Energy Efficient Causal Packet Scheduling in Wireless Fading Channels with Hard Delay Constraints

Naveed UL Hassan and Mohamad Assaad
Ecole Supérieure d’électricité (SUPELEC) 91192 Gif-sur-Yvette, France
Email: naveed.ulhassan@supelec.fr, mohamad.assaad@supelec.fr

Abstract—In this paper we study the problem of energy efficient packet scheduling in wireless fading channels with hard delay constraints. Each packet has a strict delay constraint of \( D \) \((1 < D < \infty)\) time slots and it has to be delivered before its delay deadline. Optimal scheduling of each packet now depends on future Channel State Information (CSI) which is not available. We develop the upper and lower bounds on the optimal output rate and develop a two step solution. In the first step we solve the relaxed optimization problem without bounds by using Lagrange Optimization theory and obtain an initial output rate. In the second step this output rate is adjusted such that the deadlines are successfully achieved. Simulation results show good performance in terms of energy efficiency.

I. INTRODUCTION

Wireless fading channel has time varying channel gains. Energy efficient scheduling aims at efficiently utilizing these variations in order to conserve power. Increased energy efficiency translates to longer battery life and reduced interference among different transmitters. Channel variations can best be exploited by an opportunistic scheduler which transmits at a higher rate when the channel quality is good and otherwise at a lower rate when the channel quality is poor. However, when the packets have hard delay constraints of \( D \) \((1 < D < \infty)\) time slots, opportunistic scheduling fails to meet the delay deadlines. This range of delay constraints is applicable to real time applications such as voice traffic and audio/video streaming applications. When \( 1 < D < \infty \) short term channel variations can still be exploited since a packet can now be transmitted in any of the \( D \) time slots. However, channel information for future time slots in generally not available which makes the causal scheduling a much challenging problem.

In this paper we consider the problem of causal energy efficient packet scheduling in wireless fading channels with hard delay constraints. Each packet has a strict delay constraint of \( D \) time slots. We want to develop a causal scheduler which has channel state information of the current time slot but not of the future time slots. The problem of energy efficient scheduling has been considered by various authors under different traffic models and delay constraint assumptions. In [1] the authors consider packet scheduling with strict delay constraints. They develop robust energy efficient schedulers as low pass filters.

However, their analysis is limited to AWGN channels which cannot be extended to wireless fading channels. The authors in [2], [3] exploit energy delay trade off and propose strategies to minimize average queuing delay for single-user systems. Dynamic programming was adopted in [4] to schedule packets over a time-slotted single-user wireless channel for bursty traffic. In [5]-[6], non-causal packet schedulers are proposed for wireless fading channels and heuristic variations of the non-causal algorithms are used for the causal case. Most of this work either consider average delay constraints, non-causal scheduling or AWGN channels.

We propose a two step solution to the energy efficient causal scheduling problem. We derive an optimal output data rate every time slot in such a way that delay constraints of all the packets are achieved. In order to guarantee strict delay constraints, we derive the optimal upper and lower bounds on the instantaneous data rate. In this way, we convert the delay deadline of the packets into the bounding constraints. The problem is then formulated as a constrained optimization problem. We solve this optimization problem in two steps. In the first step we consider a relaxed convex optimization problem without the bounds. This problem can be solved by solving the appropriate Lagrange KKT conditions and we obtain an output data rate. This data rate may not satisfy the bounding constraints. In the next step, we modify this data rate in such a way that the bounding constraints are always satisfied and hence the delay deadlines of all the packets are achieved.

The rest of the paper is organized as follows. In Section II we formulate the problem and develop the bounds on the instantaneous output data rates. In Section III, we solve the relaxed optimization problem while in Section IV the bounding constraints are achieved. Simulation results are presented in Section V while the paper is concluded in Section VI.

II. PROBLEM FORMULATION

We consider a discrete time slotted wireless system. At the start of each time slot \( t \), the queue of each user receives packets from some higher layer application. The number of packets arriving in the user queue at time \( t \) are denoted by \( X_t \). We assume that all the packets are of equal size and they have a delay constraint of \( D \) time slots. Let \( R_t \) be the data rate scheduled for transmission at time \( t \). The queue backlog \( B_t \)
evolves according to,
\[ B_{t+1} = B_t + X_t - R_t \]  \hspace{1cm} (1)

We derive necessary and sufficient conditions on the output data rate in order to guarantee strict delay constraints. Without any loss of generality we assume that we start at time \( t = 1 \) and that the initial backlog is zero. A valid scheduler imposes two constraints on output data rates.

A. Constraint I

Since all the packets have same delay constraint \( D \), we have,
\[ X_1 + \cdots + X_t \leq R_1 + \cdots + R_t + \cdots + R_{t+D-1} \]  \hspace{1cm} (2)

Therefore, at any time \( t \) the output rate \( R_t \) must satisfy the following constraint,
\[ R_t \geq \sum_{i=1}^{t} X_i - \sum_{i=1}^{t-D} R_i - \sum_{i=t+1}^{t+D-1} R_i, \hspace{0.5cm} \forall t \]  \hspace{1cm} (3)

As \( X_t \) represents the number of packets arriving at time \( t \), therefore, \( B_t = \sum_{i=1}^{t-1} [X_i - R_i] \) is the backlog at time \( t \) and all the information about previous scheduling decisions is concentrated in this value. Thus we get,
\[ R_t \geq B_t + X_t - \sum_{i=t+1}^{t+D-1} R_i, \hspace{0.5cm} \forall t \]  \hspace{1cm} (4)

This constraint ensures that a packet arriving at time \( t \) will be out of the buffer before \( t + D - 1 \). Eq (4) gives a lower bound on output data rate. We have to proceed sequentially in time to derive the optimal output rates. Moreover, the dependence of \( R_t \) on future allocation decisions is explicit from this constraint.

B. Constraint II

This constraint arises from the fact that a packet cannot be transmitted before its arrival. Therefore, packets arriving at time \( t \) should be transmitted either during this time slot or during future time slots i.e.,
\[ R_1 + \cdots + R_t \leq X_1 + \cdots + X_t \]  \hspace{1cm} (5)

The condition on output rate becomes,
\[ R_t \leq B_t + X_t, \hspace{0.5cm} \forall t \]  \hspace{1cm} (6)

Eq (6) gives an upper bound on \( R_t \).

We assume a time varying channel denoted by \( g_t \). The channel gain varies from one time slot to another due to fast fading phenomenon. Using the Shannon capacity formula, the instantaneous channel capacity of a wireless fading channel is given by,
\[ R_t = \log \left( 1 + \frac{p_t g_t}{N_o} \right) \text{ nats/transmission} \]  \hspace{1cm} (7)

where, \( N_o \) is the Noise power. Without loss of generality we assume \( N_o = 1 \) and from (7) we get the expression of power,
\[ p_t = \frac{e^{R_t} - 1}{g_t} \]  \hspace{1cm} (8)

It is evident from Eq (8) and Figure 1 that the transmit power depends on two factors (i) it increases exponentially w.r.t the increase in output data rate and (ii) it increases linearly w.r.t decrease in channel quality. Therefore, a small increase or decrease in the transmission rate has huge impact on transmit power compared to the channel quality. Therefore, in order to develop energy efficient schedulers, appropriate data rate should be transmitted every time slot. Packets need to be intelligently scheduled so as to avoid a situation where data rate has to be quickly increased in order to respect the delay deadlines.

The objective of our energy efficient scheduler is to minimize the total transmit power subject to strict delay constraints.

In order to derive the scheduler, we write an optimization problem to schedule packets arriving in time interval \([t, t+T]\). We call \( T \) the optimization interval. The reason behind solving the optimization problem for \( T \) time slots is to make explicit the dependence of \( R_t \) on future arrival rates. Since the delay constraint of packets arriving at time slot \( t+T \) is \( D \), hence the summation is over \( t + T + D - 1 \). During each time slot \( t \) the following optimization problem has to be solved in order to get the minimum output data rates,
\[ \min_{p_t} \sum_{i=t}^{t+T+D-1} p_i \]  \hspace{1cm} (9)

subject to,
\[ \sum_{i=t}^{t+T+D-1} R_i = B_t + \sum_{i=t}^{t+T} X_i \]  \hspace{1cm} (10)

\[ R_t \geq B_t + X_t - \sum_{d=i+1}^{i+D-1} R_d, \hspace{0.5cm} i = t, \ldots, t+T \]  \hspace{1cm} (11)

\[ R_t \leq B_t + X_t, \hspace{0.5cm} i = t, \ldots, t+T \]  \hspace{1cm} (12)

Let, \( E[p_i] \) be the expected value of power, \( E[X_i] \) be the expected value of input arrival rate and \( E[R_i] \) be the expected...
output rate. Since we assume that the optimization interval is very large thus by using the law of large numbers, which states that the average value of large number of observations of any random variable tends toward the mean of that random variable at any time slot \( t \), the above minimization problem can be written as,

\[
\min \ p_t + (T + D - 1)E[p_t] \tag{13}
\]

subject to,

\[
R_t + (T + D - 1)E[R_t] = B_t + X_t + (T)E[X_t] \tag{14}
\]

\[
R_t \geq B_t + X_t - (D - 1)E[R_t] \tag{15}
\]

\[
R_t \leq B_t + X_t \tag{16}
\]

where,

\[
\sum_{i=t+1}^{t+T+D-1} p_i = (T + D - 1)E[p_t] \tag{17}
\]

\[
\sum_{i=t+1}^{t+T+D-1} X_i = TE[X_t] \tag{18}
\]

and

\[
\sum_{i=t+1}^{t+T+D-1} R_t = (T + D - 1)E[R_t] \tag{19}
\]

This problem is not convex and is very difficult to solve because we do not have a valid starting point for current and future output rates. We propose a heuristic solution where in order to get a good starting point we solve the problem in two steps. In the first step we ignore the bounding constraints (15) and (16) and solve the resulting convex optimization problem using the Lagrange theory. At the end of this step we get an output data rate which may not be satisfying one of the two bounding constraints. In the second step, we use the data rate obtained during first step to get valid output rates satisfying both the constraints.

### III. RELAXED OPTIMIZATION PROBLEM

In this section we solve the relaxed optimization problem obtained by ignoring the bounding constraints (15) and (16). This problem can be written as,

\[
\min \ p_t + (T + D - 1)p \tag{20}
\]

subject to,

\[
\log(1 + p_t g_t) + (T + D - 1)E_g[\log(1 + pg)] = B_t + X_t + (T)X_0 \tag{21}
\]

This is a convex optimization problem since the objective and the constraint function are convex. The above optimization problem can be solved using the Lagrange optimization techniques and KKT conditions are sufficient to arrive at the final solution [7]. Let \( \beta \) be the Lagrange multiplier associated with constraint (18), the Lagrangian is,

\[
\mathcal{L}(p_t, p) = p_t + (T + D - 1)p - \beta \left\{ \log(1 + p_t g_t) + (T + D - 1)E_g[\log(1 + pg)] - B_t - X_t - (T)X_0 \right\}
\]

From KKT conditions \( \frac{\partial \mathcal{L}(p_t, p)}{\partial p_t} = 0 \) and \( \frac{\partial \mathcal{L}(p_t, p)}{\partial p} = 0 \) we get,

\[
\beta \left( \frac{g_t}{1 + p_t g_t} \right) = 1 \tag{22}
\]

\[
\beta E_g \left[ \frac{g}{1 + pg} \right] = 1 \tag{23}
\]

Let, \( f_1(p) = E_g \left[ \frac{g}{1 + pg} \right] \) and \( f_2(p) = E_g[\log(1 + pg)] \). From (20) we get,

\[
\beta = \frac{1}{f_1(p)} \tag{24}
\]

Similarly, from (19) we have

\[
\log(1 + p_t g_t) = \log(\beta g_t)
\]

so we can rewrite (18) as,

\[
\log \left( \frac{g_t}{f_1(p)} \right) + (T + D - 1)f_2(p) = B_t + X_t + (T)X_0 \tag{25}
\]

Based on the above equations we have the following algorithm to find the value of \( R_t \), provided the probability density function (pdf) of the underlying physical channel is known,

1. Numerically solve Eq (22) to get the value of \( p \).
2. For this value of \( p \), find \( \beta \) using Eq (21).
3. The output rate at time \( t \) is \( R_t = \log \left( \beta g_t \right) \).
4. The anticipated scheduled rates for future time slots at time \( t \) are, \( E[R] = \log \left( \beta g_0 \right) \), where \( g_0 \) is the mean channel gain value.

It should be noted that the scheduled rates \( E[R] \) are not the actual future output rates because their exact values cannot be determined until that future time is reached. The value of interest is the current output rate \( R_t \) which may not be satisfying the two constraints given in (15) and (16).

**Remark:** \( f_1(p) \) and \( f_2(p) \) depend on the nature of the underlying physical channel and can be determined if the probability density function (pdf) of random channel variable \( g \) is known. Thus the solution developed in this section is quite general and can be used for any type of channel as long as channel pdf is known and \( f_1(p) \) and \( f_2(p) \) are computable.

**Example:** As an example we determine the values of \( f_1(p) \) and \( f_2(p) \) by assuming the underlying channel to be Rayleigh fading. In this case, random variable \( g \) is exponentially distributed with mean \( g_0 \) and probability density function given by \( e^{-g/g_0} \). With \( f_1(p) \) and \( f_2(p) \) defined on the interval \([0, \infty)\) we have,

\[
f_1(p) = \int_{0}^{\infty} \frac{g}{g_0(1 + pg)} e^{-g} \\mathrm{d}g = \frac{g_0p - e^{1/g_0p} Ei(1/g_0p)}{g_0p^2}
\]

and

\[
f_2(p) = \int_{0}^{\infty} \frac{1}{g_0} \log(1 + pg) e^{-g} \\mathrm{d}g = e^{1/g_0p} Ei(1/g_0p)
\]
where, $E_i$ is the exponential integral function, defined as, 
$$E_i(p) = \int_{p}^{\infty} \frac{e^{-x}}{x} \, dx, \quad p > 0.$$ 

IV. BOUNDING CONSTRAINT ACHIEVEMENT 

Since the output rate has to satisfy both the upper and the lower bound constraints hence there are three possibilities for the value of $R_t$ attained by the above algorithm. Let, $x = R_t$, $y = B_t + X_t$ and $z = (D - 1)E[R]$ in the constraint equations (15) and (16), then these three cases are as follows,

A. Case I: $x \leq y$ and $x \geq y - z$

In this case both the constraints are satisfied so $R_t$ is a valid minimum rate.

B. Case II: $x > y$ and $x \leq y - z$

Constraint (16) is violated because the proposed output rate is higher than total number of packets available for transmission. The output rate is high because the channel is good compared to the mean channel gain. Therefore, valid strategy is to transmit all the available packets in this time slot. We reduce the output rate and make it equal to $y$, i.e,

$$R_t = B_t + X_t$$

It is obvious that by decreasing $R_t$ Constraint (15) is not violated because all the packets are scheduled for instantaneous transmission.

C. Case III: $x \leq y$ and $x < y - z$

In this case Constraint (15) is violated so the delay deadlines of the packets are not achieved. The output rate is less than what is required to ensure the delay constraints. Therefore, we have to increase $x$ or $z$ so that $x + z = y$. The problem can be viewed as rescheduling $y = B_t + X_t$ packets over $D$ time slots which is equivalent to the relaxed optimization problem in the interval $[t, t + D - 1]$,

$$\min \quad p_t + (D - 1)p$$

subject to,

$$R_t + (D - 1)E_y[R] = B_t + X_t$$

This problem can be solved by using the algorithm developed for the relaxed problem discussed before and the resulting value of $R_t$ is the valid output rate which satisfies both the constraints. It is important to mention here that since we are proceeding sequentially in time so we are achieving delay constraint in every time slot. Since $x + z = y$ therefore, Constraint (16) cannot be violated.

It should be noted that both constraints cannot be violated at the same time because they represent the upper and the lower bounds.

V. SIMULATION RESULTS 

We consider a time slotted system where transmission time is divided into discrete slots. The duration of each Transmission Time Interval (TTI) is assumed to be 1ms. For the purpose of simulations we assume that data packets are generated every TTI according to Poisson distribution. Each packet has a strict delay deadline and has to be delivered before its deadline arrives. Moreover, we assume that all the packets have same delay constraint. The size of each packet is assumed to be 1Kbits. The wireless channel is assumed to be Rayleigh fading and the channel gain values are exponentially distributed with mean channel gain of 1. We do not consider the effects of the path loss and shadow fading in the channel model. The noise power is assumed to be normalized and considered to be equal one. The system bandwidth is assumed to be 1MHz.

We compare the performance of our approach with Equal Rate Scheduler (ERS). Since all the input arrivals have same delay constraint of $D$ time slots the ERS divides the incoming packets $X_t$ at time $t$ into $D$ equal portions. One portion of $X_t$ along with the portions obtained during the previous time slots is then scheduled for transmission. The total packets transmitted during each time slot by this scheduler is then a moving average filter and the output rate is given by,

$$R_t(ERS) = \frac{X_t + X_{t-1} + \ldots + X_{t+1-D}}{D}$$

In Figure 2 we plot the average required power for different values of input arrival rates. We assume that all the packets have delay deadline of 10 TTI. It is evident that as the input arrival rate increase more power is required for packet transmission. However, the performance of our approach is far better than the ERS. The power consumed by our algorithm is between 9 and 10 dBs less that that by ERS for the whole range of input arrival rates. This enhanced performance of our scheduler is due to efficient exploitation of the short term channel time variations.

In Figure 3 we plot the mean power required for different values of delay constraints at the input arrival rate of 6
Packets/TTI. When the delay deadline of the packets is 1 TTI all the packets have to be transmitted upon their arrival regardless of the channel conditions. Thus there is no time diversity available for exploitation and the power consumed is almost 38 dBs. However, when the delay deadline of packets is 2 TTI our scheduler consumes 24 dBs of power. This amounts to almost 14 dBs performance gain. Moreover we can see that as the delay deadline is further increased we can have more power savings. However the power saving margin decreases with increasing the delay and the power consumed by allowing the delays of 100 TTI per packet is only 6 dBs less compared to that for 2 TTI packet delays. However it is evident that our scheduler is sensitive to varying delay constraints of the packets and is able to provide more savings as the delay deadline of the packets is increased.

VI. Conclusion

In this paper we studied the problem of energy efficient packet scheduling in wireless fading channels with hard delay constraints. We developed the appropriate upper and lower bounds on output rates and then developed a two step solution. The proposed approach not only respects the delay deadlines of the packets but also exploits the channel time diversity provided by the hard delay constraints. Simulation results show good performance and huge power savings provided by our scheduler.

REFERENCES