CONTROL OF A PWM INVERTER USING PROPORTIONAL PLUS EXTENDED TIME-DELAYED FEEDBACK

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Pulse width modulation (PWM) current-mode single phase inverters are known to exhibit bifurcations and chaos when parameters vary or if the gain of the proportional controller is arbitrarily increased. Our aim in this paper is to show, using control theory and numerical simulations, how to apply a method to stabilize the interesting periodic orbit for larger values of the proportional gain. To accomplish this aim, a time-delayed feedback controller (TDFC) is used in conjunction with the proportional controller in its simple form as well as in its extended form (ETDFC). The main advantages of those methods are the robustness and ease of construction because they do not require the knowledge of an accurate model but only the period of the target unstable periodic orbit (UPO). Moreover, to improve the dynamical performances, an optimal criterion and an adaptive law are defined to determine the control parameters.

Keywords: ETDFC; current inverter; bifurcation; UPO; chaos control.

1. Introduction

Power electronics is a discipline that has emerged from the need to convert electrical energy. Its field of application is wide and concerns industrial, commercial, residential and also aerospace environments. Power converters are basic switching circuits that are modeled by a number of linear differential equations corresponding to different topologies. The toggling between different topologies can either be done naturally due to switching characteristics or done under the force of a feedback control system. Due to the existence of various operating modes and control saturations, the overall operation is compared to a piecewise smooth nonlinear dynamical system. Early studies by Hamill and coworkers [Hamill & Jefferies, 1988; Deane & Hamill, 1990] have shown that switching power converters can exhibit several nonlinear phenomena. These include subharmonic oscillations, quasi-periodic operations, bifurcations and chaos.

Chaos in power electronics have intrigued many researchers [Hamill & Jefferies, 1988; Di Bernardo & Tse, 2002; Robert & Robert, 2002b]. In fact, engineers have frequently encountered chaos in power electronics systems, but more often than not this phenomenon was considered as strange and undesirable, hence engineers usually attempted to avoid chaos. During the last two decades, tools of analyzing bifurcations and chaos have been well developed. Therefore, the investigation of the very peculiar aspect of this phenomenon has become an attractive endeavor.

Actually, much work has focused on bifurcation and chaos due to parameter variations [Banerjee
& Chakrabarty, 1998; Di Bernardo & Vasca, 2000; Benadero \textit{et al.}, 2003]. Recently it has been shown by Robert and Robert [2002b] that control parameters themselves may lead to bifurcation and chaos if they are badly selected.

Since the seminal paper of Ott \textit{et al.} [1990] (OGY), control of chaos has been the focus of a growing literature. Knowing that a chaotic attractor contains infinitely many UPO, the OGY methods take advantage of the great sensitivity of chaotic orbits to stabilize a UPO by appropriately perturbing an accessible parameter [Grebogi & Lai, 1997; Ditto \textit{et al.}, 1995]. These methods suffer from lack of robustness to imprecise measurements and to uncontrolled parameter variations.

To overcome this deficiency, the TDFC has been proposed as an alternative method [Pyragas, 1992, 1995]. TDFC is known for its robustness and simplicity of construction. Besides, the system model need not be known but only the length $\tau$ of the UPO to be stabilized is essential. The control signal is proportional to the difference between the current state and the $\tau$-delayed state. Once the desired UPO is stabilized the control signal vanishes. The TDFC controller is then switched off. This method has been successfully applied to the control of the Duffing equation [Basso \textit{et al.}, 1997], discrete chaotic systems [Konishi & Kokame, 1998] and to the control of power converters [Batle \textit{et al.}, 1999; In & Robert, 2003a; Robert \textit{et al.}, 2004].

In this paper we propose to control a PWM current-mode H-bridge inverter using TDFC [In & Robert, 2003b] and ETDFC methods in conjunction with the proportional controller. A discrete model describing the behavior of the converter is developed and digital controllers are designed. Using the Jury criteria a stability zone in the parameter space is defined. In this study we show the effect of the TDFC and ETDFC on the bifurcation diagrams and we present two-dimensional bifurcation diagrams.

Our work will be outlined as follows. In Sec. 2 we present the discrete model of an H-bridge converter circuit. Section 3 gives an overview on the effect of the variation of the proportional gain and boosting of the chaotic behavior. Sections 4 and 5 will be devoted to the presentation of the TDFC and ETDFC controllers and their results, respectively. In Sec. 4 we show a two-degrees of freedom design procedure and the resulting two-dimensional bifurcation diagram. The ETDFC controller presented in Sec. 5 leads to a three-degrees of freedom design and improves the results obtained in the preceding sections. Section 6 includes results on sinusoidal output tracking. Finally, our conclusions and remarks are stated in Sec. 7.

2. H-Bridge Model

In order to increase dynamical performances, a growing number of applications in the field of electrical engineering require that they be fed by a precise current generator. However, all industrial power sources are voltage generators. It therefore follows that converters are necessary to adapt voltage sources to loads. Indeed, owing to the presence of many windings in electrical machines, most of them are naturally inductive and are deemed to be current sources. A convenient way to adapt sources is to add a current control to a voltage converter. In this section, we describe the converter structure and its running mode before setting up its sampled data model.

With the goal to increase the efficiency of the power stages, static converters operate by switching the load between several voltage sources. There is a wide range of conversion structures, more or less complex, whose choice depends on many parameters. By controlling the switching pattern over the operating period, it is possible to vary the average output voltage. This technique, called Pulse Width Modulation (PWM), is very widespread. Naturally, from an energy point of view, the average output voltage keeps an obvious meaning if the output current of such converter is ideally constant or if it varies slowly compared to the switching period.

A well-known structure, used in most variable speed drives, is the H-Bridge. Figure 1 illustrates the circuit of this voltage inverter i.e. a DC/AC converter. A switch is realized in this bridge by combining a bipolar transistor and an anti-parallel diode. The inverter is fed by a voltage source $E$ and it...
supplies a resistive and inductive load (L/R). The output current is controlled by a current loop. The four equivalent switches, named \( S_1, S_2, S_3 \) and \( S_4 \), are shared among two pairs (\( S_1, S_2 \) and \( S_3, S_4 \)). Pairs are controlled by the PWM modulator in a complementary way. States of the switches define two distinct topologies of the inverter, \( T_1 \) (\( S_1, S_2 \) on and \( S_3, S_4 \) off) and \( T_2 \) (\( S_1, S_2 \) off and \( S_3, S_4 \) on), and yield to two opposite voltages across the load. For example, topology \( T_1 \) implies that the voltage across the load \( v = E \).

The PWM modulator defines the switching period \( T \). It generates a high state (\( C = 1 \)), centered in the period, whose duration is defined by the current controller acting on the duty ratio \( d \). \( d_n \) denotes the duty cycle on the \( n \)th functioning period. The low state (\( C = 0 \)) is split into two parts at the beginning and the end of the period. So the normal running mode of the converter involves a sequence of three topologies: \( T_1 \rightarrow T_2 \rightarrow T_1 \). Then the voltage \( v \) evolves as depicted in Fig. 2 and \( v = -1)^C E \). This pattern avoids switchings and interferences at the sampling times because the digital controller samples current at the beginning of the switching period. At that point the sampling frequency is equal to the fundamental PWM frequency.

Figure 2 also shows different possible waveforms of the output current under a normal periodic operating mode. As explain before, it is important to reduce the current ripple within a period. For that aim (see the dashed curve), the inverter period is chosen much smaller than the electric time constant \( L/R \). This figure assumes also that the reference current \( i_n \) is constant. In fact, it is possible to track a sinusoidal current reference (see Sec. 6).

Due to the sampled nature of the current controller, a nonlinear map will be required as a model of the converter instead of a usual linear averaged state space model. For the sake of generality, the current is scaled with respect to the maximum output current \( E/R \) and the voltage is scaled to \( E \). On each interval, the one-dimensional model is described by a single linear differential equation.

\[
\frac{1}{2} \frac{di}{dt} = \alpha + (-1)^C \beta,
\]

where \( \delta = RT/2L \ll 1 \) in order to reduce the ripple current. By integrating on the three intervals and stacking up the solutions, the model becomes a nonlinear discrete time map.

\[
i_{n+1} = \alpha i_n + \beta (2\sinh(\delta n) - \sinh(\delta))
\]

where \( \alpha = e^{-2\delta} \) and \( \beta = 2e^{-\delta} \).

For the sake of clarity, it is important to note that due to boundedness of the duty cycle \( d \), the PWM modulator introduces saturations to limit too much wandering i.e. to keep the duty cycle \( d \) inside the interval \([0, 1]\). Saturated running modes are characterized by a unique topology over \( T \) namely \( T_1 \) or \( T_2 \). Saturation of the modulator requires in fact a piecewise model and can be studied in an analytical way by the normal form theory. Saturations lead to the emergence of border collision bifurcations when controller parameters vary. These phenomena have been thoroughly investigated in previous works [Robert & Robert, 2002a, 2002b]. Because our goal in this paper is to define a new controller stabilizing the normal running mode over a wide range, we focus on the standard model (2).

With the objective of the design of an experimental prototype, the constants are chosen in order to make it easier with a power that does not exceed a few kW and to satisfy the frequency condition presented above. To maintain the ripple current at a low level, we set \( L/R = 0.5 \) ms and \( T = 0.2 \) ms, hence parameters were chosen as follows:

\[
R = 40 \Omega \quad L = 20 \text{mH} \quad E = 400 \text{V},
\]

Leading to the following constants:

\[
\delta = 0.2 \quad \alpha = 670.32 \times 10^{-3} \quad \beta = 1.6375.
\]
3. Current-Programmed Inverter

Figure 3 depicts an H-Bridge current-programmed single phase inverter. The proportional corrector is the main controller used to control the switching process and is given by:

\[ \gamma_n = \gamma_{n-1} = k(I_n - i_n) \]  

where \( k \) is a proportional gain and \( I_n \) is the reference current. In this case, the PWM modulator generates the duty cycle:

\[ d_n = \frac{1}{2} + \frac{1}{2} + k(I_n - i_n) \]

As a matter of fact, when static converters are driven by a \( T \)-periodic clock, it is important to have a \( T \)-periodic output current. Using the discrete-time map given by (2) and (4), the \( T \)-periodic output corresponds to a fixed point of order one. The local stability of the \( T \)-periodic mode is analyzed by deriving the Jacobian of the discrete-time map in the neighborhood of the fixed points. By solving \( i_{n+1} = i_n \), we obtain the fixed point denoted by \( i^* \) as a function of \( k \), and thereby we can determine the loci of the eigenvalues when \( k \) varies. By combining the inverter model (2) and the proportional controller (4) we obtain a first-order
closed loop iteration with a unique eigenvalue $\lambda_P$ given by:
\[ \lambda_P = \alpha - 2\beta \kappa \cosh(\delta d^*), \]
where $d^*$ is the solution of:
\[ \begin{cases} (\alpha - 1) s^* + \beta (2 \sinh(\delta d^*) - \sinh(\delta)) = 0, \\ d^* = \frac{1}{2} + k(I_n - i^*). \end{cases} \]
When $k$ varies the eigenvalue crosses the unit circle at $\lambda_0 = -1$ with $k = k_0$. Using the fact that $\delta d^* \ll 1$ it follows that $\cosh(\delta d^*) \approx 1$ and
\[ k_0 = \alpha + \frac{1}{2\beta \kappa} = 2.55 \]
Therefore, for $k > k_0$ the $T$-periodic mode becomes unstable. The bifurcation diagram depicted in Fig. 4 was obtained by iterating Eqs. (2) and (4) and the exact value of $k_0 = 2.52$ is obtained. It has been shown in [Robert & Robert, 2002b] that the fixed point of order one (corresponding to $T$-periodic orbit) continues to exist but becomes unstable.

4. Time-Delayed Feedback Controller

4.1. Controller design

The TDFC controller is similar to a proportional corrector but referenced to the same state delayed by a time $\tau$ equal to the length of the UPO to be stabilized. The aim of this section is to stabilize the UPO of length $T$. Using the discrete-time notation, this corresponds to stabilizing the fixed point of order one for higher values of $k$ thus the TDFC expression is given by:
\[ \gamma_Dn = \eta(i_n - i_{n-1}). \]
where $\eta$ is the TDFC gain to be adjusted. Figure 5 delineates the block diagram of the TDFC controller. The whole control signal and the duty cycle become:
\[ \gamma_n = \gamma_{Pn} + \gamma_{Dn} \]
\[ d_n = \frac{1}{2} + k(i_n - i_{n-1}) + \eta(i_n - i_{n-1}) \]

4.2. Stability analysis

When TDFC is applied, the discrete system defined by (2) and (8) becomes second order since it involves $i_n$ and $i_{n-1}$. Let us define $x_n = i_{n-1} + i^*$ and $y_n = i_n - i^*$, using $x_n$ and $y_n$ as the new state variables and combining (2) and (8) yields
\[ x_{n+1} = y_n, \]
\[ y_{n+1} = \alpha y_n + (\alpha - 1) s^* + \beta (2 \sinh(\delta d_n) - \sinh(\delta)). \]

We investigate the local stability by deriving the Jacobian evaluated at the origin:
\[ J(d^*) = \begin{bmatrix} 0 & 1 \\ J_1(\eta, d^*) & J_2(\eta, k, d^*) \end{bmatrix} \]
Since at equilibrium we have $i_n = i_{n-1}$ then TDFC signal is null. Thus $d^*$ is again the solution of (6) and it follows that
\[ J_1(\eta, d^*) = -2\beta \eta \delta \cosh(\delta d^*) \]
\[ J_2(\eta, k, d^*) = \alpha + 2\beta (\eta - k) \cosh(\delta d^*) \]
The characteristic equation is given by:
\[ s^2 - J_2 s - J_1 = 0 \]
The Jury criterion is used to evaluate the stability domain in terms of $k$ and $\eta$. According to the Jury criterion, the system is stable if the following conditions are satisfied:
\[ |-J_1| < 1, \]
\[ 1 - J_2 - J_1 > 0, \]
\[ 1 + J_2 - J_1 > 0. \]
Equation (13) yields to an upper limit on $\eta$
\[ \eta < \frac{1}{2\beta \kappa \cosh(\delta d^*)} \]
Eq. (14) yields to a lower limit on $k$
\[ k > \frac{\alpha - 1}{2\beta \kappa \cosh(\delta d^*)} \]
We should note that $\alpha - 1 < 0$, then it is sufficient to choose $k > 0$, in order to satisfy condition (17) as well as the positiveness of the proportional corrector gain. The last of Jury’s conditions (15) leads to an
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Fig. 6. Stability zone in the $k - \eta$ space.

affine relation between $\eta$ and $k$:

$$\eta > \frac{k}{2} - \frac{\alpha + 1}{4\beta\delta} \cosh(\delta d^*) \quad (18)$$

Using conditions (16)–(18), a stability zone in the parameters space $k - \eta$ is defined and depicted in Fig. 6.

The shaded zone has a special interest if we suppose that the reference is constant. Indeed, Eq. (9) can be written in the following form:

$$\gamma_n = k(I_n - i_n) - \eta((I_n - i_n) - (I_n - i_{n-1}))$$

Denote $\epsilon_n = I_n - i_n$ the steady state error, then the control signal is similar to a Proportional-Derivative (P.D.) controller:

$$\gamma_n = K_P\epsilon_n + K_D(\epsilon_n - \epsilon_{n-1})$$

where $K_P = k$ and $K_D = -\eta$. Ordinarily we have $K_P > 0$ and $K_D > 0$, then the conjunction of the proportional and the TDFC controllers can be assimilated to a P.D. controller inside the shaded zone of Fig. 6. It is known from linear control theory that the larger the proportional gain $k$ the less is the steady state error $\epsilon$. In view of Fig. 6, $k$ is maximized when $\eta$ is maximized and the limit of stability is the point $M$ which is the intersection of boundary lines

$$L_1: \eta = \frac{1}{2\beta \delta} \quad \text{and} \quad L_2: \eta = \frac{k}{2} - \frac{1 + \alpha}{4\beta\delta}$$

obtained from Eqs. (16) and (18) with the approximation $\cosh(\delta d^*) \simeq 1$. The coordinates of $M$ are:

$$\eta_{\text{max}} \simeq \frac{1}{2\beta \delta} = 1.5 \quad (19)$$

$$k_{\text{max}} \simeq \frac{3 + \alpha}{2\beta\delta} = 5.6 \quad (20)$$

4.3. Results

The two-dimensional bifurcation diagram depicted in Fig. 7 delineates in brown the zone of the period one mode and this matches the stability zone shown in Fig. 6. Zones of higher periods as well as chaotic
zones are also shown as parameters $k$ and $\eta$ vary. Particularly, when $\eta = 0$ we observe that as $k$ increases the output current is of period one then becomes period two and for a short interval of $k$ becomes period four before it goes into chaotic mode. As $k$ increases further the output becomes period six and finally arrives at period three mode and this matches very well with the one-dimensional bifurcation diagram shown in Fig. 4. We also notice that as $\eta$ increases and a TDFC controller is acting then the period one mode extends for higher values of $k$ as expected (see also Fig. 8). Figure 8 shows that the fixed point of order one that was embedded in a zone of period two mode ($2.5 \leq k \leq 3.3$) and in a chaotic attractor ($3.3 \leq k \leq 5.0$) has been stabilized by the application of the TDFC in conjunction with the proportional controller.

4.4. Optimality criterion

The main concern of the previous design was to obtain stability of the fixed point of order one for larger values of $k$ which reduces the steady state error $\varepsilon_n$. In this section, we show that a drawback of TDFC is to degrade the dynamical response by increasing the settling time. Figure 9 shows the sampled and hold output current resulting from a step reference from $I_n = 0$ to $I_n = 0.5$ with $k = 2$ and different values of $\eta$. We note that the transient time is increased when the TDFC is applied. When $\eta$ tends to $\eta_{\text{max}}$, the transient time becomes too large.

To circumvent this problem we present an adaptive law to calculate a value of $\eta$ that leads to a fast response for each value of $k$. The linearization of system (10) gives

$$
\begin{bmatrix}
    x_{n+1} \\
    y_{n+1}
\end{bmatrix} =
\begin{bmatrix}
    x_n \\
    y_n
\end{bmatrix} +
\begin{bmatrix}
    f_{x0} \\
    f_{y0}
\end{bmatrix}.
$$
This implies that [Callier & Desoer, 1991]

$$\|x_{n+1}\| = \|J^{n+1}\| \|x_0\| \leq \pi(n) \sigma(J)^{n+1} \|x_0\|$$

(21)

where $\pi(n) \geq 0$ is a positive polynomial in $n$ and $\sigma(J)$ is the spectral radius of $J$. To obtain a fast response we should choose $k$ and $\eta$ that minimize $\sigma(J)$. In case of complex eigenvalues $\sigma(J) = \sqrt{-J}$, thus $\eta$ should be as small as possible. When $\eta$ is less than or equal to a critical value $\eta_c$ (i.e. $\eta \leq \eta_c$), the eigenvalues become real and the spectral radius is minimum if both eigenvalues are equal, that is when the discriminant is equal to zero.

$$J^2(\eta_c, d^*) + 4J(\eta_c, d^*) = 0$$

solving for $\eta_c$, we obtain

$$\eta_c = k + \frac{2 - \alpha - 2\sqrt{2\delta k} + 1 - \alpha}{2\delta k}.$$  

(22)

Figure 10 shows the loci of the eigenvalues when $\eta$ varies for different values of $k$. Figure 11 depicts the effect of adapting the TDFC gain $\eta$ according to the value of the proportional gain $k$. Indeed, for $k = 4$ the proportional gain on its own leads to a chaotic output as shown in Fig. 4. However, if the TDFC is arbitrarily added ($\eta = 1.3$) the fixed point of order one is stabilized and the step reference is tracked with a certain static error. Nevertheless, the settling time equal to 50 cycles (10 ms) is significantly long. Eventually, the adapted TDFC ($\eta = 0.86$) yields to the stability of the fixed point with a considerably shorter settling time equal to 7 cycles (1.4 ms).

5. Extended Time-Delayed Feedback Controller

5.1. Controller design

The ETDFC is a generalization of the TDFC and provides the designer with a third parameter $r$ hence one more degree of freedom. The ETDFC extends the effect of earlier states to the present output with a decaying weight as we go further in the past. The output signal of the ETDFC is given by:

$$\gamma_{En} = \eta \left( i_n - (1 - r) \sum_{m=1}^{n} e^{m-1} i_{n-m} \right).$$

(23)

where $k$, $\eta$ and $0 \leq r < 1$ are control parameters to be fixed to guarantee stability of the fixed point of order one. We clearly notice that (23) is equivalent to (8) if $r = 0$. The block diagram realization of the ETDFC is shown in Fig. 12. The overall control signal and the duty cycle are expressed as:

$$\gamma_n = \gamma_{p_n} + \gamma_{En}$$

(24a)

$$d_n = \frac{1}{2} + k(I_n - i_n)$$

(24b)

$$+ \eta \left( (1 - r) \sum_{m=1}^{n} e^{m-1} i_{n-m} \right).$$
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Fig. 12. ETDFC controller.

5.2. Stability analysis

To derive necessary conditions for local stability of the fixed point of order one, we start by finding the Jacobian matrix of the discrete-time map. Let us first denote

\[ z_{n+1} = \eta \left( i_n - (1 - r) \sum_{m=1}^{n} i_{n-m} \right), \]

and notice that \( z_{n+1} = \eta(i_n - i_{n-1}) + r z_n \). Thus the discrete-time inverter model can be described as follows:

\[
\begin{align*}
    i_{n+1} &= \alpha i_n + \beta (2 \sinh(\delta d_n) - \sinh(\delta)), \\
    d_n &= \frac{1}{2} + k(n - i_n) + \eta(i_n - i_{n-1}) + r z_n, \\
    z_{n+1} &= \eta(i_n - i_{n-1}) + r z_n.
\end{align*}
\]

(25a) (25b) (25c)

Since at equilibrium ETDFC vanishes then again we can define \( x_n = i_{n-1} - i^* \) and \( y_n = i_n - i^* \) where \( i^* \) and \( d^* \) are the solutions of (6). Using \( x_n, y_n \) and \( z_n \) as the new state variables, system (25) becomes

\[
\begin{align*}
    x_{n+1} &= y_n, \\
    y_{n+1} &= \alpha y_n + (\alpha - 1)/r \delta^* \\
    &+ \beta (2 \sinh(\delta d_n) - \sinh(\delta)), \\
    z_{n+1} &= \eta(y_n - x_n) + r z_n.
\end{align*}
\]

(26a) (26b) (26c)

Hence the Jacobian matrix is

\[
J = \begin{bmatrix}
0 & 1 & 0 \\
-\eta & \eta & r
\end{bmatrix}
\]

where

\[
J_1 = -2 \beta \eta \delta \cosh(\delta d^*) \\
J_2 = \alpha + 2 \beta (\eta - k) \delta \cosh(\delta d^*) \\
J_3 = 2 \beta r \delta \cosh(\delta d^*)
\]

The characteristic equation of the linearized system is:

\[ z^3 - (J_2 + r) z^2 + (r J_2 - \eta J_1 - J_1) z + \eta J_1 + r J_1 = 0 \]

(27)

Using the system description (26), the aim is to stabilize the origin of the system hence we obtain \( v_n = i_{n-1} = i^* \) and ETDFC becomes zero. The Jury stability criteria yields to conditions on control parameters \( k \), \( \eta \) and \( r \):

\[
\begin{align*}
    k > \frac{\alpha - 1}{2 \beta \delta \cosh(\delta d^*)}, \\
    \eta > (1 + r) \left( \frac{k}{2} - \frac{1 + \alpha}{4 \beta \delta \cosh(\delta d^*)} \right), \\
    \eta < rk + \frac{1 - \alpha}{2 \beta \delta \cosh(\delta d^*)}, \\
    \eta > rk - \frac{1 - \alpha}{2 \beta \delta \cosh(\delta d^*)}
\end{align*}
\]

(28) (29) (30) (31)

We recall that since \( \delta = 0.2 \) and \( 0 \leq \delta^* \leq 1 \) then we have \( \cosh(\delta d^*) \approx 1 \) and \( \alpha < 1 \). A proportional gain is usually considered positive, thus condition (28) reduces to \( k > 0 \). Let us now define three lines \( (\eta = L(k)) \)

\[
\begin{align*}
    L_3 : \eta &= (1 + r) \left( \frac{k}{2} - \frac{1 + \alpha}{4 \beta \delta} \right), \\
    L_4 : \eta &= rk + \frac{1 - \alpha}{2 \beta \delta}, \\
    L_5 : \eta &= rk - \frac{1 - \alpha}{2 \beta \delta}
\end{align*}
\]

We notice that \( L_4 \) and \( L_5 \) have the same slopes \( (\text{i.e. } r) \) therefore they are parallel. We further see that since \( r < 1 \) then \( L_3 \) is above \( L_4 \) when \( k > 0 \).

\[
\begin{align*}
    (1 + r) \left( \frac{k}{2} - \frac{1 + \alpha}{4 \beta \delta} \right) &- rk \frac{1 + \alpha}{2 \beta \delta} \\
    &= \frac{1 - r}{2} \left( k + \frac{1 - \alpha}{2 \beta \delta} \right) > 0
\end{align*}
\]

Thus, it follows that condition (31) is obviously satisfied when condition (29) is satisfied and that as \( r \) tends to 1 condition (31) tends to condition (29). Moreover, we notice that when \( r \) tends to zero \( L_4 = L_1 \) and \( L_3 = L_2 \). Figure 13 sketches the stability zone in the parameters space for a fixed value of \( r \).
5.3. Results

Figures 14-16 show two-dimensional bifurcation diagrams. It is worth noting that the period one zone shown in brown in Fig. 14 matches the zone depicted in Fig. 13. Moreover, we see that the fixed point of order one can be stabilized for higher values of $k$ when $\eta$ and $r$ increase. However, we notice that for large values of $\eta$ and $r$, small values of $k$ no longer lead to stability but to different other modes, this is clearly depicted in Fig. 14 as well as in Fig. 17.

5.4. Optimality criterion

Similarly to the case of TDFC, the ETDFC may induce significantly slow dynamic response if the parameters are badly chosen. In this section we present numerical simulations results to find optimal parameters values that lead to a fast response. The criterion considered herein is to find the shortest settling time to be within an interval of 5% around the fixed point.

For different values of $k$ and $r$, Fig. 18 shows the optimal value of $\eta$ necessary to minimize the settling time. The optimum value $\eta_{\text{opt}}$ is coded on the color-bar and we see that it increases when both $k$ and $r$ increase. Figure 19 depicts the settling time in the number of periods. We notice that as we move to the boundary of the stability zone the settling time gets longer. Moreover, we can notice that for small values of $r$ the settling time increases rapidly as we move towards the stability boundary, whereas for large values of $r$ the settling time increases gradually. Figure 20 sketches the sampled and hold output current for $k = 6$. When no TDFC is applied the response is period 6 as shown in Fig. 4. When TDFC with adaptive value $\eta$ is applied the response...
Fig. 15. 2-D bifurcation diagram ($k - r$).

Fig. 16. 2-D bifurcation diagram ($\eta - r$).
Fig. 17. 1-D bifurcation diagram with ETDFC controller.

Fig. 18. Optimal values of \( \eta \) for different values of \( k \) and \( r \).
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6. Results on Sinusoidal Output Tracking

When considering an industrial application, we should think of a sinusoidal mode functioning. In the foregoing sections we have considered a constant reference current for simplicity of the analysis.

In that case, a new frequency condition has to be met: the reference frequency must be much smaller than the inverter’s in order to guarantee a variation of the current close to a sinusoid (as shown in Fig. 21).

Fig. 19. Settling time shown in number of periods $T$.

Fig. 20. Dynamic response to the step with $k = 6$.

is again period 6. However, the ETDFC stabilizes the fixed point of order one.

$T$ has to be small enough for obtaining an output current close to a sinusoid, besides $T$ has to be sufficiently large to allow the application of a digital control and to limit the losses to an acceptable level.

Fig. 21. Input reference current and controlled current in the load.
Considering a reference frequency ranging from zero to 100 Hz and a minimal switching frequency to reference frequency ratio equal to 50, the switching period is upper limited to $T = 0.2 \text{ ms}$.

In Fig. 22 we compare in the case of $k = 2$ the performance of the proportional, TDFC and ETDFC controllers. The sampled and hold current is sketched, and we see that when the adaptive TDFC or ETDFC is applied the performance is almost unchanged. We have seen in Sec. 4 that when the proportional gain is raised to $k = 4.5$, TDFC should be applied to obtain stability of the period one mode. Although, we apply adaptive TDFC, Fig. 23 shows that the settling time can be further

![Fig. 22. Sinusoidal reference tracking with $k = 2$.](image)

![Fig. 23. Sinusoidal reference tracking with $k = 4.5$.](image)
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minimized with the action of ETDFC. In Fig. 24 we present sinusoidal reference tracking when \( k = 7.2 \) which can only be obtained with ETDFC. We note that a judicious choice of the parameters lead to faster settling time. In Fig. 25 the continuous time-domain waveform of the output current is depicted to show how the ripple is minimized in the periodic mode compared to the chaotic mode.
7. Conclusion

By modeling power electronic converters using nonlinear discrete time maps, it is possible to predict and control chaotic behaviors. The Extended Time-Delayed Feedback Control is an efficient method to stabilize unstable periodic orbits. In this work, TDFC presented in Sec. 4 and ETDFC presented in Sec. 5 have been applied to stabilize a current-programmed PWM single phase inverter. By analyzing the Jacobian matrix of the map, we have determined the stability domain of the T-periodic running mode. For each controller, we have presented 2-D bifurcation diagrams (see Figs. 14–16) corresponding to different control parameters. We have also presented the stability zones in the parameters space (see Figs. 6 and 13). It may be worthwhile to study the effect of the system parameters variations on these stability zones. This constitutes one line of future work.

It was found that the TDFC as well as ETDFC induces longer settling time if parameters are not appropriately chosen (see Figs. 9 and 19). To circumvent this problem, an adaptive TDFC has been proposed [see Eq. (22)] and numerical analysis of ETDFC was presented to give the optimal parameters that lead to minimum settling time (see Fig. 18). The range of the stable T-periodic mode of the inverter is widened and the dynamical performances are improved. Results on tracking sinusoidal reference have been presented [see Figs. 22–24] with the perspective of an experimental realization of the presented control methods.

References


