A Link-Node Nonlinear Complementarity Model for a Multiclass Simultaneous Transportation Dynamic User Equilibria

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ABSTRACT
Static transportation network equilibrium models have evolved from traditional sequential models to simultaneous (combined) models, and then to the multiclass simultaneous models to improve prediction of traffic flow. Most Dynamic Traffic Assignment (DTA) models, however, still deal only with the trip assignment step (traveler route choice) that is one of several steps in the transportation planning process. In this paper, the authors combine a dynamic link-node based discrete-time Nonlinear Complementarity Problem (NCP) DTA model with a static Multiclass Simultaneous Transportation Equilibrium Model (MSTEM) in a unified dynamic link-node based discrete-time NCP Dynamic Multiclass Simultaneous Transportation Equilibrium Model (DMSTEM) model. The new model improves the prediction process and eliminates inconsistencies that arise when the DTA or Dynamic Traffic Assignment with Departure Time (DTA-DT) is embedded in a more comprehensive transportation planning framework. An iterative solution algorithm for the proposed DMSTEM model is proposed by solving several relaxed NCPs in each iteration of the algorithm.

Keywords: Dynamic Traffic Assignment, Intelligent Transportation Systems, Multiclass Simultaneous Transportation Equilibrium Models, Nonlinear Complementarity Problem, Variational Inequality

INTRODUCTION

Dynamic Traffic Assignment (DTA) models, which aim to predict future dynamic traffic states in a short-term fashion, have been extensively studied for decades. DTA studies accelerated in the last fifteen years with the advent of intelligent transportation systems (ITS). Within DTA studies, the dynamic user equilibrium (DUE) problem, which tries to determine the distribu-
tion of time-dependent traffic flow of a traffic network assuming travelers follow rational route choices (such as to minimize travel time or other forms of cost), is one of the most challenging. Two distinct approaches have dominated the methodologies applied to DUE research: the simulation-based (microscopic/mesoscopic) approach and the analytical (macroscopic) approach. A review of these two approaches can be found in Ran and Boyce (1996); Peeta and Ziliaskopoulos (2001).

Among the analytical modeling techniques used for DUE, Variational Inequality (VI) and Nonlinear Complementarity (NC) have been shown to be the most effective approaches (Friesz et al., 1993; Ran & Boyce, 1996; Lo et al., 1996; Lo & Szeto, 2002; Chen, 1999; Heydecker & Verlander, 1999; Peeta & Ziliaskopoulos, 2001; Akamatsu, 2001; Wie et al., 2002). Ban et al. (2008) recently applied NCP to model DUE with exact flow propagation based on a specially designed discretization scheme. Friesz and Mookherjee (2006) developed a differential variational inequality (DVI) technique to model DUE in continuous time. The DVI framework has been used more recently by Friesz et al. (2010) to model both the within-day and day-to-day traffic dynamics, and by Pang et al. (2009) to model multi-class single bottleneck dynamic departure time choice problems.

In the DUE literature, elasticity of the dynamic demand (input to DUE models) is usually modeled as departure-time choices for a given total fixed demand flow between each O-D pair during the whole time horizon (Friesz et al., 1993). Multiclass DUE models have also been studied with these analytical approaches (Ran & Boyce, 1996; Belimer et al., 2003) or simulation-based approaches (Ben-Akiva et al., 2001; Mahmassani et al., 2001). The early studies, however, focused on different vehicle classes and a few social-economic factors of drivers, such as value of times (VOD) preferences (Lu et al., 2008). With this goal, Ramadurai and Ukkusuri (2010) integrate activity-based models for demand analysis into DUE. The model is solved using a super-network representation of activities and traffic networks.

By investigating further how socio-economic factors of drivers may affect trip generation, trip distribution, mode choice, and departure-times in a dynamic (i.e. time-dependent) fashion, and integrating the findings into DUE modeling, we expect to improve the practicality and accuracy of DUE models. With this objective, we combine the NCP-based DUE model in Ban et al. (2008) and the Multiclass Simultaneous Transportation Equilibrium Model (MSTEM) in Hasan and Dashti (2007) to develop a multiclass combined Trip Generation (TG)-Trip Distribution (TD)-Nested Modal Split (NMS)-Modal Split (MS)-Dynamic Trip Assignment (DTA)-Departure Time (DT) Model (NCPDUE-TG-TD-NMS-MS-DTA-DT). In the next paragraphs we provide the necessary background on static multiclass combined models as employed in the Multiclass Simultaneous Transportation Equilibrium Model (MSTEM) that we utilize in this paper.

Single-class static combined equilibrium models consider all travelers as one homogeneous group with respect to their travel-decision characteristics (particularly their money-value of time and sensitivity to travel times in choosing their origin, destination and mode of travel, etc. To be more realistic, travelers must often be divided into classes -- either by socio-economic attributes or purpose of travel. Travel-decision characteristics are thus the same within each class, but differ among classes.

The introduction of multiple classes increases the mathematical complexity of travel forecasting models. Travel costs in single class models are often separable and symmetric, allowing for convex optimization formulation. In multiclass models, travel costs of one class are affected by decisions of other classes; hence the cost structure is not separable, asymmetric, and not amenable to a convex optimization formulation (Patriksson, 2003). Boyce and Barger (2004) review the most recent multiclass combined models and provide a good detailed comparison of the implementation of four multiclass combined models. Boyce (2004) presents the most recent comprehensive review and prospects for network equilibrium models.
As summarized by Hasan and Dashti (2007), researchers in Chile began to implement multiclass combined models in 1986 -- emphasizing route choices in a congested transit network with several combinations of transit modes, as found in Santiago (de Cea et al., 2003). This research led to the development of ESTRAUS and related software, which has been extensively applied to Santiago as well as other Chilean cities. Florian, We and He (2002) proposed a variant of ESTRAUS intended to be more efficient computationally.

Although several works concerning departure time choices are reported in the technical literature, this travel decision has not been integrated yet to supply-demand equilibrium models. Dekock (2001) and Dekock et al. (2001) have described a simultaneous equilibrium model considering trip distribution, modal split and departure time choices. The basic idea of this model is that even if trip generations (and attractions) are fixed for a given period of time, users can choose the sub-period in which they travel, according to a Logit model. The ESTRAUS combined model considers a doubly constrained entropy-maximizing model, while modal split and departure time choices are modeled with a hierarchical Logit model. Based on these works, two different models, depending on the relative values of the calibration parameters of the Logit model, were developed but not used yet in any implementation.

All existing multiclass combined models, consider that the total originating and terminating flows are known, i.e., the trip generation step of transportation planning process is exogenous to the combined prediction process. This deficiency was addressed in the STEM model which is the only model that combined the trip generation in the prediction process, but it is not a multiclass model. The STEM model was extended to incorporate multiple user classes (socio-economic groups), trip purposes, and both pure and combined transportation modes, interacting over a physically unique network. The resulting Multiclass Simultaneous Transportation Equilibrium Model (MSTEM) also combined explicitly the departure time as one of the main components of the prediction process for the first time and will be considered as a new generation of new Case 6 of the multiclass model classification of Boyce and Bar-Gera (2004).

The MSTEM included all the features of ESTRAUS in addition to the other features mentioned above and a more flexible structure for demand models where the trip generation can depend upon the system’s performance through an accessibility measure that is based on the random utility theory of users’ behavior (instead of being fixed as in ESTRAUS). The trip distribution is determined by a more behaviorally richer Multinomial Logit (MNL) model based on the random utility theory (instead of being given by an entropy maximization model in ESTRAUS), and modal split and departure time are given by Multinomial Logit (MNL) models based on the random utility theory (instead of hierarchical Logit for modal split used in ESTRAUS).

In the next section we introduce the discrete-time NCP-based DUE model in Ban et al. (2008). Section 3 includes the brief description of the Multiclass Simultaneous Transportation Equilibrium Model (MSTEM) of Hasan and Dashti (2007). Section 4 gives a detailed development of discrete-time domain finite-dimensional Nonlinear Complementarity DUE Multiclass Simultaneous Transportation Equilibrium Model (DMSTEM). Section 5 covers in detail the solution algorithm of the NCP that was formulated in section 4. The where the appendix gives a detailed description of the relaxed sub-problems that should be solved by the algorithm. In Section 6 the main conclusions of the paper are presented along with some directions of future research.

**The Nonlinear Complementarity Formulation for DUE**

In this paper, we adopt the NCP model in Ban et al. (2008). In particular, we show how to extend it to DUE with multiclass trip generation, trip distribution, modal split, and departure time choices. Our focus here is the finite-dimension-
al NCP model in discrete-time domain. In order to obtain the discrete-time model, Ban et al. (2008) evenly divided the entire study period into \( K' \) time intervals by introducing the length of each time interval \( \Delta \) such that \( K' \Delta = T' \). We use index \( k \) to denote the \( k \)-th time interval \( [(k-1)\Delta, k\Delta) \) for \( 1 \leq k \leq K' \).

The notation for the discrete-time model is first listed as follows (Ban et al., 2008):

\[
\begin{align*}
\nu_{ik}^k &= u_{ik}^k((k-1)\Delta) : \text{The inflow rate to link } a \text{ towards destination } s \text{ at the beginning of time interval } k, \text{ which is also assumed to be constant during the entire time interval } k, v = (v_{ik}^k)_{a,s,k} . \\
v_{ik}^k &= v_{ik}^k((k-1)\Delta) : \text{The exit flow rate from link } a \text{ towards destination } s \text{ at the beginning of time interval } k, \text{ which also assumed to be constant during the entire time interval } k, v = (v_{ik}^k)_{a,s,k} . \\
u_{ik}^k \Delta &= \text{The inflows to link } a \text{ towards destination } s \text{ during time interval } k . \\
v_{ik}^k &= \text{The exit flows to link } a \text{ towards destination } s \text{ during time interval } k . \\
x_{ik}^k &= x_{ik}^k((k-1)\Delta) : \text{The flows of link } a \text{ towards destination } s \text{ at the beginning of time interval } k, \text{ which also assumed to be constant during the entire time interval } k, x = (x_{ik}^k)_{a,s,k} . \\
u_k &= \sum_{s \in S} u_{ik}^k : \text{The aggregated inflow rate to link } a \text{ at the beginning of time interval } k, u^k = (u_{ik}^k)_{a,k} . \\
v_k &= \sum_{s \in S} v_{ik}^k : \text{The aggregated exit flow rate from link } a \text{ at the beginning of time interval } k, v^k = (v_{ik}^k)_{a,k} . \\
x_k &= \sum_{s \in S} x_{ik}^k : \text{The aggregated flows of link } a \text{ towards destination } s \text{ at the beginning of time interval } k, \text{ which assumed to be constant during the entire time interval } k, x^k = (x_{ik}^k)_{a,k} . \\
d_k &= \sum_{s \in S} d_{is}^k((k-1)\Delta) : \text{The demand rate generated from node } i \text{ to destination } s \text{ at the beginning of time interval } k, \text{ which assumed to be constant during the entire time interval } k, d = (d_{is}^k)_{s,k,i} , \text{ and } d_s^k = \sum_{i \in S} d_{is}^k((k-1)\Delta) . \\
\pi_{ik}^k &= \pi_{ik}((k-1)\Delta) : \text{The minimum travel time from node } i \text{ to destination } s \text{ at the end of time interval } k, \pi = (\pi_{ik}^k)_{i,s,k,i} . \\
\phi_{ik}^k &= \phi_{ik}((k-1)\Delta) + C_{ik}^k-1(u) : \text{The exit time for vehicles entering link } a \text{ at the beginning of time interval } k, \text{ a function of } u, e = (\phi_{ik}^k)_{a,k} .
\end{align*}
\]

Note that in the above notation, \( u, v, x, \) and \( d \) are defined at the beginning of a time interval, while \( C \) and \( \pi \) are defined at the end of the time interval. This is to associate the inflow to a link at the beginning of any time interval \( k \) with the cost (travel time) at the end of the time interval in DUE route choice. As already discussed in Heydecker and Verlander (1999), this way of associating inflow and cost will lead to predictive DUE that has more plausible solution properties compared with reactive DUE. For more details of the concepts of predictive and reactive DUE, one can refer to Heydecker and Verlander (1999), Han (2000), and Han (2003).

Ban et al. (2008) showed that the above discretization scheme leads to the following NCP formulation for DUE with fixed demand: find \((u, \pi)\) such that:

\[
\begin{align*}
0 &\leq u \perp \Phi_u(u, \pi) \geq 0 \\
0 &\leq \pi \perp \Phi_\pi(u, \pi) \geq 0
\end{align*}
\]
Box 1.

\[
\Phi_a(u, \pi) = \left\{ \begin{array}{l}
C_a^k(u) + \\
l - 1 \leq e_a^{k+1}(u) / \Delta < l
\end{array} \right., \forall a, s, k
\]

\[
\Phi_{\pi}(u, \pi) = \left\{ \begin{array}{l}
\sum_{a \in A(i)} u_a - d_i - \sum_{a \in B(i)} e_a(u) \sum_{a \in B(l)} e_a'(u) \left[ \lambda_a^{k+1}(u) \lambda_a^{2,k,e}(u) - (1 - \lambda_a^{2,k,e}(u)) \lambda_a^{1+1}(u) \right]
\end{array} \right., \forall k, s, i = s, k
\]

where the two functions \( \Phi_a(u, \pi) \) and \( \Phi_{\pi}(u, \pi) \) are defined as seen in Box 1, where \( e_a^k(u) = (k-1)\Delta + C_a^k(u) \) is the exit time of vehicles entering \( a \) at the beginning of the \( k \)-th interval, which is a function of the inflow vector \( u \) since \( C_a^k(u) \) is so. He also defines three types of indicator functions on \( u \): \( \lambda_a^{1,k'}(u) \), \( \lambda_a^{2,k,k'}(u) \) and \( \lambda_a^{3,k,q}(u) \). First, flow propagation constraints can be represented by \( \lambda_a^{1,k'}(u) \), which is defined as:

\[
\lambda_a^{1,k'}(u) = \frac{\Delta}{C_a^{k'}(u) - C_a^{(k-1)}(u) + \Delta}, \forall a, k'
\]

The relation between link inflow and exit flow rates can be represented by both \( \lambda_a^{1,k'}(u) \) and \( \lambda_a^{2,k,k'}(u) \) with the latter defined as:

\[
\lambda_a^{2,k,k'}(u) = \frac{e_a^k(u) + (k' + 1 - k)\Delta}{C_a^{k'}(u) - C_a^{(k-1)}(u) + \Delta}, \forall a, k, k'
\]

\[
e_a^{k'}(u) \leq (k-1)\Delta < e_a^{(k+1)}(u), \forall a, k'
\]

Similarly, \( \lambda_a^{3,q}(u) \) is used to discretize (and interpolate) the minimum travel time \( \pi_{h,i} \) in equation (1). It can be defined as:

\[
\lambda_a^{3,q}(u) = q - k - \frac{C_a^k(u)}{\Delta}, \forall q - 1 \leq \frac{e_a^{(k+1)}(u)}{\Delta} < q
\]

The three indicator functions satisfy the following:

\[
0 < \lambda_a^{3,q}(u) \leq 1, \forall a, k', \lambda_a^{2,k,k'}(u) \leq (k-1)\Delta < e_a^{(k+1)}(u)
\]

\[
0 < \lambda_a^{3,k,q}(u) \leq 1, \forall q - 1 \leq \frac{e_a^{(k+1)}(u)}{\Delta} < q
\]

\( \Phi_u \) is a vector function for any combination of link \( a \), destination \( s \), and time interval \( k \). Each of its component represents, for the given \( a; s; k \), the difference of the minimum travel times via two sets of paths from the starting node (or tail node) of link \( a \) to destination \( s \) at \( k \). The first set of paths must traverse link \( a \), which is a subset of the second set that includes all paths from the starting node of \( a \) to destination \( s \). This difference should be zero if link \( a \) is selected (i.e. the inflow to link \( a \) at time \( k, u_a^k \), is nonzero). \( \Phi_{\pi} \), on the other hand, simply represents the flow conservation constraint at any node \( i \), destination \( s \neq i \) and time.
Detailed discussion on how $\Phi_u$ and $\Phi_\pi$ are derived can be found in Ban et al. (2008), which also showed (Theorem 3) that $NCPDUE$ has a nonempty and compact solution set under certain conditions.

**MULTICLASS SIMULTANEOUS TRANSPORTATION EQUILIBRIUM MODEL (MSTEM)**

In this section we give a brief description of MSTEM model that developed by Hasan and Dashti (2007). The notation for MSTEM model is first listed as follows:

$(N, A) =$ A multimodal traffic network consisting of a set of $N$ nodes and a set of $A$ links

$l =$ User class (e.g., income level, car availability, etc.)

$L =$ Set of all user classes

$o =$ Trip purpose (e.g., home-based-work, home based-shopping, etc.)

$O =$ Set of all trip purpose

$I_{lo} =$ Set of origin nodes for user class $l$ and trip purpose $o$

$i =$ An origin node in the set $I_{lo}$ for user class $l$ with trip purpose $o$

$D_{i}^{lo} =$ Set of destination nodes that are accessible from a given origin $i$ for user class $l$ with trip purpose $o$

$s =$ A destination node in the set $D_{i}^{lo}$ for user class $l$ with trip purpose $o$

$R_{lo} =$ Set of origin-destination pairs $is$ for user class $l$ with trip purpose $o$, i.e., the set of all origins $i \in I_{lo}$ and destinations $s \in D_{i}^{lo}$

$m =$ Any transportation mode in the urban area

$n =$ Nest of transportation modes $m$ that has a specific characteristics (e.g., pure modes including private and public or combined modes) those are available for user class $l$ with trip purpose $o$ travel between origin-destination pairs $is$

$A_{lo} =$ Set of all nests of modes $n$ that are available for user class $l$ with trip purpose $o$ travel between origin-destination pairs $is$

$M_{lo}^{n} =$ Set of all transportation modes $m$ in the nest $n$ for user class $l$ with trip purpose $o$ travel between origin-destination pairs $is$

$k =$ Departure time period for user class $l$ with trip purpose $o$ using mode $m$ in the nest $n$ to travel between origin-destination pairs $is$

$K_{m}^{lo} =$ Time horizon of the departure time periods $k$ for users of class $l$ with trip purpose $o$ using mode $m$ between origin-destination pairs $is$

$a =$ A link in the set $A$ in the multimodal network $(N, A)$

$C_{a}^{lo} = \left( u_{l}^{lo} m \right) =$ the travel time of vehicles entering link $a$ at the end of the $kth$ time interval (=Departure time period $k$) for user class $l$ with trip purpose $o$ using mode $m$ in the nest $n$ to travel between origin-destination pairs $is$

$A_{ws}^{lo} =$ the value of the $wth$ socio-economic variable that influences trip attraction at destination $s$ for users of class $l$ with trip purpose $o$

$A_{s}^{lo} =$ $A_{s}^{lo} = \sum_{w=1}^{W} q_{w}^{lo} d_{w}^{lo} (A_{ws}^{lo}) =$ a composite measure of the effect that socio-economic variables, which, are exogenous to the transport system, have on trip attraction at destination $s$ for users of class $l$ with trip purpose $o$. 

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\( g_w(A_w^l) \) = a given function specifying how the \( w \)th socio-economic variable \( A_w^l \) influences trip attraction at destination \( s \) for users of class \( l \) with trip purpose \( o \), and The quantities \( \theta_i^o \) and \( \theta_{iw}^l \) for \( w = 1, 2, \ldots, W \) are coefficients to be estimated, where \( \theta_i^o > 0 \).

During the time period required to achieve the short-run equilibrium predicted in the model, socio-economic activities in the system will remain essentially unchanged; the composite effect \( A_j^l \) for users of class \( l \) with trip purpose \( o \) of these activities is assumed to be a fixed constant.

\( q_i^o(E_i^l) \) = a given function specifying how the \( w \)th socio-economic variable, \( E_i^l \), influences the number of trips generated from origin \( i \) for users of class \( l \) with trip purpose \( o \), and

\[
E_i^l = \sum_{\omega=1}^{\Omega} \alpha_{\omega i}^l q_i^o(E_i^l) = a \text{ composite measure of the effect the socio-economic variables, which are exogenous to the transport system, have on the number of trips generated from origin } i \text{ for users of class } l \text{ with trip purpose } o.
\]

The quantities \( \alpha_i^o \) and \( \alpha_{\omega i}^l \) for \( \omega = 1, 2, \ldots, \Omega \) are coefficients to be estimate \( \forall l \in L, \forall o \in O \).

Similar to \( A_s^l \), \( E_s^l \) is assumed to be a fixed constant during the time period required to achieve short-run equilibrium.

\( n_s^{lom} \) = the inflow rate to link \( a \) towards destination \( s \) at the beginning of the \( k \)th time interval for user class \( l \) with trip purpose \( o \) using mode \( m \) in the nest \( n \), which is assume to be constant during the entire \( k \)th time interval \( \pi_{s}^{lomk} \) = the minimum travel time from node \( i \) to \( s \) destination at the end of time interval \( k \) for user class \( l \) with trip purpose \( o \) using mode \( m \) in the nest \( n \).

\( S_i^o \) = the accessibility of origin \( i \in I_s^o \) as perceived from user of class \( l \) with trip purpose \( o \) traveling from that origin.

\( G_i^o \) = the number of trips generated from origin \( i \) for users of class \( l \) with trip purpose \( o \) traveling from the origin node \( \in I_s^o \) to the destination node \( s \in D_i^o \) and whose already chose the mode of transport \( m \in M_i^s \) from the nest of modes \( n \in \Lambda_s^o \) and start their trip at the time interval \( k \in K_m^o \).

\( d_{is}^{lom} \) = the number of trips of users of class \( l \) with trip purpose \( o \) traveling from the origin node \( i \in I_s^o \) to the destination node \( j \in D_i^o \) and whose already chose the mode of transport \( m \in M_i^s \) from the nest of modes \( n \in \Lambda_s^o \) and whose already chose the nest of modes \( n \in \Lambda_s^r \).

\( d_{is}^{lo} \) = the number of trips of users of class \( l \) with trip purpose \( o \) traveling from the origin node \( i \in I_s^o \) to the destination node \( i \in I_s^o \) and whose already chose the nest of modes \( n \in \Lambda_s^o \).  

\( d_{is}^{lo} \) = the number of trips of users of class \( l \) with trip purpose \( o \) traveling from the origin node \( i \in I_s^o \) to the destination node \( s \in D_i^o \) and whose already chose the nest of modes \( n \in \Lambda_s^o \).

Using the above notations the MSTEM model can be expressed by Box 2.
Box 2. MSTEM

\[
S^{lo}_{is} = \max \left\{ 0, \ln \sum_{s \in D^o_i} \sum_{n \in N^o_i} \sum_{m \in M^o_i} \sum_{k \in K^o_i} \exp(-\theta_i^{lo} n_{is}^{lomnk} + A_s^{lo}), \forall i \in I^{lo}, \forall l \in L, \forall o \in O \right\}
\]

\[
G_i^{lo} = \alpha_i^{lo} S_i^{lo} + E_i^{lo}, \forall i \in I^{lo}, \forall l \in L, \forall o \in O
\]

\[
d^{lo}_{is} = G_i^{lo} \sum_{s \in D^o_i} \sum_{n \in N^o_i} \sum_{m \in M^o_i} \sum_{k \in K^o_i} \exp(-\theta_i^{lo} n_{is}^{lomnk} + A_s^{lo}), \forall i \in I^{lo}, \forall l \in L, \forall o \in O
\]

\[
d^{lo}_{is} = d^{lo}_{is} \sum_{n \in N^o_i} \sum_{m \in M^o_i} \sum_{k \in K^o_i} \exp(-\theta_i^{lo} n_{is}^{lomnk} + A_s^{lo}), \forall i \in I^{lo}, \forall l \in L, \forall o \in O
\]

\[
d^{lo}_{is} = d^{lo}_{is} \sum_{m \in M^o_i} \sum_{k \in K^o_i} \exp(-\theta_i^{lo} n_{is}^{lomnk} + A_s^{lo}), \forall i \in I^{lo}, \forall l \in L, \forall o \in O
\]

\[
d^{lo}_{is} = d^{lo}_{is} \sum_{k \in K^o_i} \exp(-\theta_i^{lo} n_{is}^{lomnk} + A_s^{lo}), \forall k \in K^o_i, \forall m \in M^o_i, \forall n \in N^o_i, \forall i \in I^{lo}, \forall l \in L, \forall o \in O
\]

\[
d^{lo}_{is} = (\alpha_i^{lo} S_i^{lo} + E_i^{lo}) \sum_{j \in D^o_i} \sum_{n \in N^o_i} \sum_{m \in M^o_i} \sum_{k \in K^o_i} \exp(-\theta_i^{lo} n_{is}^{lomnk} + A_s^{lo}), \forall i \in I^{lo}, \forall l \in L, \forall o \in O
\]

See Hasan and Dashit (2007) for more details about MSTEM model.

**DISCRETE-TIME DOMAIN FINITE-DIMENSIONAL NONLINEAR COMPLEMENTARITY DUE MULTICLASS SIMULTANEOUS TRANSPORTATION EQUILIBRIUM MODEL (DMSTEM)**

Following the same thought of Ban et al. (2008), the dynamic user equilibrium traffic assignment (NCPDUE-DTA) can be represented by Box 3.

To extend the DTA model above to model simultaneous route and departure time, we assume the early/late arrival penalty function for each user class \(l\) with trip purpose \(O\) using mode \(m\) in the nest \(n\) to travel between origin-destination pairs is departing at time \(k\) (denoted as \(F_{is}^{lomnk}\)) as:

\[
F_{is}^{lomnk} = 0.5 \left( k \nabla + \pi_{is}^{lomnk} - T_{is}^{lomnk} \right)^2, \forall i, s, k, i \neq s \text{ and } \forall l, o, n, m
\]

where \(T_{is}^{lomnk}\), \(\forall i, s, k, i \neq s\) is the desired arrival time for user class \(l\) with trip purpose \(O\) using mode \(m\) in the nest \(n\) to travel between origin-destination \(is\) departing at time \(k\). Therefore, \(k \nabla + \pi_{is}^{lomnk} - T_{is}^{lomnk}\) represents early or late arrival for travelers of class \(l\) with trip purpose \(O\) using mode \(m\) in the nest \(n\) departing \(i\) to \(s\) at time \(k\). In other words, the cost (disutility) for such a traveler at time \(k\) is
Further, we denote the minimum disutility between \(i\) and \(s\) for user class \(l\) with trip purpose \(o\) using mode \(m\) in the nest \(n\) (for all time intervals) as

\[
\mu_{ls} = \min_{k} \psi_{ls}^{L_{mn}}, \quad \forall i, s, k, i \neq s, \forall l, o, n, m
\]

Therefore, the departure time choice condition can then be expressed as:

\[
d_{ls}^{L_{mn}} \geq 0, \quad \text{if } \psi_{ls}^{L_{mn}} = \mu_{ls}, \quad \forall i, s, k, i \neq s, \forall l, o, n, m
\]

This condition simply states that travelers are trying to minimize their disutility when choosing departure times.

Combining equations (6) and (7) with the following two NCPs we can combined the dynamic traffic assignment with the departure time (NCPT-DUE-MS-DTA-DT as seen in Box 4).

Similarly, combining equations (6)-(9) with the following two NCPs we can combined the Dynamic Traffic Assignment, Departure Time and Modal Split (NCPT-DUE-MS-DTA-DT as seen in Box 5).

Combining equations (6)-(11) with the following two NCPs we can combined the Dynamic Traffic Assignment, Departure Time,
Modal Split, and Nested Modal Split (NCPDUE-NMS-MS-DTA-DT as seen in Box 6).

Combining equations (6)-(13) with the following two NCPs we can combined the Dynamic Traffic Assignment, Departure Time, Modal Split, Nested Modal Split and Trip Distribution (NCPDUE-TD-NMS-MS-DTA-DT as seen in Box 7).

Combining equations (6)-(15) with the following two NCPs we can combined the Dynamic Traffic Assignment, Departure Time, Modal Split, Nested Modal Split, Trip Distribution, and Trip Generation (NCPDUE-TG-TD-NMS-MS-DTA-DT as seen in Box 8). where the decision variables are defined as follows:

\[
\begin{align*}
    &\mathbf{u}^{lonm} = \left( \mathbf{u}^{lonm}_{us}, \forall a, s, k \right), \forall l, o, n, m, \\
    &\pi^{lonm} = \left( \pi^{lonm}_{is}, \forall i, s, k, i = s \right), \forall l, o, n, m \\
    &\mathbf{d}^{lonm} = \left( \mathbf{d}^{lonm}_{is}, \forall i, s, k, i = s \right), \forall l, o, n, m, \\
    &\mathbf{\mu}^{lonm} = \left( \mathbf{\mu}^{lonm}_{is}, \forall i, s, i = s \right), \forall l, o, n, m \\
\end{align*}
\]

Similarly to the formulation of Ban et al. (2008), all \( \Phi \) s functions can be defined as seen in Box 9.

Box 6.

\[
0 \leq d^{lo} \perp \Phi_{d^{lo}} \left( \mathbf{u}^{lonm}, \pi^{lonm}, \mathbf{d}^{lonm}, \mathbf{\mu}^{lonm}, \mathbf{d}^{lonm}, \mathbf{\mu}^{lonm}, \mathbf{d}^{lo}, \mathbf{\mu}^{lo}, \mathbf{d}^{lo}, \mathbf{\mu}^{lo}, \mathbf{S}^{lo}_{R^{lo}}, \mathbf{\mu} \right) \geq 0, \forall l, o \tag{12}
\]

\[
0 \leq \mathbf{\mu}^{lo} \perp \Phi_{\mathbf{\mu}^{lo}} \left( \mathbf{u}^{lonm}, \pi^{lonm}, \mathbf{d}^{lonm}, \mathbf{\mu}^{lonm}, \mathbf{d}^{lonm}, \mathbf{\mu}^{lonm}, \mathbf{d}^{lo}, \mathbf{\mu}^{lo}, \mathbf{d}^{lo}, \mathbf{\mu}^{lo}, \mathbf{S}^{lo}_{R^{lo}}, \mathbf{\mu} \right) \geq 0, \forall l, o \tag{13}
\]

Box 7.

\[
0 \leq d^{lo}_{R^{lo}} \perp \Phi_{d^{lo}_{R^{lo}}} \left( \mathbf{u}^{lonm}, \pi^{lonm}, \mathbf{d}^{lonm}, \mathbf{\mu}^{lonm}, \mathbf{d}^{lonm}, \mathbf{\mu}^{lonm}, \mathbf{d}^{lo}, \mathbf{\mu}^{lo}, \mathbf{d}^{lo}, \mathbf{\mu}^{lo}, \mathbf{S}^{lo}_{R^{lo}}, \mathbf{\mu} \right) \geq 0, \forall l, o \tag{14}
\]

\[
0 \leq \mathbf{\mu}^{lo}_{R^{lo}} \perp \Phi_{\mathbf{\mu}^{lo}_{R^{lo}}} \left( \mathbf{u}^{lonm}, \pi^{lonm}, \mathbf{d}^{lonm}, \mathbf{\mu}^{lonm}, \mathbf{d}^{lonm}, \mathbf{\mu}^{lonm}, \mathbf{d}^{lo}, \mathbf{\mu}^{lo}, \mathbf{d}^{lo}, \mathbf{\mu}^{lo}, \mathbf{S}^{lo}_{R^{lo}}, \mathbf{\mu} \right) \geq 0, \forall l, o \tag{15}
\]

Box 8.

\[
0 \leq S^{lo}_{R^{lo}} \perp \Phi_{S^{lo}_{R^{lo}}} \left( \mathbf{u}^{lonm}, \pi^{lonm}, \mathbf{d}^{lonm}, \mathbf{\mu}^{lonm}, \mathbf{d}^{lonm}, \mathbf{\mu}^{lonm}, \mathbf{d}^{lo}, \mathbf{\mu}^{lo}, \mathbf{d}^{lo}, \mathbf{\mu}^{lo}, \mathbf{S}^{lo}_{R^{lo}}, \mathbf{\mu} \right) \geq 0, \forall l, o \tag{16}
\]

\[
0 \leq \mathbf{\mu} \perp \Phi \left( \mathbf{u}^{lonm}, \pi^{lonm}, \mathbf{d}^{lonm}, \mathbf{\mu}^{lonm}, \mathbf{d}^{lonm}, \mathbf{\mu}^{lonm}, \mathbf{d}^{lo}, \mathbf{\mu}^{lo}, \mathbf{d}^{lo}, \mathbf{\mu}^{lo}, \mathbf{S}^{lo}_{R^{lo}}, \mathbf{\mu} \right) \geq 0, \forall l, o \tag{17}
\]
Theorem: The NCPDUE-TG-TD-NMS-MS-DTA-DT model (6) - (17) has a nonempty and compact solution set if the following four conditions are satisfied.

a. The link travel time is positive and finite for any finite u;

b. $\lambda^k$ is bounded from above;

c. $T^{l_{omk}}_{s_{is}} \forall i, s, k, i \neq s$ and $\forall I, o, n, k$ is positive and bounded from above; and

d. $\Phi_{d_{is}}^{lsomk}$, $\Phi_{\pi_{is}}^{l_{omk}}$, $\Phi_{\lambda_{is}}^{l_{omk}}$, $\Phi_{\mu_{is}}^{l_{omk}}$, $\Phi_{\alpha_{is}}^{l_{omk}}$, $\Phi_{\beta_{is}}^{l_{omk}}$, $\Phi_{\gamma_{is}}^{l_{omk}}$, $\Phi_{\delta_{is}}^{l_{omk}}$, $\Phi_{\epsilon_{is}}^{l_{omk}}$, $\Phi_{\phi_{is}}^{l_{omk}}$, $\Phi_{\psi_{is}}^{l_{omk}}$, $\Phi_{\chi_{is}}^{l_{omk}}$, $\Phi_{\delta_{is}}^{l_{omk}}$, $\Phi_{\epsilon_{is}}^{l_{omk}}$, $\Phi_{\phi_{is}}^{l_{omk}}$, $\Phi_{\psi_{is}}^{l_{omk}}$, and $\Phi_{\mu_{is}}^{l_{omk}}$ are continuous with respect to

\[
\begin{align*}
(u_{lomk}, \lambda_{lomk}, \mu_{lomk}, d_{lomk}, \mu_{lomk}, \lambda_{lomk}, \mu_{lomk}, d_{lomk}, \mu_{lomk}, \lambda_{lomk}, \mu_{lomk}, S_{l_{omk}}^{l_{omk}}, \mu_{l_{omk}}) = \\
(\sum_{i \neq j} \frac{d_{l_{omk}}^{lsomk}}{\sum_{s \neq o}} \left[\left[ \sum_{i \neq j} \left[ \lambda_{l_{omk}} \left( u_{l_{omk}}^{lomk} \right) \lambda_{l_{omk}} \left( u_{l_{omk}}^{lomk} \right) \lambda_{l_{omk}} \left( u_{l_{omk}}^{lomk} \right) \right] \right] \right] \right) \forall I, o, n, m
\end{align*}
\]
Box 10.

\[
\Phi_{d_{is}}(u_{lom}, \pi_{lom}, d_{lom}, \mu_{lom}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, \mu_{lo}, \mu_{lo}, S_{lo}, \mu) = \\
\left(\left[\mu_{lom} - \mu_{lo}\right], \forall i, s, m, i = s\right) \\
\forall l, o, n
\]

\[
\Phi_{i_{lo}}(u_{lom}, \pi_{lom}, d_{lom}, \mu_{lom}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, \mu_{lo}, \mu_{lo}, S_{lo}, \mu) = \\
\left(\left[\mu_{lom} - \mu_{lo}\right], \forall i, s, n, i = s\right) \\
\forall l, o
\]

\[
\Phi_{d_{is}}(u_{lom}, \pi_{lom}, d_{lom}, \mu_{lom}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, \mu_{lo}, \mu_{lo}, S_{lo}, \mu) = \\
\left(\left[\mu_{lom} - \mu_{lo}\right], \forall i, s, m, i = s\right) \\
\forall l, o, n
\]

\[
\Phi_{s_{lo}}(u_{lom}, \pi_{lom}, d_{lom}, \mu_{lom}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, S_{lo}, \mu) = \\
\left(\left[\mu_{lom} - \mu_{lo}\right], \forall i, s, n, i = s\right) \\
\forall l, o
\]

\[
(d_{is} - d_{lo}) \left(\sum_{n \in N_{is}^l} \sum_{m \in M_{is}^l} \sum_{k \in K_{is}^l} \exp(-\theta_{is}^l \pi_{lom} \mu + A_{is}^l)\right) = 0, \forall n \in N_{lo}, \forall s \in R_{is}, \forall l \in L, \forall o \in O
\]

\[
(d_{lo} - (\alpha_{lo} S_{lo} + E_{lo})) \left(\sum_{n \in N_{lo}} \sum_{m \in M_{lo}} \sum_{k \in K_{lo}} \exp(-\theta_{lo} \pi_{lom} \mu + A_{lo}^l)\right) = 0, \forall s \in R_{lo}, \forall l \in L, \forall o \in O
\]

\[
(|S_{lo} - \max\left\{0, \ln \sum_{n \in D_{lo}} \sum_{m \in M_{lo}} \sum_{k \in K_{lo}} \exp(-\theta_{lo} \pi_{lom} \mu + A_{lo}^l)\right\}| = 0, \forall i \in I_{lo}, \forall l \in L, \forall o \in O
\]

mainly due to the fact, as shown in the definitions of functions \(\Phi_{u_{lom}}, \Phi_{i_{lo}}, \Phi_{d_{is}}, \Phi_{i_{lo}}\), \(\Phi_{d_{lo}}, \Phi_{\mu_{lo}}, \Phi_{p_{lo}}, \Phi_{i_{lo}}, \Phi_{s_{lo}}, \) and \(\Phi_{\mu}\) in Equations (18)-(19), that the model is not close-formed. However, for a given demands vectors \(\tilde{d}_{lom}, \tilde{d}_{lom}, \tilde{d}_{lo}, \tilde{d}_{lo}, \tilde{S}_{lo}\) (called base demand) and its resulting inflow vector \(\tilde{u}_{lom}\) (called base inflow), all the indicator functions and the exit times (e) can be calculated and fixed via a so-called dynamic network loading procedure (see Ban et al. 2008). This leads to the so-called relaxed sub-problems that are explained in details in the appendix.
**DSTEM Algorithm**

**Step 1:** Initialization. Assign a base demands vectors \( \mathbf{d}_{\text{lonm}}^{(0)}, \mathbf{d}_{\text{lo}}^{(0)}, \mathbf{d}_{\text{lom}}^{(0)} \), and a base inflow vector \( \mathbf{u}_{\text{lonm}}^{(0)} \).

**Step 2:** Major Iteration. Set \( \nu = 0 \).

**Step 2.1:** Construct and solve RNCPDUE-DTA

(A1) - (A2). Repeat this \( \omega_1 \) times. For the first time, use \( \mathbf{u}_{\text{lonm}}^{(0)} \) as the base inflow. For other times, use inflow from the previous solve as the base inflow. Denote the final inflow (after the \( \nu \) solves) as \( \mathbf{u}_{\text{lonm}}^{(\nu)} \).

**Step 2.2:** Construct and solve RNCPDUE-DTA

(A5) - (A8). Repeat this \( \omega_2 \) times. For the first time, use \( \mathbf{u}_{\text{lonm}}^{(0)} \) as the base inflow and use \( \mathbf{d}_{\text{lonm}}^{(0)} \) as the initial demand. For other times, use inflow and demand obtained from the previous solve as the base inflow and initial demand. Denote the final inflow and demand (after the \( \omega_2 \) solves) as \( \mathbf{u}_{\text{lonm}}^{(\nu)} \) and \( \mathbf{d}_{\text{lonm}}^{(\nu)} \) respectively.

**Step 2.3:** Construct and solve RNCPDUE-DTA

(A10) - (A15). Repeat this \( \omega_3 \) times. For the first time, use \( \mathbf{u}_{\text{lonm}}^{(0)} \) and \( \mathbf{d}_{\text{lonm}}^{(0)} \) as the base inflow and base demand respectively. For other times, use inflow and demand obtained from the previous solve as the base inflow and initial demand. Denote the final inflow and demands (after the \( \omega_3 \) solves) as \( \mathbf{u}_{\text{lonm}}^{(\nu)}, \mathbf{d}_{\text{lonm}}^{(\nu)}, \) and \( \mathbf{d}_{\text{lon}}^{(\nu)} \) respectively.

**Step 2.4:** Construct and solve RNCPDUE-DTA

(A17) - (A24). Repeat this \( \omega_4 \) times. For the first time, use \( \mathbf{u}_{\text{lonm}}^{(0)}, \mathbf{d}_{\text{lonm}}^{(0)}, \) and \( \mathbf{d}_{\text{lo}}^{(0)} \) as the base inflow and base demands respectively. For other times, use inflow and demand obtained from the previous solve as the base inflow and initial demand. Denote the final inflow and demands (after the \( \omega_4 \) solves) as \( \mathbf{u}_{\text{lonm}}^{(\nu)}, \mathbf{d}_{\text{lonm}}^{(\nu)}, \mathbf{d}_{\text{lon}}^{(\nu)}, \) and \( \mathbf{d}_{\text{lo}}^{(\nu)} \) respectively.

**Step 2.5:** Construct and solve RNCPDUE-DTA

(A26) - (A35). Repeat this \( \omega_5 \) times. For the first time, use \( \mathbf{u}_{\text{lonm}}^{(0)}, \mathbf{d}_{\text{lonm}}^{(0)}, \mathbf{d}_{\text{lon}}^{(0)}, \) and \( \mathbf{d}_{\text{lo}}^{(0)} \) as the base inflow and base demands respectively. For other times, use inflow and demand obtained from the previous solve as the base inflow and initial demand. Denote the final inflow and demands (after the \( \omega_5 \) solves) as \( \mathbf{u}_{\text{lonm}}^{(\nu)}, \mathbf{d}_{\text{lonm}}^{(\nu)}, \mathbf{d}_{\text{lon}}^{(\nu)}, \mathbf{d}_{\text{lo}}^{(\nu)}, \) and \( \mathbf{d}_{\text{lo}}^{(\nu)} \) respectively.

**Step 2.6:** Construct and solve RNCPDUE-DTA

(A37) - (A48). Repeat this \( \omega_6 \) times. For the first time, use \( \mathbf{u}_{\text{lonm}}^{(0)}, \mathbf{d}_{\text{lonm}}^{(0)}, \mathbf{d}_{\text{lon}}^{(0)}, \mathbf{d}_{\text{lo}}^{(0)}, \mathbf{d}_{\text{lo}}^{(0)}, \) and \( \mathbf{d}_{\text{lo}}^{(0)} \) as the base inflow and base demands respectively. For other times, use inflow and demand obtained from the previous solve as the base inflow and initial demand. Denote the final inflow and demands (after the \( \omega_6 \) solves) as \( \mathbf{u}_{\text{lonm}}^{(\nu)}, \mathbf{d}_{\text{lonm}}^{(\nu)}, \mathbf{d}_{\text{lon}}^{(\nu)}, \mathbf{d}_{\text{lo}}^{(\nu)}, \mathbf{d}_{\text{lo}}^{(\nu)}, \) and \( \mathbf{S}_{\text{lo}}^{(\nu)} \) respectively. Call them the candidate solution.

**Step 2.7:** Convergence Test. If certain convergence criterion is satisfied at the candidate solution, go to Step 3; otherwise, go to Step 2.8.

**Step 2.8:** Update and Move. Set

\[
\begin{align*}
\mathbf{u}_{\text{lonm}}^{(\nu+1)} &= \mathbf{u}_{\text{lonm}}^{(\nu)} + \sigma(\mathbf{u}_{\text{lonm}}^{(0)} - \mathbf{u}_{\text{lonm}}^{(\nu)}) \\
\mathbf{d}_{\text{lonm}}^{(\nu+1)} &= \mathbf{d}_{\text{lonm}}^{(\nu)} + \sigma(\mathbf{d}_{\text{lonm}}^{(0)} - \mathbf{d}_{\text{lonm}}^{(\nu)}) \\
\mathbf{d}_{\text{lon}}^{(\nu+1)} &= \mathbf{d}_{\text{lon}}^{(\nu)} + \sigma(\mathbf{d}_{\text{lon}}^{(0)} - \mathbf{d}_{\text{lon}}^{(\nu)}) \\
\mathbf{d}_{\text{lo}}^{(\nu+1)} &= \mathbf{d}_{\text{lo}}^{(\nu)} + \sigma(\mathbf{d}_{\text{lo}}^{(0)} - \mathbf{d}_{\text{lo}}^{(\nu)}) \\
\mathbf{S}_{\text{lo}}^{(\nu+1)} &= \mathbf{S}_{\text{lo}}^{(\nu)} + \sigma(\mathbf{S}_{\text{lo}}^{(0)} - \mathbf{S}_{\text{lo}}^{(\nu)})
\end{align*}
\]
Set \( n = n + 1 \) and go to Step 2.1.

**Step 3:** Find an optimal solution \( \bar{u}^{(\text{lonm})}, \bar{d}^{(\text{lonm})}, \bar{d}^{(\text{lo} \text{m})}, \bar{S}^{(\text{lo} \text{m})} \), and \( \bar{S}^{(\text{lo} \text{m})} \).

In Step 2.1 and 2.2 of DSTEM Algorithm, the calculation of the indicator functions and exit times are done via dynamic network loading, as detailed in Nie and Zhang (2005) and Ban et al. (2008). The relaxed sub-problems \( \text{RNCPDUE-DTA} \) and \( \text{RNCPDUE-DTA-DT} \), \( \text{RNCPDUE-DTA-MS} \), \( \text{RNCPDUE-DTA-DT-MS} \), \( \text{RNCPDUE-DTA-DT-MS-NMS} \), \( \text{RNCPDUE-DTA-DT-MS-NMS-TD} \), and \( \text{RNCPDUE-DTA-DT-MS-NMS-TD-TG} \) are solved directly by the PATH solver in GAMS (Ferris and Munson 1998). For detailed discussions, one can refer to Ban et al. (2008)

In Step 2.7, there are several commonly used gap functions that can be used in the convergence test. The first one is based on the difference of the candidate solution and the base solution. In particular, we can check whether the following condition holds (Box 11).

Especially if \( \bar{u}^{(\text{lonm})} = \bar{u}^{(\text{lonm})} \),

\[
\bar{d}^{(\text{lonm})} = \bar{d}^{(\text{lonm})}, \quad \bar{d}^{(\text{lo} \text{m})} - \bar{d}^{(\text{lo} \text{m})}, \quad \bar{d}^{(\text{lo} \text{m})} = \bar{d}^{(\text{lo} \text{m})},
\]

we obtain a fix point solution to the original problem \( \text{NCPDUE-DTA-DT-MS-NMS-TD-TG} \).

We can also check directly the conditions of route choice, departure time choice, nest mode choice, mode choice from the nest, trip distribution choice, and trip generation choice by defining the second gap function (Box 12).

Similarly, if \( \text{Gap}_2 = 0 \), both conditions are satisfied, the obtained candidate solution is indeed an optimal solution. In (29) and (30), \( \varepsilon_1 \) and \( \varepsilon_2 \) are both small scalars representing user-defined convergence accuracy.

**CONCLUSIONS AND FUTURE RESEARCH**

In this paper we presented a link-node based discrete-time NCP model for DUE that combined multiclass trip generation, trip distribution, nested modal split, specific modal split, dynamic trip assignment and departure time (DMSTEM) in a unified formulation. This model combined the dynamic link-node based discrete-time NCP model that developed by Ban et al. (2008) and the static Multiclass Simultaneous Transportation Equilibrium Model (MSTEM) that developed by Hasan and Dashti.

---

**Box 11.**

\[
\text{Gap}_1 = \left( \left\| \bar{u}^{(\text{lonm})} - \bar{u}^{(\text{lonm})} \right\|_2 + \left\| \bar{d}^{(\text{lonm})} - \bar{d}^{(\text{lonm})} \right\|_2 + \left\| \bar{d}^{(\text{lo} \text{m})} - \bar{d}^{(\text{lo} \text{m})} \right\|_2 + \left\| \bar{S}^{(\text{lo} \text{m})} - \bar{S}^{(\text{lo} \text{m})} \right\|_2 \right) < \varepsilon_1 \forall l, o \quad (29)
\]

**Box 12.**

\[
\text{Gap}_2 = \left( \bar{u}^{(\text{lonm})} \Phi^{(\text{lo} \text{e} \text{w})} + \bar{d}^{(\text{lonm})} \Phi^{(\text{lo} \text{e} \text{w})} + \bar{d}^{(\text{lo} \text{m})} \Phi^{(\text{lo} \text{m})} + \bar{d}^{(\text{lo} \text{m})} \Phi^{(\text{lo} \text{m})} + \bar{S}^{(\text{lo} \text{m})} \Phi^{(\text{lo} \text{m})} + \bar{S}^{(\text{lo} \text{m})} \Phi^{(\text{lo} \text{m})} \right) < \varepsilon_2 \forall l, o \quad (30)
\]
We also developed an iterative solution algorithm for the proposed model by solving several relaxed NCP in each iteration. Most of the recent DUE models deal only with the traffic assignment and fixed demand flows between each O-D pair at each time period of the time horizon or an elastic dynamic demand (input to DTA models) that is usually modeled as departure-time choice for single user (traveler) class. To apply DTA with departure-time choice (DTA-DT) in practice, we require fixed demand flows between each O-D pair during the whole time horizon and the DTA-DT to be embedded in more comprehensive transportation planning framework that results in an inconsistence prediction process. The DMSTEM model will generate a better solution than this in consistent prediction process within a comprehensive planning and operations framework.

For future research, we would apply the DMSTEM and its solution algorithm to a prototype example to get more insight and implication of its predictions and validity. This prototype solution will encourage applying the model to real world transportation network. A comparative study between DMSTEM as a dynamic model and MSTEM as static model in term of their prediction accuracy and solution complexity is very desirable in the transportation planning in general and in particular, the Intelligence Transportation System (ITS).

Transportation network analysis tools can be broadly categorized as static analysis tools and dynamic analysis tools. The former focuses on long-term, steady traffic states, often referred as the four-step demand analysis models (Sheffi, 1985), while the latter focuses on short-term, dynamic traffic states, often called dynamic network analysis or dynamic traffic assignment (DTA, see Peeta and Ziliaskopoulos (2001)). Traditionally, these two types of models have been applied in quite distinct scenarios: static tools for transportation planning purposes and dynamic tools for traffic operation purposes. It would be ideal to combine the planning and operation tools to address transportation problems in a more comprehensive manner. In practice, such combination has been recently experimented and tested. Examples include the corridor management plan demonstration project (CCIT, 2006) and the state-wide corridor system management plan that is currently underway in California, which aims to integrate long-term planning tools (such as the static four-step demand analysis models) with operational tools such as microscopic traffic simulation and dynamic traffic assignment. The ICM program by FHWA also focuses on combining different transportation analysis tools for integrated corridor planning and operations.

Combining planning and DTA models in theory is not trivial because the two types of models are based on quite distinct assumptions. In this paper we present a combined model to simultaneously consider the origin, destination, mode, departure, and route choices of multiclass travelers.

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APPENDIX

To start our relation problems, we assume that \( \pi_{l0m} \) can be computed based on free flow travel time via a so-called dynamic network loading procedure (see Ban et al. 2008) and \( \widetilde{d}_{l0m}, \widetilde{d}_{l0m}, \tilde{d}_{l0m}, \tilde{d}_{l0m}, \tilde{S}_{l0m} \) can be computed accordantly from MSTEM model as follows:

\[
\begin{align*}
\widetilde{d}_{l0m} &= (\tilde{d}_{is_{l0m}}, \forall i, s, k, i \not= s), \forall l, o, n, m; \\
\widetilde{d}_{l0m} &= (\tilde{d}_{is_{l0m}}, \forall i, s, m, i \not= s), \forall l, o, n
\end{align*}
\]

\[
\begin{align*}
\tilde{d}_{l0} &= (\tilde{d}_{is_{l0}}, \forall i, s, n, i \not= s), \forall l, o; \forall l, o; \\
\tilde{S}_{l0} &= (\tilde{S}_{l0}, \forall l, o)
\end{align*}
\]

\[
\begin{align*}
\bar{S}_{l0} &= \max \left\{ 0, \ln \sum_{s \in D_{l0}} \sum_{m \in M_{l0}} \sum_{k \in K_{l0}} \exp(-\theta_{l0m}^{\pi}_{is} + A_{s}^{lo}) \right\} \quad \forall i \in I_{lo}, \forall l \in L, \forall o \in O
\end{align*}
\]

\[
\begin{align*}
\bar{d}_{l0} &= (\alpha_{l0}^{lo} \bar{S}_{l0}^{lo} + E_{l0}^{lo}) \quad \frac{\sum_{s \in D_{l0}} \sum_{m \in M_{l0}} \sum_{k \in K_{l0}} \exp(-\theta_{l0m}^{\pi}_{is} + A_{s}^{lo})}{\sum_{s \in D_{l0}} \sum_{m \in M_{l0}} \sum_{k \in K_{l0}} \exp(-\theta_{l0m}^{\pi}_{is} + A_{s}^{lo})} \quad \forall i \in I_{lo}, \forall l \in L, \forall o \in O
\end{align*}
\]

\[
\begin{align*}
\bar{d}_{l0} &= \frac{\sum_{m \in M_{l0}} \sum_{k \in K_{l0}} \exp(-\theta_{l0m}^{\pi}_{is} + A_{s}^{lo})}{\sum_{m \in M_{l0}} \sum_{k \in K_{l0}} \exp(-\theta_{l0m}^{\pi}_{is} + A_{s}^{lo})}, \forall n \in N_{l0}, \forall i \in I_{lo}, \forall l \in L, \forall o \in O
\end{align*}
\]

\[
\begin{align*}
\bar{d}_{l0} &= \frac{\sum_{m \in M_{l0}} \sum_{k \in K_{l0}} \exp(-\theta_{l0m}^{\pi}_{is} + A_{s}^{lo})}{\sum_{m \in M_{l0}} \sum_{k \in K_{l0}} \exp(-\theta_{l0m}^{\pi}_{is} + A_{s}^{lo})}, \forall m \in M_{n}, \forall n \in N_{l0}, \forall i \in I_{lo}, \forall l \in L, \forall o \in O
\end{align*}
\]

\[
\begin{align*}
\bar{d}_{l0} &= \frac{\sum_{k \in K_{l0}} \exp(-\theta_{l0m}^{\pi}_{is} + A_{s}^{lo})}{\sum_{k \in K_{l0}} \exp(-\theta_{l0m}^{\pi}_{is} + A_{s}^{lo})}, \forall k \in K_{l0}, \forall m \in M_{n}, \forall n \in N_{l0}, \forall i \in I_{lo}, \forall l \in L, \forall o \in O
\end{align*}
\]

The fixed demand relaxed sub-problem fixes both the base demand and the inflow. We denote it as RNCPEU-DTA which can be defined as:

\[
\begin{align*}
0 \leq u_{l0m}^{lm} &\perp \Phi_{u^{lm}^{l0m}, \pi^{l0m}} (u_{l0m}^{lm}, \pi_{l0m}) \geq 0, \forall l, o, n, m & (A1) \\
0 \leq \pi_{l0m}^{l0m} &\perp \Phi_{\pi^{l0m}, \pi^{l0m}} (u_{l0m}^{lm}, \pi_{l0m}) \geq 0, \forall l, o, n, m & (A2)
\end{align*}
\]

Where \( \Phi_{u^{lm}, \pi^{l0m}} (u_{l0m}^{lm}, \pi_{l0m}) \) and \( \Phi_{\pi^{l0m}, \pi^{l0m}} (u_{l0m}^{lm}, \pi_{l0m}) \) are defined at the base inflow and demands:

\[
\Phi_{\pi^{l0m}} (u_{l0m}^{lm}, \pi_{l0m}) = (\tilde{d}_{is}^{l0m} u_{l0m}^{lm}) + \sum_{q=1}^{\lambda_{l0m}^{l0m,q}} \lambda_{l0m}^{l0m,q} u_{l0m}^{lm} \]

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\begin{equation}
[1 - \lambda^3_{a} \lambda_{k} \lambda_{m} \left( \bar{u}_{l} \right) \pi_{l} \left( \bar{u}_{l} \right) \delta \left( \bar{u}_{l} \right) ] - \pi_{l} \left( \bar{u}_{l} \right), \forall a, s, k, \forall l, o, n, m \tag{A3}
\end{equation}

\begin{equation}
\Phi_{\text{RNCPDUE}} \left( u_{l}, \pi_{l}, \mu_{l} \right) = \\
\left( \sum_{a \in A(l)} u_{a} - \bar{d}_{a} \right) - \sum_{a \in B(l)} \left( \sum_{k \in \left( \pi_{l} \right) \left( k \right) - \left( \delta \left( \bar{u}_{l} \right) \right) \left( \pi_{l} \right) \left( k \right) - \left( \delta \left( \bar{u}_{l} \right) \right) \right) \lambda_{a} \left( \bar{u}_{l} \right) \lambda_{a} \left( \bar{u}_{l} \right) u_{a} - \\
\lambda_{a} \left( \bar{u}_{l} \right) \left( 1 - \lambda_{a} \left( \bar{u}_{l} \right) \right) u_{a} \left( k + 1 \right) \right), \forall a, s, k, \forall l, o, n, m \tag{A4}
\end{equation}

The departure time relaxed sub-problem fixed the base in flow and the demands \(d_{l}, d_{l}^{io}, d_{l}^{fo}, S_{l}^{io} \). We denote it as RNCPDUE-DTA-DT, which can be defined as:

\begin{equation}
0 \leq u_{l} - \Phi_{\text{RNCPDUE}} \left( u_{l}, \pi_{l}, d_{l}, \mu_{l} \right) \geq 0, \forall l, o, n, m \tag{A5}
\end{equation}

\begin{equation}
0 \leq \pi_{l} - \Phi_{\text{RNCPDUE}} \left( u_{l}, \pi_{l}, d_{l}, \mu_{l} \right) \geq 0, \forall l, o, n, m \tag{A6}
\end{equation}

\begin{equation}
0 \leq d_{l} - \Phi_{\text{RNCPDUE}} \left( u_{l}, \pi_{l}, d_{l}, \mu_{l} \right) \geq 0, \forall l, o, n, m \tag{A7}
\end{equation}

\begin{equation}
0 \leq \mu_{l} - \Phi_{\text{RNCPDUE}} \left( u_{l}, \pi_{l}, d_{l}, \mu_{l} \right) \geq 0, \forall l, o, n, m \tag{A8}
\end{equation}

Where \( \Phi_{\text{RNCPDUE}} \left( u_{l}, \pi_{l}, d_{l}, \mu_{l} \right) \) has the same expressions as in (A3) and

\begin{equation}
\Phi_{\text{RNCPDUE}} \left( u_{l}, \pi_{l}, d_{l}, \mu_{l} \right) = \\
\left( \sum_{a \in A(l)} u_{a} - \bar{d}_{a} \right) - \sum_{a \in B(l)} \left( \sum_{k \in \left( \pi_{l} \right) k - \left( \delta \left( \bar{u}_{l} \right) \right) k - \left( \delta \left( \bar{u}_{l} \right) \right) \right) \lambda_{a} \left( \bar{u}_{l} \right) \lambda_{a} \left( \bar{u}_{l} \right) u_{a} - \\
\lambda_{a} \left( \bar{u}_{l} \right) \left( 1 - \lambda_{a} \left( \bar{u}_{l} \right) \right) u_{a} \left( k + 1 \right) \right), \forall a, s, k, \forall l, o, n, m \tag{A9}
\end{equation}

The mode \( m \) relaxed sub-problem fixed the base in flow and the demands \( d_{l}^{io}, d_{l}^{fo}, S_{l}^{fo} \). We denote it as RNCPDUE-DTA-DT-MS, which can be defined as:

\begin{equation}
0 \leq u_{l} - \Phi_{\text{RNCPDUE}} \left( u_{l}, \pi_{l}, d_{l}, m_{l} \right) \geq 0, \forall l, o, n, m \tag{A10}
\end{equation}

\begin{equation}
0 \leq \pi_{l} - \Phi_{\text{RNCPDUE}} \left( u_{l}, \pi_{l}, d_{l}, m_{l} \right) \geq 0, \forall l, o, n, m \tag{A11}
\end{equation}

\begin{equation}
0 \leq d_{l} - \Phi_{\text{RNCPDUE}} \left( u_{l}, \pi_{l}, d_{l}, m_{l} \right) \geq 0, \forall l, o, n, m \tag{A12}
\end{equation}

\begin{equation}
0 \leq \mu_{l} - \Phi_{\text{RNCPUE}} \left( u_{l}, \pi_{l}, d_{l}, m_{l} \right) \geq 0, \forall l, o, n, m \tag{A13}
\end{equation}

\begin{equation}
0 \leq d_{l} - \Phi_{\text{RNCPUE}} \left( u_{l}, \pi_{l}, d_{l}, m_{l} \right) \geq 0, \forall l, o, n, m \tag{A14}
\end{equation}
\[ 0 \leq \mu \perp \Phi(\mu, \pi, \pi, \pi, \pi) \geq 0, \forall l, o, n \] (A15)

Where \( \Phi(\mu, \pi, \pi, \pi, \pi) \) has the same expressions as in (A3), \( \Phi(\pi, \pi, \mu, \mu, \mu) \) has the same expressions as in (A9), \( \Phi(\mu, \pi, \mu, \mu, \mu) \) has the same expressions as in (20), and

\[ \Phi = \sum_{i \leq k < l} d_{is} - \sum_{i \neq k, i \neq l} d_{is} \] (A16)

The nest mode relaxation sub-problem fixed the base in flow and the demands. We denote it as RNCPDUE-DTA-DT-MS-NMS, which can be defined as:

\[ 0 \leq u \perp \Phi(\mu, \pi, \mu, \mu, \mu, \mu) \geq 0, \forall l, o, n, m \] (A17)

\[ 0 \leq \pi \perp \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \geq 0, \forall l, o, n, m \] (A18)

\[ 0 \leq d \perp \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \geq 0, \forall l, o, n, m \] (A19)

\[ 0 \leq \mu \perp \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \geq 0, \forall l, o, n, m \] (A20)

\[ 0 \leq \mu \perp \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \geq 0, \forall l, o, n \] (A21)

\[ 0 \leq \mu \perp \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \geq 0, \forall l, o, n \] (A22)

\[ 0 \leq \mu \perp \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \geq 0, \forall l, o \] (A23)

\[ 0 \leq \mu \perp \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \geq 0, \forall l, o \] (A24)

Where

\( \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \) has the same expressions as in (A3),
\( \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \) has the same expressions as in (A9),
\( \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \) has the same expressions as in (20),
\( \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \) has the same expressions as in (21),
\( \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \) has the same expressions as in (22), and

\[ \Phi = \sum_{i \leq k < l} d_{is} - \sum_{i \neq k, i \neq l} d_{is} \] (A25)

The Trip Distribution (TD) relaxed sub-problem fixed the base in flow and the demands. We denote it as RNCPDUE-DTA-DT-MS-NMS-TD, which can be defined as:

\[ 0 \leq u \perp \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \geq 0, \forall l, o, n, m \] (A26)

\[ 0 \leq \pi \perp \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \geq 0, \forall l, o, n, m \] (A27)

\[ 0 \leq d \perp \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \geq 0, \forall l, o, n, m \] (A28)

\[ 0 \leq \mu \perp \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \geq 0, \forall l, o, n, m \] (A29)

\[ 0 \leq \mu \perp \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \geq 0, \forall l, o, n \] (A30)

\[ 0 \leq \mu \perp \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \geq 0, \forall l, o, n \] (A31)

\[ 0 \leq \mu \perp \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \geq 0, \forall l, o \] (A32)

\[ 0 \leq \mu \perp \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \geq 0, \forall l, o \] (A33)

Where

\( \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \) has the same expressions as in (A3),
\( \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \) has the same expressions as in (A9),
\( \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \) has the same expressions as in (20),
\( \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \) has the same expressions as in (21),
\( \Phi(\mu, \mu, \mu, \mu, \mu, \mu) \) has the same expressions as in (22), and

\[ \Phi = \sum_{i \leq k < l} d_{is} - \sum_{i \neq k, i \neq l} d_{is} \] (A34)
Where
\[ \Phi_{\sigma,\tau}(u_{l}, \pi, d, \mu, d_{lo}, \mu_{lo}, d_{lo}^{R}, \mu_{lo}^{R}) \] has the same expressions as in (A3),
\[ \Phi_{\sigma,\tau}(u_{l}, \pi, d, \mu, d_{lo}, \mu_{lo}, d_{lo}^{R}, \mu_{lo}^{R}) \] has the same expressions as in (A9),
\[ \Phi_{d,\tau}(u_{l}, \pi, d, \mu, d_{lo}, \mu_{lo}, d_{lo}^{R}, \mu_{lo}^{R}) \] has the same expressions as in (20),
\[ \Phi_{\sigma,\tau}(u_{l}, \pi, d, \mu, d_{lo}, \mu_{lo}, d_{lo}^{R}, \mu_{lo}^{R}) \] has the same expressions as in (21),
\[ \Phi_{d,\tau}(u_{l}, \pi, d, \mu, d_{lo}, \mu_{lo}, d_{lo}^{R}, \mu_{lo}^{R}) \] has the same expressions as in (22),
\[ \Phi_{\sigma,\tau}(u_{l}, \pi, d, \mu, d_{lo}, \mu_{lo}, d_{lo}^{R}, \mu_{lo}^{R}) \] has the same expressions as in (24), and
\[ \Phi_{\sigma,\tau}(u_{l}, \pi, d, \mu, d_{lo}, \mu_{lo}, d_{lo}^{R}, \mu_{lo}^{R}) = \left\{ \sum_{n \in s} d_{s,n}^{lo} \right\}, \forall i, s, i \neq s \] \( \forall l, o \) (A36)

The Trip Generation (TG) relaxed sub-problem fixed the base in flow. We denote it as RNC-PDUE-DTA-DT-MS-NMS-TD-TG, which can be defined as:

\[ 0 \leq u_{l} \leq \Phi_{\sigma,\tau}(u_{l}, \pi, d, \mu, d_{lo}, \mu_{lo}, d_{lo}^{R}, \mu_{lo}^{R}, \mu_{lo}^{R}) \geq 0, \forall l, o, n, m \] (A37)
\[ 0 \leq \Phi_{\sigma,\tau}(u_{l}, \pi, d, \mu, d_{lo}, \mu_{lo}, d_{lo}^{R}, \mu_{lo}^{R}) \geq 0, \forall l, o, n, m \] (A38)
\[ 0 \leq \Phi_{d,\tau}(u_{l}, \pi, d, \mu, d_{lo}, \mu_{lo}, d_{lo}^{R}, \mu_{lo}^{R}) \geq 0, \forall l, o, n, m \] (A39)
\[ 0 \leq \Phi_{\sigma,\tau}(u_{l}, \pi, d, \mu, d_{lo}, \mu_{lo}, d_{lo}^{R}, \mu_{lo}^{R}) \geq 0, \forall l, o, n, m \] (A40)
\[0 \leq d_{lo} \perp \Phi_{\pi_{lo}}(u_{lo}, \pi_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, S_{p_{lo}}^{lo}, \mu) \geq 0, \forall l, o, n \quad (A41)\]
\[0 \leq \mu_{lo} \perp \Phi_{\mu_{lo}}(u_{lo}, \pi_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, S_{p_{lo}}^{lo}, \mu) \geq 0, \forall l, o, n \quad (A42)\]
\[0 \leq d_{lo} \perp \Phi_{\pi_{lo}}(u_{lo}, \pi_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, S_{p_{lo}}^{lo}, \mu) \geq 0, \forall l, o \quad (A43)\]
\[0 \leq \mu_{lo} \perp \Phi_{\mu_{lo}}(u_{lo}, \pi_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, S_{p_{lo}}^{lo}, \mu) \geq 0, \forall l, o \quad (A44)\]
\[0 \leq d_{lo} \perp \Phi_{\pi_{lo}}(u_{lo}, \pi_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, S_{p_{lo}}^{lo}, \mu) \geq 0, \forall l, o \quad (A45)\]
\[0 \leq \mu_{lo} \perp \Phi_{\mu_{lo}}(u_{lo}, \pi_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, S_{p_{lo}}^{lo}, \mu) \geq 0, \forall l, o \quad (A46)\]
\[0 \leq S_{p_{lo}}^{lo} \perp \Phi_{S_{p_{lo}}^{lo}}(u_{lo}, \pi_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, S_{p_{lo}}^{lo}, \mu) \geq 0, \forall l, o \quad (A47)\]
\[0 \leq \mu \perp \Phi_{\mu_{lo}}(u_{lo}, \pi_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, S_{p_{lo}}^{lo}, \mu) \geq 0, \forall l, o \quad (A48)\]

Where

\[\Phi_{\pi_{lo}}(u_{lo}, \pi_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, S_{p_{lo}}^{lo}, \mu)\] has the same expressions as in (A3),
\[\Phi_{\mu_{lo}}(u_{lo}, \pi_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, S_{p_{lo}}^{lo}, \mu)\] has the same expressions as in (A9),
\[\Phi_{\pi_{lo}}(u_{lo}, \pi_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, S_{p_{lo}}^{lo}, \mu)\] has the same expressions as in (20),
\[\Phi_{\pi_{lo}}(u_{lo}, \pi_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, S_{p_{lo}}^{lo}, \mu)\] has the same expressions as in (21),
\[\Phi_{\pi_{lo}}(u_{lo}, \pi_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, S_{p_{lo}}^{lo}, \mu)\] has the same expressions as in (22),
\[\Phi_{\pi_{lo}}(u_{lo}, \pi_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, S_{p_{lo}}^{lo}, \mu)\] has the same expressions as in (23),
\[\Phi_{\pi_{lo}}(u_{lo}, \pi_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, S_{p_{lo}}^{lo}, \mu)\] has the same expressions as in (26),

\[
\Phi_{\pi_{lo}}(u_{lo}, \pi_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, d_{lo}, \mu_{lo}, S_{p_{lo}}^{lo}, \mu) = \left\{ \sum_{i \in D_{p_{lo}}} d_{is}^{lo} - \bar{c}_{is}^{lo} \right\}_{i, \forall i \neq s} \forall l, o \quad (A49)\]