SPECKLE REDUCTION IN MEDICAL ULTRASOUND IMAGES USING A NEW MULTISCALE BIVARIATE BAYESIAN MMSE-BASED METHOD

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ABSTRACT

This paper introduces a new wavelet-based speckle reduction method for medical ultrasound images, which extends a recently emerged homomorphic Bayesian technique. The new method exploits interscale dependency of wavelet coefficients. For this purpose, bivariate alpha-stable distributions are proposed, which are able to better capture the heavy-tailed nature of the data. Using this new statistical model, we design a bivariate Bayesian estimator to effectively remove speckle from wavelet coefficients. Better results are obtained using the oriented two-dimensional dual-tree complex wavelet transform (2-D DTCWT) which offers improved directional selectivity and near shift invariance property.

To assess the performance of the proposed method, results are compared with some related earlier techniques. Both numerical and visual comparisons indicate improved speckle reduction while preserving structural features, as desired for better diagnosis in medical images.

1. INTRODUCTION

As in any coherent imaging system, medical ultrasound images are affected by speckle phenomenon. Speckle is a kind of multiplicative noise which degrades the image quality and affects the task of human interpretation and diagnosis. Early attempts to suppress speckle noise were implemented by averaging uncorrelated images of the same tissue recorded under different spatial positions \cite{1}. Although these techniques are effective for speckle reduction, they involve hardware modifications that can be expensive and inconvenient. This has led to various post-processing algorithms, such as Median, Lee \cite{2}, Frost \cite{3}, and Kuan \cite{4} filtering techniques. However, it is recognized that these conventional noise filtering methods often result in blurred image features \cite{5}.

Recently, the wavelet transform has been proposed as a useful processing tool for signal recovery. There are two main reasons for the choice of multiscale decomposition: First, the statistics of many natural signals, when decomposed into wavelet bases are significantly simplified. Second, by using multiscale decomposition, we will be able to process noise and signal components at different scales and orientations, proportionally.

In this paper, we further extend a wavelet-based Bayesian approach introduced by Achim \textit{et al} \cite{6}. They showed that wavelet coefficients of logarithmically transformed ultrasound images have significantly non-Gaussian statistics that are best described by families of heavy-tailed distributions such as the alpha stable. Within this framework, they designed a Bayesian MMSE estimator based on independent assumption of wavelet coefficients at different scales. Here, we extend their work in two aspects: i) in the data modeling component of our processor, we propose \textit{bivariate alpha-stable distributions} to exploit interscale dependency of wavelet coefficients. The efficacy of this model for the special case of bivariate Cauchy distribution has been already demonstrated in image denoising \cite{7}. However we use a general form of alpha-stable distributions and apply it to ultrasound despeckling, ii) for the multiscale decomposition step, we use the recently introduced oriented 2-D \textit{dual-tree complex wavelet transform} which offers improved directional selectivity and near shift invariance property.

This paper is organized as follows. In Section 2, the speckle statistics are investigated. Section 3 introduces the DTCWT. In Section 4, the proposed approach is developed. Experimental results are presented in Section 5 and the paper is concluded in Section 6.

2. STATISTICAL CHARACTERISTICS OF SPECKLE NOISE

Speckle is the result of constructive and destructive summation of coherent backscattered waves form several scatterers within a resolution cell. If there are a large number of scatterers within each resolution element, and the received phases are uniformly distributed from 0 to \(2\pi\) radians, the phasor amplitude of backscattered waves \(A_s\) obeys a Rayleigh probability density function (PDF) given by \cite{1}

\[
p(A_s) = \frac{A_s}{\sigma^2} \exp \left( -\frac{A_s^2}{2\sigma^2} \right)
\]
for $A_i > 0$, while $p(A_i) = 0$ for $A_i < 0$.

Now, if we consider the average value of $A_i$ as the signal component, speckle can be modeled as a multiplicative noise with a unit mean Rayleigh distribution. When speckle is reduced through averaging over $N$ independent sample images, its PDF may be obtained by $N$ successive convolutions of (1) [8].

In order to convert the multiplicative noise into an additive one, we have to take a logarithmic transform. It has been shown that when an image is logarithmically transformed, speckle can be approximately modeled as a Gaussian additive noise [8]. At this stage, we can apply any conventional additive-noise suppression technique.

3. THE DUAL-TREE COMPLEX WAVELET TRANSFORM

The wavelet transform performs the decomposition of a signal onto the family of functions generated by dilation and translation of a prototype mother wavelet $\psi(x)$. The mother wavelet is constructed from the scaling function $\phi(x)$. In the case of the discrete wavelet transform (DWT), Mallat proposed a filter bank scheme using filter coefficients instead of the explicit forms for $\psi(x)$ and $\phi(x)$ [9].

The dual-tree complex wavelet transform is a relatively recent enhancement to DWT. DTCWT is implemented as two real DWTs in parallel; the first DWT gives the real part of the transform while the second DWT gives the imaginary part. The two sets of filters are jointly designed so that the overall transform is approximately analytic while the perfect reconstruction conditions are satisfied. Within this framework, a 2-D wavelet transform that is both oriented and complex (approximately analytic) can be easily developed. The 2-D DTCWT is four-times expansive, but it achieves important additional properties compared with 2-D DWT: It is nearly shift invariant and directionally selective. Standard 2-D DWT offers the feature selectivity in only 3 directions with poor directionally selective. Standard 2-D DWT offers the feature selectivity in only 3 directions with poor directionally selective. Standard 2-D DWT offers the feature selectivity in only 3 directions with poor directionally selective. Standard 2-D DWT offers the feature selectivity in only 3 directions with poor directionally selective. Standard 2-D DWT offers the feature selectivity in only 3 directions with poor directionally selective. Standard 2-D DWT offers the feature selectivity in only 3 directions with poor directionally selective. Standard 2-D DWT offers the feature selectivity in only 3 directions with poor directionally selective. Standard 2-D DWT offers the feature selectivity in only 3 directions with poor directionally selective. Standard 2-D DWT offers the feature selectivity in only 3 directions with poor directionally selective.

4. PROPOSED BIVARIATE BAYESIAN PROCESSOR

Wavelet adjacent scales are strongly dependent and these interscale dependencies can be exploited for better signal processing results [11]. In this section we design a bivariate Bayesian processor which uses these statistics to provide more accurate signal estimation from noisy coefficients.

4.1. Interscale wavelet model

In the wavelet domain, noise level decreases rapidly along scales, while signal structures are strengthened with scale increasing [12]. So we use coarser scale (parent) information to improve finer scale (child) estimation. For every two adjacent levels, (2) can be written in vectorial form as

$$d = s + n$$

where $d = (d_i, d_{i+1})$, $s = (s_i, s_{i+1})$, and $n = (n_i, n_{i+1})$, while subscripts $i$ and $i+1$ represents the child and parent coefficients of the same spatial location, respectively. At this stage, we can assume a zero-mean bivariate Gaussian model for the noise component $n$ as

$$p_n(n) = \frac{1}{2\pi\sigma_n^2}exp \left( -\frac{n_i^2 + n_{i+1}^2}{2\sigma_n^2} \right)$$

where $\sigma_n^2$ is the noise variance. For the signal component $s$, we assume an isotropic bivariate alpha-stable distribution whose characteristic function has the form [13]

$$\phi_s(w) = exp \left( i(\delta_i w_i + \delta_{i+1} w_{i+1}) - \gamma (w_i^2 + w_{i+1}^2)^{\alpha/2} \right)$$

where $\alpha$ is the characteristic exponent, taking values $0 < \alpha \leq 2$, $\delta_i$ and $\delta_{i+1}$ ($0 < \delta < \infty$) are the location parameters, $\gamma$ ($\gamma > 0$) is the dispersion of the distribution and $w = (w_i, w_{i+1})$. Since our further developments are in the framework of wavelet analysis, we will assume that $(\delta_i, \delta_{i+1}) = (0, 0)$.

4.2. Parameter estimation

A robust estimate of noise variance is obtained using the median absolute deviation (MAD) of the coefficients at the first level of decomposition [14].
For the purpose of bivariate alpha-stable parameter estimation from noisy observations, a method based on characteristic functions is developed. The characteristic function of noisy coefficients is given by the product of the characteristic functions of the signal and noise as \( \phi_n(w) = \phi_s(w) \cdot \phi_n(w) \), where \( \phi_n(w) \) is the characteristic function corresponding to noise

\[
\phi_n(w) = \exp \left( -\frac{\sigma_n^2(w_i^2 + w_{i+1}^2)}{2} \right)
\]

Now we can estimate \( \alpha \) and \( \gamma \) in the least square sense

\[
\{ \hat{\alpha}, \, \hat{\gamma} \} = \underset{\alpha, \gamma}{\arg \min} \int \left[ \phi_d(w) - \phi_{d,\alpha}(w) \right]^2 d\omega
\]

where \( \phi_{d,\alpha}(w) \) indicates the empirical characteristic function.

### 4.3. Bivariate Bayesian MMSE estimator

Having estimated the necessary signal and noise distribution parameters from the data, our goal is to design a Bayes risk estimator. By minimizing a quadratic cost function, the bivariate MMSE estimator is given by the marginal conditional mean of \( s_i \), given \( d \)

\[
\hat{s}_i(d) = \int s_i \ p_{s|d}(s|d) \ ds
\]

Using Bayes’ theorem we get

\[
\hat{s}_i(d) = \frac{\int p_n(d-s) \ p_s(s) \ s_i \ ds}{\int p_n(d-s) \ p_s(s) \ ds}
\]

where \( p_s(s) \) is the PDF of wavelet coefficients corresponding to the signal. Since the processor does not have a closed-form expression, it should be computed numerically. In the proposed processor, the shrinkage of a coefficient is conditioned on both its amplitude (as the one proposed in [6]) and the value of the corresponding coefficient at the next decomposition level (parent value). Exploiting these interscale dependencies, we achieve better denoising results.

### 4.4. Image reconstruction

After estimating the signal component of the noisy coefficients in wavelet domain, we invert the multiscale decomposition and take an exponential to reconstruct the noise-free image.

### 5. EXPERIMENTAL RESULTS

In order to evaluate the efficacy of the proposed method, the speckled images were simulated. A speckle image \( I_{x,y} \) is commonly modeled as \( I_{x,y} = S_{x,y} \times N_{x,y} \), where \( S_{x,y} \) is a reference noise-free image, \( N_{x,y} \) is the multiplicative speckle noise, and \( x \) and \( y \) are variables of spatial locations, \( (x, y) \in \mathbb{Z}^2 \). Speckle can be simulated by low-pass filtering a complex Gaussian random field and taking the magnitude of the filtered output. The magnitude will approximately obey a Rayleigh distribution as (1). By changing the variance of the underlying complex Gaussian random field, images with different noise levels can be generated [15].

For quantitative evaluation, the proposed algorithm was tested on medical ultrasound images in which a natural speckle noise was previously suppressed with spatial compounding technique [16]. The resulting images were considered as reasonable approximations of speckle-free test images. The experiments were conducted on several images, at different noise levels. The DTCWT was performed at five levels of decomposition. To assess the performance of the proposed method, results were compared with homomorphic Wiener filter using a window of size \( 3 \times 3 \), soft thresholding technique with a threshold \( t = 1.5\sigma_d \), where \( \sigma_d \) is the standard deviation of noisy coefficients, and Achim’s approach [6]. To quantify the achieved performance improvements, the values of two quality metrics, signal-to-noise ratio (SNR), and edge preservation index (\( \beta \)) [17], were computed using original noise-free image and the denoised image. The numerical values for these parameters, computed at various noise levels, are given in Table I. For visual comparison, the denoised images are shown in Fig. 2. The results indicate that our method reduces speckle while preserving object boundaries and enhancing fine signal details. Soft thresholding method seems to over-smooth the image and the Wiener filter looses fine signal details while the image is still noisy.

### 6. CONCLUSION

In this paper, a new bivariate Bayesian MMSE-based method for speckle removal in ultrasound images was proposed. The main innovation was the use of bivariate alpha-stable distributions to model the complex wavelet coefficients of adjacent scales in a logarithmically transformed medical ultrasound image. These models were exploited to develop a bivariate Bayesian MMSE estimator. The performance of the method was studied, using simulated speckled images and real ultrasound images. Simulation results showed that our method achieves state-of-the-art performance in comparison with some related recent algorithms.
7. ACKNOWLEDGMENT

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8. REFERENCES


