by employing a delay locked loop (DLL). If a wrong delay, which does not contain the desired user’s signal, passes the threshold test, the tracking loop will detect it and declare a false alarm state after a relatively long period of processing time. Thus, it is extremely important to reduce the probability of false alarm of the detection scheme. From (2), the average correlation energy at the $k$th delay offset is

$$Y(k) = \frac{1}{N_A} \sum_{n=1}^{N_A} |h_k(n)|^2 = \tilde{p}(k) + \sigma^2(k),$$

where $\sigma^2(k)$ and $\tilde{p}(k)$ are, respectively, estimates of the interference power $\sigma^2(k)$ and the user power $p(k) = E_b\phi_1(k)$ obtained from $N_A$ independent search results. In (3), it is assumed that the interference and the desired user’s signal are independent. This is a valid assumption since each multipath component fades independently. It can be shown that both variables $\tilde{\sigma}^2(k)$ and $\tilde{p}(k)$ have a central Chi-square distribution with $2N_A$ degrees of freedom. The first and second order statistics of these variables are $E\{\tilde{\sigma}^2(k)\} = \sigma^2(k)$, $var\{\tilde{\sigma}^2(k)\} = \frac{\sigma^4(k)}{N_A}$, $E\{\tilde{p}(k)\} = p(k)$, and $var\{\tilde{p}(k)\} = \frac{\sigma^4(k)}{N_A}$.

When $Y(k)$ exceeds the threshold, the $k$th delay offset will be detected. Furthermore, if the RAKE receiver has $L$ fingers, the $k$th delay offset will be assigned to one finger if it is one of the $L$ maximum components in the search window. Apparently, when the SINR is low, some of the actual multipath components will be ‘masked’ by other delay offsets with strong interference power, and consequently wrong paths will pass the threshold declaring a false alarm state. The probability of false alarm for the EMDS is derived next.

A. Probability of False Alarm and Probability of Detection

When the desired user is not transmitting, the decision metric is given as

$$Y(k) = \tilde{\sigma}^2(k).$$

It can be shown that $Y(k)$ has a central Chi-square distribution with $2N_A$ degrees of freedom whose cumulative distribution function (cdf) of $Y(k)$ is given as [10]

$$F_{Y_k}(y) = 1 - e^{-N_Ay/\tilde{\sigma}^2(k)} \sum_{i=0}^{N_A-1} \frac{1}{i!} \left( \frac{N_Ay}{\tilde{\sigma}^2(k)} \right)^i.$$  

Using (5), the probability that the $k$th delay offset produces a false alarm state is

$$P_{fa}(k) = Pr[Y(k) > \eta] = e^{-N_A\eta/\tilde{\sigma}^2(k)} \sum_{i=0}^{N_A-1} \frac{1}{i!} \left( \frac{N_A\eta}{\tilde{\sigma}^2(k)} \right)^i,$$

where $\eta$ denotes a threshold. The average probability of false alarm, denoted by $P_{fa}$, is the probability that any delay offset in the search window causes a false alarm state, which is given
by
\[ P_{FA} = \frac{1}{K} \sum_{k=1}^{K} P_{fa}(k). \] (7)

We note from (6) that the probability of false alarm increases as the interference power increases, as expected. The probability of detecting a multipath component in the desired user’s channel is basically the probability that the average correlation energy corresponding to that component is greater than the threshold. If the desired user’s signal impinges at the receiver front end from \( L \) different paths \( \{ l_1, l_2, \ldots, l_L \} \), the average correlation energy corresponding to any one of these paths is given by (3). Since the decision metric is a function of two independent random variables, namely \( \hat{p}(k) \) and \( \tilde{\sigma}_j^2(k) \), we formulate its conditional cdf as
\[ F_{Y_k}(y \mid \hat{p}(k)) = 1 - e^{-N_A y/|\tilde{\sigma}_j^2(k)+\hat{p}(k)|}. \] (8)

The probability of detecting the \((l_j)^{th}\) multipath component, for \( j = 1, 2, \ldots, L \), is given as
\[ P_D(l_j) = \int_{0}^{\infty} F_{Y_k}(y \mid \hat{p}(k))f_{l_j}(\tilde{\sigma}_j^2(l_j))d\tilde{\sigma}_j^2(l_j), \] (9)
where
\[ f_{l_j}(\tilde{\sigma}_j^2(l_j)) = \frac{1}{\Gamma(N_A)} \left( \frac{N_A \tilde{\sigma}_j^2(l_j)}{E_{\sigma_1} \phi(l_j)} \right)^{N_A-1} \cdot e^{-N_A \tilde{\sigma}_j^2(l_j)/E_{\sigma_1} \phi(l_j)}, \quad \tilde{\sigma}_j^2(l_j) \geq 0 \] (10)

The probability of detection can be found following the same steps used to find the probability of false alarm. Averaging the conditional probability of detecting the \((l_j)^{th}\) multipath component over the received signal’s envelope results in the probability of detecting that component which is given by
\[ P_D(l_j) = \int_{0}^{\infty} Q\left( \frac{\sqrt{N_A \alpha} \hat{p}(l_j)}{\sqrt{\sigma_j^2(l)}} \right) f_{l_j}(\tilde{\sigma}_j^2(l))d\tilde{\sigma}_j^2(l). \] (14)

It is important to note that while the probability of false alarm at a certain threshold level depends on the interference power, the probability of detection is a function of the received per path SINR: \( \hat{p}(l_j)/\tilde{\sigma}_j^2(l) \).

V. PRACTICAL REALIZATION OF THE IMDS

Although the interference variance estimator in (11) is optimum in the sense that it is MVU, it assumes perfect knowledge of the desired user’s channel coefficients. At the data demodulation stage, this is a reasonable assumption since the channel coefficients are needed by the RAKE receiver to implement coherent combining of the assigned fingers and a channel estimation algorithm is usually implemented at the receiver for that purpose. At the delay detection stage, however, the estimator requires the knowledge of the channel coefficients for all the \( K \) delay offsets in the search window. When the search window is large, estimating the channel at each delay offset becomes prohibitive. To account for the hardware limitation of the receiver, a modified low complexity version of the previous estimator should be implemented at the detection stage. This can be accomplished as follows.

To obtain an estimate of the interference power, the \( N_A \) search results at each delay offset in the search window will pass through an FIR filter. The filtered version of the search Consequently, under the IMDS, the paths with maximum \( Z \) will be acquired. Comparing the decision metric given in (12) and the one in (3), we observe that the proposed metric depends on the desired signal power as opposed to the conventional metric which considers the composite power (signal plus interference) in the detection process. The probabilities of false alarm and detection for the IMDS are derived next.

A. Probability of False Alarm and Probability of Detection

The detection metric can be written compactly as \( Z(k) = \hat{p}(k) + [\tilde{\sigma}_j^2(k) - \hat{\sigma}_j^2(k)] \). Clearly, the detection metric consists of the signal component \( \hat{p}(k) \) and estimation error given by the term \( \tilde{\sigma}_j^2(k) - \hat{\sigma}_j^2(k) \). Since the estimator in (11) is MVU, the estimation error is Gaussian distributed with zero mean and variance \( \sigma_j^2(k)/N_A \) [11]. When the desired user is idle, the detection metric equals the estimation error. Using this result, the probability that the estimation error for the \( k^{th} \) delay offset exceeds a given threshold \( \epsilon \) can be found to be
\[ P_{fa}(k) = Q\left( \frac{\sqrt{N_A \alpha}}{\sqrt{\sigma_j^2(k)}} \right), \] where \( Q(\cdot) \) is the \( Q \)-function. The average probability of false alarm is then given by
\[ P_{FA} = \frac{1}{K} \sum_{k=1}^{K} Q\left( \frac{\sqrt{N_A \alpha}}{\sqrt{\sigma_j^2(k)}} \right). \] (13)

The probability of detection can be found following the same steps used to find the probability of false alarm. Averaging the conditional probability of detecting the \((l_j)^{th}\) multipath component over the received signal’s envelope results in the probability of detecting that component which is given by
\[ P_D(l_j) = \int_{0}^{\infty} Q\left( \frac{\sqrt{N_A \alpha \hat{p}(l_j)}}{\sqrt{\sigma_j^2(l)}} \right) f_{l_j}(\tilde{\sigma}_j^2(l))d\tilde{\sigma}_j^2(l). \] (14)

IV. IMPROVED MULTIPATH DETECTION SCHEME (IMDS)

The EMDS scheme can be modified through estimating the interference power in each path as it will be shown below. The \( N_A \) search results at each delay offset in the search window can be used to estimate the interference variance as follows.
\[ \tilde{\sigma}_j^2(k) = \frac{1}{N_A} \sum_{n=1}^{N_A} [\hat{h}_k(n) - \sqrt{\hat{p}(k)\alpha_{1k}(n)}]^2. \] (11)

With the assumption that the channel fading coefficients are known at the receiver, this estimator can be shown to be Minimum Variance Unbiased (MVU) estimator [11]. Using the estimates computed using (11), the new detection metric is
\[ Z(k) = Y(k) - \tilde{\sigma}_j^2(k). \] (12)
results is $\hat{h}_k(n) = \sum_{m=0}^{K_f-1} f(m) h_k(n-m)$, where $f(\cdot)$ is the filter shaping function and $K_f$ is the filter length. Using the filtered search results, a simple per-path interference variance estimator can be expressed as $\hat{\sigma}_I^2(k) = \frac{1}{N} \sum_{n=1}^{N_A} |\hat{h}_k(n) - \hat{h}_k(n)|^2$. Clearly, this estimator uses the output of the FIR filter as rough channel estimates in order to estimate the interference variance and the signal power.

In Fig. 1, we present a schematic diagram of the proposed IMDS structure where the simplified estimator is incorporated into the detection scheme. While the upper branch is used to compute the decision metric of the EMDS, the lower branch is leased to estimate the interference variance. It is worth mentioning that the increase in complexity is rather modest compared to the EMDS implementation.

![Schematic diagram of the proposed IMDS structure](image)

**Fig. 1.** A realization of the improved multipath detection scheme.

**VI. NUMERICAL RESULTS AND DISCUSSION**

In our simulations we use a CDMA up-link channel with a total of 21 users. The desired user has 4 paths with a uniform power delay profile. The receiver operational characteristic (ROC) curves obtained from the simulations are compared with those calculated using the developed expressions for both detection schemes. Fig. 2 shows sample results of the conducted comparison when $N_A = 25$ search results are used for both detection schemes at $E_b/N_0 = 0$ dB and $SIR = 0$ dB. As shown in the figure, the simulation results are in good agreement with the theoretical results especially in the range of interest for most practical systems ($P_{FA} < 10^{-2}$). The small discrepancy between both theory and simulations when the probability of false alarm is small is attributed to the correlation between the interference components seen at each delay offset. This effect has not been taken into consideration in deriving the theoretical expressions. However, increasing the number of interfering users sufficiently assures better agreement as suggested by the central limit theorem.

To examine the effectiveness of the IMDS scheme on the receiver performance, we examine the ROC curves obtained using the EMDS and the IMDS as a function of the number of search results $N_A$. Fig. 3 shows the ROC for both schemes at $p/\sigma_I^2 = 0$ dB with different numbers of search results $N_A$. As shown in the figure, the IMDS has a superior performance compared to the EMDS. We observe that the proposed detection scheme is far more efficient in utilizing the number of search results. For instance, IMDS needs only 10 search results in order to draw the same detection performance as the EMDS with $N_A = 30$. This attribute of the IMDS is of special importance as it reduces the mean acquisition time which depends on the probability of false alarm, probability of detection, and number of search results used to accumulate the signal energy.

To illustrate the efficacy of the practical realization of the IMDS (see Fig. 1), we compare the detection capability of the IMDS with that of the EMDS scheme through simulations. The model considered here consists of a multiaccess environment with 5 interferers with equal power. BPSK spreading using PN sequences of spreading factor $N = 128$ is assumed. The Pilot channel gain with respect to the traffic channel is set to 0 dB. The desired user has $L = 4$ paths at relative delays $\{0, 2, 4, 6\}$ chips with uniform PDP $\phi(l) = 1/L$. The normalized Doppler frequency is set to 0.01. The number of delay offsets in the search window $K = 32$ and the search step is 0.5 chip. The correlation time is $T_{id} = 128$ chips and

![Theoretical and simulation ROC curves of the EMDS and the IMDS](image)

**Fig. 2.** Theoretical and simulation ROC curves of the EMDS and the IMDS at $E_b/N_0 = 0$ dB and $SIR = 0$ dB using $N_A = 25$ search results per data frame.

![Theoretical and simulation ROC curves of the EMDS and the IMDS](image)

**Fig. 3.** The theoretical ROC for the EMDS and the IMDS for different $N_A$ at $p/\sigma_I^2 = 0$ dB.
again $N_A = 25$ search blocks are used in each frame. Finally, the length of the FIR filter: $K_f = 10$ (rectangular window).

We investigate the ROC when the receiver operates according to both detection schemes. Simulation results for the EMDS and IMDS at $SIR = -10$ dB and $E_b/N_0 = 12$ dB are shown in Fig. 4. It is evident that the proposed detection scheme has extended the operational region of the receiver by reducing the interference effect on the probability of detection. For example, at $P_{FA} = 0.01$, the proposed scheme is capable of detecting a particular multipath component 82% of the time as compared to 67% for the EMDS. Furthermore, for a given high probability of detection, the EMDS results in a high probability of false alarm. For instance, at $P_D = 0.90$, the EMDS will result in more than 20% of them being false alarms. On the other hand, the IMDS maintains the probability false alarm at 5% for the same probability of detection.

![Fig. 4. Comparison between the ROC of the EMDS and the simplified IMDS.](image)

The probability of accurately finding all the multipath components is depicted in Fig. 5 as a function of $E_b/N_0$ at different SIRs for both detection schemes. As shown in the figure, the IMDS has maintained the superior performance with respect to the EMDS. Although at low $E_b/N_0$ the $PAA$ is poor, the performance improves exponentially as $E_b/N_0$ increases. Also, it can be observed that the IMDS results in a gain that ranges from 4 dB (at low $E_b/N_0$) to 12 dB (at large $E_b/N_0$) at $SIR = -10$ dB. The same behavior was observed at other Doppler rates.

![Fig. 5. Probability of accurate assignment of all 4 paths.](image)

**VII. CONCLUDING REMARKS**

In this paper, we have presented an improved multipath detection scheme for CDMA systems. With the proposed scheme, the multipath detection is modified such that the interference component is taken into account in detecting and assigning potential multipath components to the RAKE fingers. The performance of the proposed scheme was analyzed and compared with that of the energy-based scheme. We have shown that the proposed scheme outperforms the conventional one in different ways. In terms of the probability of detection and probability of false alarm, the proposed scheme has shown a great potential in enhancing the receiver operational characteristics. Also, the proposed scheme has proved to be more efficient as it requires less pilot power and reduces the acquisition time as compared to the energy-based scheme. Furthermore, we have presented a simplified structure for the proposed scheme.

**REFERENCES**


