Classification of Temporal Information in Situation Analysis

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Abstract - A situation is continuously subject to changes. Therefore, all the representations and the mechanisms used in order to build up a situation model, must take the temporal factor into account. In order to model the current situation, information is collected from different sources, by different ways, and at different degrees of precision. Making such an information useful in a reasoning mechanism, requires a mechanism that is able to fuse information at different levels of uncertainty. The proposed model aims at classifying and representing the temporal information that may be used in the situation analysis process. The combination of multi-source fuzzy information is also addressed.

Keywords: situation analysis, temporal classification, temporal information fusion, fuzzy logic.

1 Introduction

In this document, we aim at classifying temporal information related to occurrence of events and duration of states. These elements has been largely discussed in the literature [1-3] as the basis of representation of changes in dynamic environments. A model is proposed in an effort to define practical solutions for temporal representation and reasoning. Examples of temporal information are used to illustrate the proposed representation model.

In Section 2, we propose a model for event occurrence times. In Section 3, we propose a model to represent state changes and durations. Section 4 presents an extension to the previous models, in order to support fuzzy temporal information. Section 5 presents combination rules for fuzzy temporal information. We end this document by a final conclusion in Section 6.

2 Occurrence of events

There are many definitions of the event concept in the literature [4, 5], but, simply put, an event can be defined as the origin of a state change. In a dynamic environment, the occurrence of events permits objects to switch from one state to another. In our definition, an event has no duration, it is instantaneous [3]. In Allen’s definition [4], an event takes time to occur. For example, the event ‘open the door’ may take several seconds to occur as a result of an action. In our work, we only focus at changes of knowledge state. In the above example, we are only interested in knowing when the door becomes open, and when it stops from being closed. Of course, the period of time between these two events corresponds to the occurrence time in Allen’s definition. However, it is important to determine the state of the door at any time (this is further discussed in Section 2).

Determining the occurrence time of an event may be a real challenge, especially if events occur in a very dynamic environment. Very often, available information about such an environment is incomplete, uncertain, and even worse, it could be erroneous. In order to represent the occurrence time of an event, we intuitively use an instant-based representation. We do not discuss in this document the theoretical foundations and problems of this representation. We are only interested in classifying temporal information related to occurrence of events and duration of states. These elements has been largely discussed in the literature [1-3] as the basis of representation of changes in dynamic environments. A model is proposed in an effort to define practical solutions for temporal representation and reasoning. Examples of temporal information are used to illustrate the proposed representation model.
reasoning, if all the events occurrences are well-known. In the case where at least one date is unknown, it is possible to use relative dates in reasoning. In the case where event occurrence times are known in a full linear order, it is also possible to use pseudo-dates (the occurrence time of an event is its rank in the ordering) [2]. In the following sections, we describe each temporal class and give examples.

2.1 Absolute Date (AD)

A date is absolute if it is defined according to an absolute temporal scale. The scale defines the granularity of the temporal information (e.g., year, month, day, hour, etc.). If the granularity is of one second for example, then the minimum distance between two instants \( t_1 \) and \( t_2 \) is one second (\( \min(\text{dist}(t_1,t_2)=1) \)). It is important to notice that an event has no duration. Therefore, the granularity value is not an event duration, it is the precision of the temporal information we are able to reason about. Hence, within a second of time, we are able to consider only one event occurrence. We don’t know exactly when this event happens within this second. Different granularity values can be used in order to express a date. The granularity value of this date will be the minimum of these values. For example, the date ‘in 3 days and 4 hours’ uses two granularity values (day and hour). The granularity value for this date is one hour.

In general, a date may or not be precise according to a granularity value. For example, the date ‘23 May 00’ is precise, whereas the date ‘Between August and September 99’ isn’t. Given a granularity value of one month, it can be ‘August 99’ or ‘September 99’. This is a fuzzy date. In the following sections, we give more detailed definitions of the precise and fuzzy absolute dates.

2.1.1 Precise Absolute Date (PAD)

A precise absolute date corresponds to a point in the temporal space \( X \) being used (ex. ‘23 May 00’) [Figure 1]. Having a set of PADS, it is possible to define a total ordering relation ‘\(<\)’ on this set: \( \forall d \neq d’, \, d < d’ \) or \( d > d’ \).

![Figure 1: Precise Absolute Date representation.](image)

A PAD can be represented by a time-point and a granularity value: \( \text{PAD} := (\text{granularity}, \text{time-point}) \). In the above definition (put in the Backus-Naur Form), a PAD is defined by two terminals. The date ‘23 May 00’ can be represented by the PAD (day, \( \Delta + 23 \)), where \( \Delta \) is the number of days from the origin until the 1st of May 00. The date ‘In September 1999’ can be represented by the PAD (month, \( \Delta’ + 9 \)), where \( \Delta’ \) is number of months from the origin to the beginning of 1999.

2.1.2 Fuzzy Absolute Date (FAD)

A fuzzy absolute date can take several positions in the temporal space \( X \) being used [Figure 2]. For example, ‘By the summer 99’ is a fuzzy date if the granularity value is one month, it is a precise date if the granularity value is a season.

![Figure 2: Fuzzy Absolute Date representation.](image)

A FAD can be represented by a granularity value and an interval of time-points: \( \text{FAD} := (\text{granularity}, \text{interval}) \). For example, the date ‘By the summer 99’ can be represented by the FAD (month, \( [\Delta+7, \Delta+9] \)), where \( \Delta \) is number of months from the origin to December 98. It is important to notice that changing the granularity value may transform a FAD into a PAD, and vice-versa. For example, the date ‘In September 1999’ represented in Section [2.1.1] can also be expressed by the FAD (day, \( [\Delta, \Delta+30] \)), where \( \Delta \) is the number of days from the origin to the end of August 99. Here, changing the granularity value from one month to a day has transformed a PAD into a FAD.

2.2 Relative Date (RD)

Contrary to an absolute date, a relative date is defined according to another date called the origin. The distance between the two dates is called the offset. In some cases, the origin may be unknown. For example, the relative date ‘two hours after he came in’ is defined according to the time of an event occurrence (he came in), that can be known or not. It is important to mention that in a relative date context, the granularity value is defined according to the offset. In the example given above, the granularity value is one hour. Here again, some relative dates are precise and others are fuzzy (see the following sections for more details).

2.2.1 Precise Relative Date (PRD)

A precise relative date is a relative date where the origin is a PAD, and the offset is well-known. For example, the ‘next week’ is a PRD because the origin is ‘now’ and the offset is well-known [Figure 3].

![Figure 3: Precise Relative Date representation.](image)

A PRD can be defined as the following: \( \text{PRD} := (\text{granularity}, \text{PAD}, n) \) where \( n \) is the number of units of time that compose the offset.

For example, the date ‘Ten days after’ can be represented by the PRD (day, (day, tp), 10), where (day, tp) is a PAD, and tp is a time-point corresponding to the origin. The date ‘Last month’ can also be expressed by a PRD, (month, (month,
now), -1), where (month, now) is a PAD corresponding to the origin (current month).

### 2.2.2 Fuzzy Relative Date (FRD)

A fuzzy relative date is a relative date where either the origin is a FAD, or the offset is fuzzy. For example, ‘within the same month’ is a FRD because the origin could be a PAD (the first or the last day of the month), but the offset is not well-defined if the granularity value is less than a month (Figure 4). If the granularity value is a month, then this is a PRD.

A FRD can be defined as follows:

\[
FRD ::= (\text{granularity}, \text{PAD}, n, m) \mid (\text{granularity}, \text{FAD}, n) \mid (\text{granularity}, \text{FAD}, n, m).
\]

In this definition, if the origin is fuzzy then it is represented by a FAD, and if the offset is fuzzy then it is represented by two integers, \( n \) and \( m \), indicating the minimum and maximum distances to the origin, respectively. For example, the date ‘Within January 2000’ can be represented by the FRD (day, (day, ∆), 1, 31), where \( ∆ \) is the number of days from the origin to the end of 1999.

In some cases, the offset is completely unknown. For example, in ‘several months ago’, we know nothing about the number of months. Hence, for a given granularity value, default values for \( n \) and \( m \) should be defined. This date can be represented by the FRD (month, (month, origin), -4, -10), where 4 and 10 are the minimum and maximum number of months.

### 2.3 Relationships between dates

Representation models for event occurrences (dates) have been presented in the previous sections. In these models, the granularity value is a measure of the precision for dates. In reasoning about dates, it is important to be able to maintain the same precision for all temporal information. For example, if the required granularity is \( g \), then a temporal information received with another granularity value \( g' \) must be transformed before it is used by the reasoning module. A preprocessor is in charge of performing all needed transformations. In the following sections, we demonstrate all possible transformations that could be applied to PADS, PRDs, FADs and FRDs.

#### 2.3.1 PAD vs. PRD

A PAD is of the form (\( g \), \( tp \)), where \( g \) is the granularity value and \( tp \) is a time-point. In the following, we prove that the PAD and PRD representations are equivalent by demonstrating that any PAD can expressed as a PRD, and vice-versa.

\[
(g, tp) = (g, (tp-\Delta)) = (g, (g, tp)), \Delta \text{ is a PAD, then (g, (g, tp)), \Delta is a PRD (Section 2.2.1).}
\]

A PRD is of the form (\( g, pad, n \)), where \( pad \) is a PAD and \( n \) is an integer.

\[
(g, pad, n) = (g, (g', tp), n) = (g, tp*g'/g+n) \text{ (see Figure 6).}
\]

In the first case, (\( g, pad, n \)) = (\( g, (g', tp) \), \( n = (g, tp*g'/g+n) \) is a time-point according to \( g \), hence, (\( g, tp*g'/g+n \)) is a PAD (Section 2.1.1).

\[
\begin{align*}
\text{Figure 4: Fuzzy Relative date representation.} \\
\text{Figure 5: Transformation of a PAD into a PRD.} \\
\text{Figure 6: Transformation of a PRD into a PAD.}
\end{align*}
\]

#### 2.3.2 FAD vs. FRD

In this section, we also demonstrate that a FAD can always be expressed as a FRD, and vice-versa, and hence prove the equivalence of the two representations.

A FAD is of the form (\( g, [tp_1, tp_2] \)), where \( tp_1, tp_2 \) are time-points (see Figure 7).

\[
(g, [tp_1, tp_2]) = (g, (g, tp_1)), 0, tp_2-tp_1 = (g, (g, [tp_1, tp_2])) = (g, pad, n, m) \text{ FRD (Section 2.2.2).}
\]

\[
\text{Figure 7: Transformation of a FAD into a FRD.}
\]

A FAD can take the forms:

- (\( g, pad, n, m \)), where \( pad \) is a PAD, \( n, m \) are integers
- (\( g, fad, n \)), where \( fad \) is a FAD
- (\( g, fad, n, m \)).

In the first case, (\( g, pad, n, m \)) = (\( g, (g', tp) \), \( n = (g, tp*g'/g+m) \) = FAD (Section 2.1.2) (see Figure 8).

\[
\text{Figure 8: Transformation of FRD of the form (g, pad, n, m) into a FAD.}
\]

In the second case, (\( g, pad, n, m \)) = (\( g, (g', tp) \), \( n = (g, tp*g'/g+n) \) = FAD (Section 2.1.2) (see Figure 9).
A PAD takes the form (g, tp). For all g' < g, tp corresponds to an interval [tp1, tp2], (g, tp) = (g', [tp1, tp2]), where tp1 = (tp-1)*g' and tp2 = tp*g'. This is a FAD according to the definition given in Section 2.1.2. If the granularity value increases, a PAD remains a PAD because the time-point tp remains a time-point according to the new granularity value (tp' = tp*g'/g').

A FAD takes the form (g, I). For all g' > g, (g, I) = (g', [I' - , I' + ]) (the time-points I' - and I' + remain time-points and become I'-, I'+, respectively). In addition, if I' = I' then (g', [I' - , I' + ]) = (g', tp') = PAD (Section 2.1.1) else (g', [I' - , I' + ]) remains a FAD (Section 2.1.2). If the granularity value decreases, I becomes an interval [I1, I2] and I' also, [I'1, I'2]. In this case, (g, I) = (g', [I1, I2]) = FAD (Section 2.1.2).

2.3.4 PRD vs. FRD

We demonstrated in Sections 2.3.1 and 2.3.2 that any PRD can be expressed as a PAD, and also that any FRD can be expressed as a FAD. Hence, by using the results of Section 2.3.3 we can deduce that by diminishing the granularity value, a PRD becomes a FRD and conversely, by augmenting this value, a FRD becomes a PRD.

2.3.5 Discussion

In the previous sections, we have demonstrated that it is possible to express any date by a PAD, a PRD, a FAD or a FRD. However, the precision of this date depends on the granularity value. The smaller this value is, the higher the precision of the date is. Hence, if a date is fuzzy, we need to augment the granularity value in order to transform it into a precise one. This doesn’t mean that the date in its new form is more precise than the old one. This depends on the context in which the date is needed. The granularity value (precision) is related to this context. For example, if we need to know the year when an event happened, then a granularity value of a year is enough. The date 'In 1999' defined by the PAD (year, 1999) would be enough. If we choose a granularity of one month, then we cannot define this date by a PAD, but only by a FAD, (1 month, [January 99, December 99]). Within the same context (knowing the year when an event happened), both forms have the same power of expression. However, two different mechanisms of reasoning are needed in order to process these forms. Especially in the second case, the mechanism used must support the processing of fuzzy information. For this reason, the choice of a granularity value is central in the representation of temporal information. This choice depends on the type of temporal information, and the kind of processing that takes this information into account (fusion, propagation of temporal constraints, inference, etc.).

3 Duration of states

In general, states are defined in order to capture the characteristics of a system during a period of time. The definition of a state definition is tightly related to that of an event, since an event permits to switch from one state to another. There are many definitions of the state concept in the literature. Very often, a state correspond to a set of properties that are verified under certain conditions. These properties can be defined as propositions, as in first order logic, or as facts, as in rule-based approaches. We say that the system is in a specific state during a period of time, if the corresponding properties hold during this period of time. In this document, we focus on the definition of the duration of a state, rather than the definition of a state, since the latter is application- or domain-dependent.

In Section 2 the ‘open the door’ event poses a problem, that is, what would be the state of the door in the period between when it was closed and when it becomes open. In order to resolve this problem, we introduce a new state, namely the close-open state, which means that the door is neither closed, nor open in this period. This example leads us to introduce the notion of stability for states. A state can be described as stable or not. In the reasoning mechanism, all states will be considered at the same level. However, at the semantic level, an unstable state of an object means that the corresponding object is being transformed by a specific process (e.g., the process of opening the door).

Contrary to dates (punctual temporal information), periods are continuous. From a theoretical point of view, many problems arise. In particular, one such problem is the definition of the limits of a duration (beginning and end). In practice, a duration can be defined by an interval. Hence, the upper and lower bounds are time-points. Each state has a duration (e.g., an interval). Since an event is punctual (it takes no time), it is difficult to represent a state change. This problem is called the ‘Dividing Instant Problem’ [6]. In our work, we take granularity into account. A duration is
represented by a granularity value and an interval (totally ordered list of time-points). In our representation, all intervals are closed. There is no need to use open intervals since they can be represented by closed ones. For example, given a granularity value g, \([a, b]=[a+1, b-1]\) (a unit of time corresponds to g). For \(g=1\) second, \([5', 8']=[6', 7']\). As for dates, a period may be precise or fuzzy. It may also be absolute or relative.

3.1 Absolute Period (AP)

A period is absolute if it is defined according to absolute dates.

3.1.1 Precise Absolute Period (PAP)

A precise absolute period is an absolute period defined by a granularity value and an interval of time-points:

\[
PAP ::= (\text{granularity}, \text{interval}).
\]

For example, ‘from 23 May to 23 June 2000’ is a PAP, it can be represented by \((1 \text{ day}, (1 \text{ day}, [\Delta+23, \Delta+55]))\), where \(\Delta\) is the number of days from the origin (\(\Delta=0\) if the origin is the first of May). We can remark that a FAD and a PAP have the same syntax. This is due to the fact that a FAD is a date localized within a PAP.

3.1.2 Fuzzy Absolute Period (FAP)

A fuzzy absolute period is a period having a fuzzy beginning or/and a fuzzy end. In addition, the origin and the end of the period are disjoint:

\[
FAP ::= (\text{granularity}, \text{time-point}, \text{interval}) \mid (\text{granularity}, \text{interval}, \text{time-point}) \mid (\text{granularity}, \text{interval}, \text{interval}).
\]

For example, ‘from the 15th of March to sometime in May’ is a FAP. It can be represented by \((1 \text{ day}, \Delta+15, [\Delta+46, \Delta+77])\), where \(\Delta\) is the number of days from the origin to the 1st of March.

3.2 Relative Period (RP)

A relative period is defined according to a date. It has a date as its origin, and an offset.

3.2.1 Precise Relative Period (PRP)

A precise relative period is defined by a granularity value, a PAD and an integer defining the offset:

\[
PRP ::= (\text{granularity}, \text{PAD}, n).
\]

For example, ‘since fall of 1999’ can be represented by the PRP \((\text{season}, (\text{season}, \text{now}), -9)\), where now is winter 2001. The period ‘60 days’ can also be represented by a PRP, \((\text{day}, (\text{day}, \Delta), 60)\), where \(\Delta\) corresponds to the occurrence time of an event. Here again, we can notice that a PRP has the same syntax as a PRD. Indeed, a PRD is a date localized within a PRP.

3.2.2 Fuzzy Relative Period (FRP)

A fuzzy relative period has fuzzy bounds. It is defined as follows:

\[
FRP ::= (\text{granularity}, \text{PAD}, n, m) \mid (\text{granularity}, \text{FAD}, n) \mid (\text{granularity}, \text{FAD}, n, m).
\]

In this definition, if the origin of the period is precise (PAD), then the number of units of time from this origin is between \(n\) and \(m\). The number of units of time can be precise, whereas the origin can be fuzzy (FAD). At last, both the origin and the end of the period can be fuzzy. A FRP has the same syntax as a FRD. The period ‘Within 3 to 5 days’ can be represented by the FRP \((\text{day}, (\text{day}, \Delta), 3, 5)\), where \(\Delta\) is the occurrence time of an event. The period ‘At least 4 months’ can be represented by the FRP \((\text{month}, (\text{month}, \text{now}), 4, +\infty)\), where now is the current month.

3.3 Relationships between periods

As for dates, a change of the granularity value of a period may be necessary. All possible transformations that could be applied to periods are demonstrated in the following sections.

3.3.1 PAP vs. PRP

A PAP is of the form \((g, \{tp_1, tp_2\})\). The origin of this period \(tp_1\) can be represented by the PAD \((g, \text{tp}_1)\). The distance from this origin to \(tp_2\) is \(tp_2-tp_1\). Hence, \((g, \{tp_1, \text{tp}_2\}) = (g, (g, \text{tp}_1), \text{tp}_2-\text{tp}_1) = \text{PRP} \text{ (Section 3.2.1)}\).

A PRP is of the form \((g, \text{pad}, n)\), where \text{pad} is a PAD and \(n\) is an integer. In order to transform this PRP into a PAP, we should represent the origin and the end of this period as time-points. The origin of this period is \(\text{pad} = (g', \text{tp})\), where \(g' \neq g\). According to \(g\), this origin corresponds to the time-point \(tp*g'/g\). The end of the period is at \(n\) units of time (according to \(g\)) from tp. It corresponds to the time-point \(tp*g'/g+n\). Hence, the period \((g, \text{pad}, n) = (g, (g', \text{tp}), n) = (g, [tp*g'/g, tp*g'/g+n])\). According to the definition given in Section 3.1.1 this is a PAP.

3.3.2 FAP vs. FRP

A FAP is of the form \((g, \{tp_1, \text{tp}_2\})\). The origin of this period \(\text{tp}_1\) can be represented by the PAD \((g, \text{tp}_1)\). The distance from this origin to \(\text{tp}_2\) is \(\text{tp}_2-\text{tp}_1\). Hence, \((g, \{\text{tp}_1, \text{tp}_2\}) = (g, (g, \text{tp}_1), \text{tp}_2-\text{tp}_1) = \text{PRP} \text{ (Section 3.2.1)}\).

A FRP is of the form \((g, \text{pad}, n, m)\), where \text{pad} is a PAD and \(n, m\) is an integer. In order to transform this FRP into a PAP, we should represent the origin and the end of this period as time-points. The origin of this period is \(\text{pad} = (g', \text{tp})\), where \(g' \neq g\). According to \(g\), this origin corresponds to the time-point \(tp*g'/g+n\). Hence, the period \((g, \text{pad}, n, m) = (g, (g', \text{tp}), n, m) = (g, [tp*g'/g, tp*g'/g+n])\). According to the definition given in Section 3.1.1 this is a PAP.
In the second case, if we take \((g, tp)\) as the origin of the period, the end of this period is from \(tp - I^-\) to \(tp - I^+\) units of time before the origin (see Figure 12). Hence, \((g, I, tp) = (g, (g, tp), tp - I^-, tp - I^+) = FRP\) (Section 2.2.2).

![Figure 12: Transformation of a FAP of the form \((g, I, tp)\) into a FRP.](image)

In the third case, \((g, I, I') = (g, \{I^-, I^+\}, \{I'^-, I'^+\})\). We take the FAP \((g, I)\) as the origin of this period. The minimum distance from the origin to the end of the period is \(I'^- - I^+\) (the intervals are disjoint). The maximum distance between the origin and the end of the period is \(I'^+ - I^-\) (see Figure 13). Hence, \((g, I, I') = (g, \{I^-, I^+\}, \{I'^-, I'^+\}) = FRP\) (Section 3.2.2).

![Figure 13: Transformation of a FAP of the form \((g, I, I')\) into a FRP.](image)

A FRP is of the form \((g, pad, n, m), (g, fad, n), (g, fad, n, m)\), where pad is a PAD, \(n\) and \(m\) are integers and fad is a FAD. In the first case, \((g, pad, n, m) = (g, (g', tp), n, m)\), where pad = \((g', tp)\). The origin of the period is the time-point \(tp * g'/g\) (according to \(g\)) (see Figure 14). The end of the period is within the interval \([tp * g'/g + n, tp * g'/g + m]\). Hence, \((g, pad, n, m) = (g, tp * g'/g, \{tp * g'/g + n * g, tp * g'/g + m * g\}) = FAP\) (Section 3.1.2).

![Figure 14: Transformation of a FRP of the form \((g, pad, n, m)\) into a FAP.](image)

In the second case, \((g, fad, n) = (g, (g', I), n) = (g, (g', \{I^-, I^+\}), n)\), where fad = \((g', I)\). The origin of the period is fad. According to \(g\), fad = \((g', \{I'^-, I'^+\})\). The time-point \(tp\) remains a time-point \(tp'\) according to \(g'\). The upper and lower bounds of interval \(I\) remain time-points, and the interval \(I\) becomes \(I'\). \(I' = \{I'^-, I'^+\}\), if \(I'^- = I'^+\) then the interval \(I'\) is a time-point. In this case, \((g', tp', I') = (g', \{tp', tp'\'}) = PAD\) (see Section 2.1.1). If \(tp' = tp'\), then \((g', \{tp', tp'\'}) = (g', \{tp', tp'\'}) = PAD\) (see Section 3.1.1).

The second case \((g, I, tp)\) is the same as the first case. In the last case, the interval \(I\) (resp. \(I'\)) may remain an interval \(I_1\) (resp. \(I'_1\)) or become a time-point \(tp\) (resp. \(tp'\)). If we combine all these possibilities, we have the following results:

- \((g, I, I') = (g', \{tp', tp'\'}) = PAP\) (see Section 3.1.1).
- \((g, I, I') = (g', \{tp', tp'\'}) = PAP\) (see Section 3.1.1).
- \((g, I, I') = (g', I_1, I'_1) = PAP\) (see Section 3.1.1).
- \((g, I, I') = (g', I_1, I'_1) = PAP\) (see Section 3.1.1).

If the granularity value decreases, the time-point \(tp\) is transformed into interval \(I_0\), and the lower bound of the interval \(I, I'\), is transformed into the interval \([I^-, I'_1]\), and the upper bound \(I'_0\), is transformed into the interval \([I'^-, I'_1]\). In this case, we have the following results:

- \((g, I, I') = (g', I_0, I'_0) = PAP\) (see Section 3.1.2).
- \((g, I, I') = (g', I_0, I'_0) = PAP\) (see Section 3.1.2).

3.3.3 PAP vs. FAP

As for dates, changing the granularity value, may transform a PAP into a FAP, and vice-versa.

A PAP is of the form \((g, I) = (g, \{I^-, I^+\})\). \(\forall g' < g\), \(I^-\) can be written as the interval \([I_1^-, I_2^-]\), and \(I^+\) as the interval \([I_1^+, I_2^+]\). Hence, \((g, \{I^-, I^+\}) = (g', \{I_1^-, I_2^-\}, \{I_1^+, I_2^+\})\). According to the definition given in Section 3.1.2 this is a FAP.

If the granularity value increases \((g' > g)\), then \((g, \{I^-, I^+\}) = (g', \{I'^-, I'^+\}, \{I'^-, I'^+\})\). This remains a PAP. If in addition, \(I'^- = I'^+\), then the PAP is a point period.

A FAP is of the form \((g, tp, I), (g, I, tp)\) or \((g, I, I')\). If \(g' > g\), then in the first case, \((g, tp, I) = (g', tp', I')\). The time-point \(tp\) remains a time-point \(tp'\) according to \(g'\). The upper and lower bounds of interval \(I\) remain time-points, and the interval \(I\) becomes \(I'\). \(I' = \{I'^-, I'^+\}\), if \(I'^- = I'^+\) then the interval \(I'\) is a time-point. In this case, \((g', tp', I') = (g', \{tp', tp'\'}) = PAP\) (see Section 3.1.1). If \(tp' = tp'\), then \((g', \{tp', tp'\'}) = (g', \{tp', tp'\'}) = PAD\) (see Section 2.1.1).

The second case \((g, I, tp)\) is the same as the first case. In the last case, the interval \(I\) (resp. \(I'\)) may remain an interval \(I_1\) (resp. \(I'_1\)) or become a time-point \(tp\) (resp. \(tp'\)). If we combine all these possibilities, we have the following results:

- \((g, I, I') = (g', \{tp', tp'\'}) = PAP\) (see Section 3.1.1).
- \((g, I, I') = (g', \{tp', tp'\'}) = PAP\) (see Section 3.1.1).
- \((g, I, I') = (g', I_1, I'_1) = PAP\) (see Section 3.1.1).
- \((g, I, I') = (g', I_1, I'_1) = PAP\) (see Section 3.1.1).

If the granularity value decreases, the time-point \(tp\) is transformed into interval \(I_0\), and the lower bound of the interval \(I, I'\), is transformed into the interval \([I^-, I'_1]\), and the upper bound \(I'_0\), is transformed into the interval \([I'^-, I'_1]\). In this case, we have the following results:

- \((g, I, I') = (g', I_0, I'_0) = FAP\) (see Section 3.1.2).
- \((g, I, I') = (g', I_0, I'_0) = FAP\) (see Section 3.1.2).
- \((g, I, I') = (g', I_0, I'_0) = FAP\) (see Section 3.1.2).
- \((g, I, I') = (g', I_0, I'_0) = FAP\) (see Section 3.1.2).
3.3.4 PRP vs. FRP

By using the results of the previous sections, it is easy to demonstrate that a PRP can be transformed into a FRP, and vice-versa: PRP ≡ PAP (Section 3.3.1), a PAP can be transformed into FAP, and vice-versa (Section 3.3.3), and FAP ≡ FRP (Section 3.3.2).

3.3.5 Discussion

Here again, the granularity value is a measure of precision for periods. Some definitions of dates and periods are similar. This is due to the fact that we need the same primitives to represent both of them. Precise dates are based on time-points, fuzzy dates and precise periods are based on intervals and, finally, fuzzy periods are based on fuzzy intervals that can be represented by either an interval and a time-point, or two intervals. The use of similar representations imply the use of similar temporal reasoning mechanisms. For example, the reasoning on a fuzzy date will be the same as the reasoning on a precise period. Only the interpretation of the results will differ.

4 Fuzzy membership functions

In the previous sections, FADs, FRDs, FAPs, and FRPs have been defined according to crisp constraints, that is, all of the possible values are equiprobable. For example, the FRD ‘In the past few months’ is represented by (month, (month, now), -4, -10) where the origin is now (current month) for instance. The possible values for this date are in the set A = {now-4, now-5, now-6, now-7, now-8, now-9, now-10}. All of these values have the same degree of possibility. Let this degree be defined by the function $\mu = 1$, that is, $\forall t \in X, \mu(t) = 1$ if $t \in A$ and $\mu(t) = 0$ if $t \notin A$.

In some contexts, $\mu$ can be defined differently so that it has a different degree of possibility for each value of X (temporal space). In this case, $\mu$ is called the membership function, and this is widely used in the fuzzy sets theory [7, 8]. For instance, if the date is now-3, it can be referred to by ‘In the last few months’. In order to represent that this value has a lower degree of possibility of being the date, we extend A to cover more values, and we change the definition of $\mu$ as it is shown in Figure 17-b.

In Figure 17-b, we can see that the degree of possibility of now-3 is of 0.5 ($\mu$(now-3) = .5), while it is of 1 for now-6 ($\mu$(now-6) = 1). The interpretation of such a result is that now-6 is more probable than now-3; hence it is more appropriate for the description of ‘In the last few months’.

The definition of such membership functions is very powerful when it is used in a reasoning scheme. In the next section we will see how fuzzy temporal information can be combined in order to derive new temporal information. Before that however, we need a redefinition of FADs, FRDs, FAPs and FRPs. As each FRD (resp. FRP) corresponds to a FAD (resp. FAP) (Sections 2.3.2 and 3.3.2), then it is sufficient to redefine FAD and FAP:

$$\text{FAD} ::= (\text{granularity}, \text{interval}, \text{membership-function})$$

$$\text{FAP} ::= (\text{granularity}, \text{time-point}, \text{interval}, \text{membership-function})$$

$$\text{FAP} ::= (\text{granularity}, \text{interval}, \text{time-point}, \text{membership-function})$$

$$\text{FAP} ::= (\text{granularity}, \text{interval}, \text{interval}, \text{membership-function}, \text{membership-function}).$$

Figure 17: Crisp (17-a) and trapezoidal (17-b) membership functions.

In the previous definitions given in Sections 2.1.2 and 3.1.2, the membership function $\mu$ was constantly equal to 1; it was thus omitted from the representations. In the definitions given above, a membership function is added. In the FAP definition, two membership functions are used in the case of both a fuzzy start and a fuzzy end of a period. Note also that whatever the membership function is, a FRD (resp. FRP) can be written in its FAD (resp. FAP) form following the definitions given above1. If the granularity value must change, the membership functions are redefined over the new intervals. An interpolation of this function can be used in the case where the granularity value decreases.

5 Fuzzy information combination

An interesting matter with using membership functions is the possibility of combining several fuzzy temporal information. This combination can be used in a context of multi-source information with different degree of uncertainty. It is important to mention, that the temporal information to be combined may have different granularity values. Before the combination, all the information must be transformed in order to have the same granularity value. In general, this value corresponds to maximum of the granularity values of all the temporal information to be combined. The combination rules are defined in [9, Chapter

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1 The membership function can be added/changed at any time. Hence the preprocessing of Section 3 is independent of such a function.
among others. Let’s suppose that we have received, from two different sources, two FADs corresponding to the occurrence of the same event, the entrance of a group of people in a sensitive zone, for example. The first FAD is defined by \((g, A, \mu_A)\) and the second one by \((g, B, \mu'_B)\). A and B are intervals. \(\mu_A\) (resp. \(\mu_B\)) defines the degree of possibility that an element of A (resp. B) is the occurrence time of the event. The idea is to combine these FADs in order to form a new consistent FAD.

The degree of possibility that the occurrence time is in A and B (\(A \cap B\)) is defined by: \(\mu_{A \cap B}(d) = \min[\mu_A(d), \mu_B(d)]\).

The degree of possibility that the occurrence time is in A or in B (\(A \cup B\)) is defined by: \(\mu_{A \cup B}(d) = \max[\mu_A(d), \mu_B(d)]\).

In the above definitions, min and max functions are used because they are the most widely used T-norm and T-conorm in the literature and the applications (see [9, Chapter 11] for definitions). There is an infinite number of T-norms and T-conorms that can be used in the above definitions instead of min and max [10]. Examples of combinations are illustrated in Figure 18. In Figures Figure 18a and Figure 18b are represented the membership functions \(\mu_A\) and \(\mu_B\). In Figure 18c is represented the membership function \(\mu_{A \cap B}\). At last, Figure 18d illustrates the membership function \(\mu_{A \cup B}\).

The use of \(\mu_{A \cap B}\) and \(\mu_{A \cup B}\) depends on whether the new incoming information is considered as additional information (the max of the membership functions is then chosen) or as a new temporal constraint that must be respected (the min of the membership functions is then chosen). Hence, it is important to know what kind of information is to be combined. If both sources of information are reliable, then \(\mu_{A \cap B}\) can be used in order to restrict the degree of possibility, and \(\mu_{A \cup B}\) can be used in order to maximize the degree of possibility.

Figure 18. Combination of multi-source information.

6 Conclusions

In this document, we have presented a model for temporal representations that could be used in the situation analysis process. Since temporal information could have different degrees of precision, it is important that the model supports such a flexibility in the representation. In our model, the representations are based on the granularity value. Models have been proposed to represent dates (occurrence times for events) and periods (duration of states). These concepts can be precise or fuzzy, they can also be absolute, or relative. We used such models in a specific application, the fusion of fuzzy temporal information. These models are also well-adapted to large number of applications where the temporal factor is of importance, such as temporal constraint satisfaction, simulation, reasoning, and knowledge representation.

Presently, we are investigating the use of fuzzy temporal constraint networks that could be used in order to monitor scenarios of events.

7 References