IMAGE SEGMENTATION IN A KERNEL-INDUCED SPACE

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ABSTRACT
A novel level set multiphase image segmentation method combined with kernel mapping is presented. A kernel function maps implicitly the original data into data of a higher dimension so that the piecewise constant model becomes applicable. The goal is to consider several types of noise by a single model. Gradient flow equations are iteratively derived in order to minimize the segmentation functional with respect to the partition, in a first step, and the regions parameters in a second step. Using a common kernel function, we verified the effectiveness of the method by a quantitative and comparative performance evaluation over experiments on synthetic images, as well as a variety of real images such as medical, SAR, and natural images.

Index Terms— Image segmentation, kernel mapping, mean shift, level set, multiphase

1. INTRODUCTION
Image segmentation is a central problem in computer vision and has been the subject of a considerable number of studies [1, 2]. Variational formulations [3], which express image segmentation as the minimization of a functional, have resulted in the most effective algorithms. This is mainly because they are amenable to the introduction of constraints on the solution. Conformity of region data to statistical models and smoothness of region boundaries are typical constraints. The piecewise constant model [3, 2] and the piecewise Gaussian generalization [4] have been the focus of most studies and applications because the ensuing algorithms reduce to iterations of computationally simple updates of segmentation regions and their model parameters. The more general Weibull model has also been investigated. Although they can be useful, these models are not generally applicable. For instance, synthetic aperture radar (SAR) images, of great importance in remote sensing, require the Rayleigh distribution model [5] and polarimetric images, common in remote sensing and medical imaging, the Wishart or the complex Gaussian model [6].

The use of accurate models in image segmentation is problematic for several reasons. First, modeling is notoriously difficult and time consuming [7]. Second, models are learned using a sample from a class of images and, therefore, are generally not applicable to the images of a different class. Finally, accurate models are generally complex and, as such, are computationally onerous, more so when the number of segmentation regions is large [6]. An alternative approach, which would not be prone to such problems, would be to transform the image data so that the piecewise constant model becomes applicable. This is typically what kernel functions can do, as several pattern classification studies have shown [8, 9]. A kernel function maps implicitly the original data into data of a higher dimension so that linear separation algorithms can be applied [8]. The mapping is implicit because the dot product, the Euclidean norm thereof, in the higher dimensional space of the transformed data can be expressed via the kernel function without explicit evaluation of the transform. Several studies [8] have shown evidence that the prevalent kernels in pattern classification are capable of properly clustering data of complex structure. In the view that image segmentation is spatially constrained clustering of image data [10], kernel mapping should be quite effective in segmentation of various types of images.

This study investigates level set multiphase image segmentation by kernel mapping and piecewise constant modeling of the image data thereof. The method uses an active curve objective functional containing two terms: an original term which evaluates the deviation of the mapped image data within each segmentation region from the piecewise constant model and a classic length regularization term for smooth region boundaries. Functional minimization is carried out by iterations of two consecutive steps: (1) minimization with respect to the partition by curve evolution via the Euler-Lagrange descent equations and (2) minimization with respect to the regions parameters via fixed point iteration. The latter leads, interestingly, to a mean shift update of the regions parameters. Using a common kernel function, we verified the effectiveness of the method by a quantitative and comparative performance evaluation over a large number of experiments on synthetic images, as well as diverse real images.

2. SEGMENTATION FUNCTIONAL IN A KERNEL-INDUCED SPACE
Let \( I : \Omega \subset \mathbb{R}^2 \rightarrow I \subset \mathbb{R}^+ \) be an image function. Segmenting the image \( I \) into \( N \) regions consists of finding a partition \( \{ R_i \}_{i=1}^N \) of the image domain such that each region is homogeneous with respect to some image characteristics. The image characteristics of a region are most commonly given in terms of statistical parametric models. To explain the role of the kernel function in the segmentation functional of this kernel method, and describe clearly the ensuing algorithm, we treat the case of a segmentation into two regions first (Section 2.1 and 2.2). In Section 2.3, a multiregion extension is described.

2.1. Two-region segmentation
The image data is generally non linearly separable. The basic idea in using a kernel function to transform the image data for image segmentation is as follows: rather than seeking accurate image models and addressing a non linear problem, we transform the image data implicitly via a kernel function so that the piecewise constant model becomes applicable and, therefore, solve a (simpler) linear problem.

Let \( \phi(\cdot) \) be a non-linear mapping from the observation space \( \mathcal{J} \) to a higher (possibly infinite) dimensional feature space \( \mathcal{F} \). Let...
\( \gamma(s) : [0, 1] \to \Omega \) be a closed planar parametric curve. \( \gamma \) divides the image domain into two regions: the interior of \( \gamma \) designated by \( \mathbf{R}_1 = \mathbb{R}^+_\mathbf{c} \), and its exterior \( \mathbf{R}_2 = \mathbb{R}^c_{\gamma} \). Solving the problem of segmentation in the kernel-induced space with curve evolution consists of evolving \( \gamma \) in order to minimize the functional \( \mathcal{F}_K \). The functional \( \mathcal{F}_K \) measures a kernel-induced non-Euclidian distances between the observations and the regions parameters \( \mu_1 \) and \( \mu_2 \):

\[
\mathcal{F}_K = \int_{\mathbf{R}_1} \| \phi(I(x)) - \phi(\mu_1) \|^2 dx + \int_{\mathbf{R}_2} \| \phi(I(x)) - \phi(\mu_2) \|^2 dx + \lambda \int ds.
\]

Following the Mercer’s theorem conditions [9], we do not have to know explicitly the mapping \( \phi \). Instead, we can use a continuous, symmetric, positive semi-definite kernel function, \( K(y, z) \), verifying:

\[
K(y, z) = \phi(y)^T \cdot \phi(z), \quad \forall (y, z) \in \mathbb{I}^2.
\]

Substitution of the kernel functions gives, for \( \mu \in \{ \mu_1, \mu_2 \} \):

\[
J_K(I(x), \mu_i) = \| \phi(I(x)) - \phi(\mu) \|^2 = K(I(x), I(x)) + K(\mu, \mu) - 2K(I(x), \mu).
\]

\( \mathcal{F}_K \) depends on \( \gamma \) and on the regions parameters \( \mu_1 \) and \( \mu_2 \). In general, the dependance of the integrand in the data term on \( \gamma \) can result in additional terms in the Euler-Lagrange equations [12]. Here, we adopt a two-step algorithm. The first step consists of fixing the curve and optimizing \( \mathcal{F}_K \) with respect to the parameters. The second step consists of evolving the curve with the parameters fixed.

2.1.1. Step 1

For a fixed partition of the image domain, the derivatives of \( \mathcal{F}_K \) with respect to \( \mu_i, i \in \{1, 2\} \) yield the following equations:

\[
\frac{\partial \mathcal{F}_K}{\partial \mu_i} = \int_{\mathbf{R}_i} \frac{\partial}{\partial \mu_i} \left[ K(\mu_i, \mu_i) - 2K(I(x), \mu) \right] dx.
\]

In all our experiments we used the radial basis function (RBF) kernel, a kernel which has been prevalent in pattern data clustering [11]:

\[
K(y, z) = \exp(-||y - z||^2/\sigma^2).
\]

With such kernel function, the necessary conditions for a minimum of \( \mathcal{F}_K \) with respect to region parameters are:

\[
\mu_i - g_{u_i}(\mu_i) = 0, \quad i \in \{1, 2\}
\]

where

\[
g_{u_i}(\mu_i) = \frac{\int_{\mathbf{R}_i} I(x)K(I(x), \mu_i)dx}{\int_{\mathbf{R}_i} K(I(x), \mu_i)dx}, \quad i \in \{1, 2\}
\]

The solution of (5) can be obtained by fixed point iterations. This consists of iterating

\[
\mu_{i,n+1} = g_{u_i}(\mu_{1,n}), \quad i \in \{1, 2\}, \quad n = 1, 2, \ldots
\]

The update of the region parameters obtained in (7) is a mean-shift update [13]. It is quite interesting that a mean-shift correction appears in this context of active curve segmentation. This correction occurs in the minimization with respect to the region parameters due to the kernel induced data term, via the RBF kernel. The effectiveness and flexibility of this kernel formulation and the ensuing mean-shift update will be confirmed by an extensive experimentation in section 3.

2.1.2. Step 2

With the region parameters fixed, this step consists of minimizing \( \mathcal{F}_K \) with respect to \( \gamma \). The Euler-Lagrange descent equation corresponding to \( \mathcal{F}_K \) is derived by embedding the curve \( \gamma \) into a family of one-parameter curves \( \gamma(s, t) : [0, 1] \times \mathbb{R}^+ \to \Omega \) and solving the following partial differential equation:

\[
\frac{d\gamma}{dt} = -\partial_{\mathcal{F}_K} \nabla \mathcal{F}_K.
\]

Segmentation regions \( \mathbf{R}_1 \) and \( \mathbf{R}_2 \) are obtained from curve \( \gamma \) at convergence, i.e., when time \( t \to \infty \). Using derivative results in [1], we have:

\[
\frac{d\gamma}{dt} = \left( J_K(I(x), \mu_1) - J_K(I(x), \mu_2) - \lambda \kappa \right) \bar{n},
\]

where \( \kappa \) is the mean curvature function of \( \gamma \).

2.2. Level set implementation

To implement the curve evolution in (9), we use the well-known level set method [14]. The evolving curve \( \gamma(t) \) is implicitly represented by the zero level set of a function \( u : \mathbb{R}^2 \to \mathbb{R} \): the curve \( \gamma(t) = \{ x \in \Omega | u(x, t) = 0 \} \). This representation is numerically stable and handles automatically topological changes of the evolving curve.

Following the results in [14], the level set function evolution corresponding to (9) is given by

\[
\frac{\partial u}{\partial t}(x, t) = \left( J_K(I(x), \mu_1) - J_K(I(x), \mu_2) - \lambda \kappa \right) \| \nabla u \|.
\]

2.3. Multiregion segmentation

Multiregion segmentation using several active curves can lead to ambiguity when two or more curves intersect. The main issue is to guarantee that the curves converge to define a partition of the image domain. There are several ways of generalizing a two-region segmentation functional to a multiregion functional to guarantee such a partition. Here, we use the implementation of the generalization described in [15] and used in other applications [5].

3. EXPERIMENTATION

To validate the proposed method, we first give a quantitative and comparative performance evaluation over a large number of experiments on synthetic images with various noise models and contrast parameters. The percentage of misclassified pixels (PMP) was used as a measure of segmentation accuracy.

3.1. Performance evaluation

The piecewise constant segmentation method and the piecewise Gaussian generalization have been the focus of most studies and applications [2] because of their tractability. In the following, evaluation of the proposed method, referred to as Kernelized Method (KM), is systematically supported by comparisons with the Piecewise Gaussian Method (PGM).

We first show two typical examples of our extensive testing with synthetic images and define the measures we used for performance analysis: the contrast and the percentage of misclassified pixels (PMP). Figs. 2(a)-(b) depict two versions of a two-region synthetic image, each perturbed with an exponential noise. Different noise...
parameters result in different amounts of overlap between the intensity distributions within the regions (Fig. 1). The larger the overlap, the more difficult the segmentation [16].

![Image intensity distributions: (a) small overlap; (b) significant overlap.](image1)

Fig. 1. Image intensity distributions: (a) small overlap; (b) significant overlap.

![Segmentation of two exponentially noisy images with different contrasts: (a)-(b) noisy images with different contrasts; (a1)-(b1) segmentation results with PGM; (a2)-(b2) segmentation results with KM. Image size: 128 x 128. λ = 2 for both methods.](image2)

Fig. 2. Segmentation of two exponentially noisy images with different contrasts: (a)-(b) noisy images with different contrasts; (a1)-(b1) segmentation results with PGM; (a2)-(b2) segmentation results with KM. Image size: 128 x 128. λ = 2 for both methods.

Figs. 2(a1), (b1) depict the segmentation results with the PGM. Because the actual noise model is exponential, segmentation quality obtained with the PGM was significantly affected with the second image in Fig. 2(b1). However, the KM yielded approximately the same result for both images (Fig. 2 a2, b2), although the second image undergoes a relatively significant overlap between the intensity distributions within the two regions (Fig. 1 b).

To demonstrate that the KM is a flexible and effective alternative to image modeling, we proceeded to a quantitative and comparative performance evaluation over a very large number of experiments. We run more than 100 experiments with a large set of synthetic tow-region images generated from various noise models and contrast values. The noise models we used include the Gaussian, exponential and Gamma distributions. Each image was generated from a combination of a noise model and a contrast value. The latter was measured by the Bhattacharyya distance between the intensity distributions within the two regions of the actual image [16]

\[
B = - \ln \int_{\mathbf{x} \in \mathbf{Z}} \sqrt{P_1(x)P_2(x)} \, dx,
\]

(11)

where \(P_1\) and \(P_2\) denote the intensity distributions within the two regions. Note that the higher the overlap, the lower the contrast. Each image was segmented by three methods: the PGM, the KM and segmentation with the correct model, i.e., the noise model used to generate the actual image. First, we applied both PGM and KM to the subset of images perturbed with a Gaussian noise, and plotted the PMP as a function of the contrast (Fig. 3 a). The higher the PMP, the higher the segmentation error. The KM yielded approximately the same error as segmentation with the correct model, i.e., the Gaussian model in this case. Second, we segmented all the images perturbed with an exponential noise with both PGM, KM and segmentation with the correct model, i.e., the exponential model in this case. We plotted the PMP as a function of the contrast in Fig. 3 (b). Both segmentation methods (KM and PGM) undergo high error gradients at some Bhattacharyya distance. Those results are consistent with the experiments in [17]. When \(B\) is superior to 0.8, both methods yield a low segmentation error with a PMP less than 1%. However, the KM outperforms the PGM for a considerable range of Bhattacharyya distance values. Furthermore, the KM yielded a performance similar to segmentation with the correct model (refer to Fig. 3 b) until the contrast becomes very small (\(B < 0.6\)). Similar experiments were run with the subset of images perturbed with a Gamma noise, and a similar behavior was noticed (refer to Fig. 3 c). These results demonstrate the ability of the KM to deal with various classes of image noises for a large range of contrast values, which relaxes assumptions as to the correct noise model.

![Evaluation of segmentation error for different methods: comparisons over the subset of synthetic images perturbed with a Gaussian noise in (a), the exponential noise in (b), and the Gamma noise in (c).](image3)

Fig. 3. Evaluation of segmentation error for different methods: comparisons over the subset of synthetic images perturbed with a Gaussian noise in (a), the exponential noise in (b), and the Gamma noise in (c).

The ability of the KM to deal with different noise models allows segmenting regions which require different models. To illustrate this important advantage, we consider a synthetic image of three regions with different noise models as shown in Fig. 4(a) with the initial curves in black and white. The clearer region is generated with a Gaussian noise, the grey region is derived from the Rayleigh distribution, and the darker region from the Poisson distribution. The final position of the curves following the KM is displayed in Fig. 4(b), and final segmentation, in Fig. 4(c), where each region is represented by its mean intensity value. As shown in Figs. 4(d)-(e), the Gaussian model gives incorrect results as expected. The results demonstrate the ability of our kernel method to discriminate different distributions within the same image.

![Image with different noise models: (a) initialization; (b) final position of curves; (c) final segmentation; (d)-(e) results with the PGM. Image size: 163 x 158. λ = 2.](image4)

Fig. 4. Image with different noise models: (a) initialization; (b) final position of curves; (c) final segmentation; (d)-(e) results with the PGM. Image size: 163 x 158. λ = 2.

### 3.2. Real Data

In this section, we test the proposed method with a representative sample of real images including natural, medical and satellite data.
Segmentation of a natural plane image into two regions is depicted in Fig. 5. Initial, intermediate and final positions of the evolving curve are displayed, respectively, in Figs. 5 (a), (b) and (c).

Fig. 5. A real plane image: (a) initialization; (b) intermediate curve evolution step; (c) final segmentation. Image size: 110 x 70. \( \lambda = 2 \).

Fig. 6. Monolook SAR image: (a) initialization; (b) final position of the curve; (c-d) final segmentations with KM and PGM. Image size: 151 x 361. \( \lambda = 2 \).

Fig. 6 depicts the result with a monolook SAR image characterized by a high multiplicative speckle noise. The noise level depends on the image data: the higher the intensity, the stronger the noise. Segmentation of this class of images is acknowledged as a difficult problem [5]. The final position of the evolving curve is shown in Fig. 6 (b). Both segmented regions, represented by their parameters, are displayed in Fig. 6(c).

Medical image segmentation is challenging and of a rapidly growing interest. The brain image shown in Fig. 7(a1) was segmented into three regions. The choice of the number of regions is based on prior medical knowledge. Fig. 7(a2) depicts very narrow human vessels with very small contrast in some spots. The curves obtained at convergence are displayed, for both images, in Figs. 7(b1) and (b2). Segmentation regions, represented by their parameters at convergence, are shown in Figs. 7(c1) and (c2).

4. CONCLUSION

In this study, we investigated image segmentation with kernel mapping. A kernel function is used in stead of several image data models. We presented several experiments on synthetic and real data which showed the effectiveness and the flexibility of the method.

5. REFERENCES