MODEL-INDEPENDENT ADAPTIVE CONTROL OF CHUA’S SYSTEM WITH CUBIC NONLINEARITY

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This paper proposes a linear adaptive state feedback controller which achieves asymptotic tracking of the controlled cubic Chua’s system. The gain vector is tuned using an adaptation law derived using the Lyapunov stability theory. The aim is to design an adaptive controller that mimics a predetermined input–output linearizing controller known for its effective performance in output tracking. We show that the system model is not necessary to construct the controller. We also show that synchronization of two cubic Chua’s systems can be achieved. Numerical simulations are presented to illustrate the efficiency of the proposed scheme.

Keywords: Chua’s system; adaptive control; numerical simulations; synchronization.

1. Introduction

The interest in controlling chaotic systems has been revived over the last decade. After the pioneering work of Ott et al. [1990] several strategies to control chaos have been developed [Rajasekar et al., 1997; Ramesh & Narayanan, 1999; Chen, 1993].

A widely considered controlling method consists in adding an input signal to the chaotic system to attempt to stabilize an unstable equilibrium point or to track a smooth orbit. These aims have been approached using linear control [Hwang et al., 1997; Parmananda, 1998] as well as nonlinear control methods [Murali et al., 1995; Arecchi et al., 1998; Yang et al., 2002; Tian et al., 2002]. In most of these works, the knowledge of the exact model of the chaotic system is essential to establish the control scheme.

Recently, there has been a great interest in the design of robust controlling methods with respect to parameters uncertainties. Variable structure control [Yu, 1997] and adaptive control [Bernardo, 1996; Zhang et al., 1998; Chen & Lü, 2002] are shown to be successful and effective.

Chua’s system is a typical chaotic system that has been thoroughly studied and many of the foregoing mentioned methods have been applied to the control of Chua’s system [Hwang et al., 1997] and its modified versions [Yassen, 2003; Jang et al., 2002]. In this paper we aim to design a simple linear adaptive controller in order to yield to output tracking of the cubic Chua’s system [Hwang et al., 1996; Wu & Chen, 2002; Yassen, 2003; Jang et al., 2002].

In this paper we propose a linear adaptive state feedback controller which achieves asymptotic tracking of the controlled cubic Chua’s system. The gain vector is tuned using an adaptation law derived using the Lyapunov stability theory. The aim is to design an adaptive controller that mimics a predetermined input–output linearizing controller known for its effective performance in output tracking. We show that the system model is not necessary to construct the controller. We also show that synchronization of two cubic Chua’s systems can be achieved. Numerical simulations are presented to illustrate the efficiency of the proposed scheme.

Keywords: Chua’s system; adaptive control; numerical simulations; synchronization.
and prove the output tracking, stabilization and synchronization of two Chua’s systems. Section 3 is devoted to illustrate our results with numerical simulations. Finally, in Sec. 4 we include some concluding remarks.

2. Adaptive Control Design

The cubic Chua’s circuit is represented by the following dynamical system

$$\dot{x}_m = f(x_m)$$

where

$$f(x_m) = \begin{pmatrix} \frac{1}{l} \left(2 x_{m1}^3 - x_{m1} - x_{m2} + x_{m3}\right) \\ -q x_{m2} \\ x_{m1} - x_{m2} + x_{m3} \end{pmatrix}.$$  

$x_{m1}$ and $x_{m2}$ represent the voltages across the capacitors and $x_{m3}$ is the current through the inductor. $p$ and $q$ are two positive constants which are a function of the circuit parameters. Typical values $(p, q) = (10, 100/7)$ lead to chaotic behavior of the system and to three unstable equilibrium points $E_0 = (0, 0, 0)$ and $E_{\pm} = (\pm \sqrt{0.5}, 0, \mp \sqrt{0.5})$. To control the Chua’s circuit a voltage source $u$ is added in series with the inductor, thus the dynamics of the controlled system is described by

$$\dot{x}_c = f(x_c) + B u, \quad B = (0 \ 0 \ 1)^T$$  

The design of a feedback control aims to force an output signal $y$ of the controlled system to track a smooth (infinitely differentiable) reference trajectory $y_r$.

2.1. Control design when $y = x_{c1}$

2.1.1. Output tracking of a smooth orbit

When $y = x_{c1}$, it can be easily verified that the relative degree [Isidori, 1995, Chap. 4] of system (2) is $\rho = 3$, hence the system is exact input–output linearizable. Let us define the state transformation $\Phi_1(x_c)$ as follows:

$$\Phi_1(x_c) = \begin{bmatrix} x_{c1} \\ p \left( x_{c2} - \frac{1}{7} (2 x_{c1}^3 - x_{c1}) \right) \\ p \left( \frac{q}{7} (x_{c2} - \frac{1}{7} (2 x_{c1}^3 - x_{c1})) (1 - 6 x_{c1}^2) + (x_{c1} - x_{c2} + x_{c3}) \right) \end{bmatrix}$$

Using $z_c$ as state variable, system (2) becomes:

$$\dot{z}_{c1} = z_{c2},$$

$$\dot{z}_{c2} = z_{c3},$$

$$\dot{z}_{c3} = g(z_c) + pu.$$  

$g(z_c)$ is a nonlinear function independent of $u$. Since our aim is to let $x_{c1} = z_{c1} = y_r$, we define a tracking error

$$e_1 = y_r - z_{c1}, \quad e_2 = \dot{y}_r - z_{c2},$$

$$e_3 = \ddot{y}_r - z_{c3},$$

and choose an input–output linearizing controller

$$u_{io} = \frac{1}{p} (-g(z_c) + \dot{y}_r + K^T e),$$

$$K^T = \begin{pmatrix} k_1 & k_2 & k_3 \end{pmatrix}.$$  

The resulting closed loop system becomes linear

$$\dot{e} = A_c e$$

with

$$A_c = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_1 & -k_2 & -k_3 \end{bmatrix}.$$  

Should $K$ be a Hurwitz vector, that is all the roots of the polynomial

$$p(s) = s^3 + k_3 s^2 + k_2 s + k_1$$

have negative real parts, then the error becomes exponentially stable at the origin, i.e. $\lim_{t \to \infty} e(t) = 0$ and in particular

$$\lim_{t \to \infty} x_{c1}(t) = \lim_{t \to \infty} z_{c1}(t) = y_r.$$  

We notice that if the analytic expression of $y_r$ is known then so is $\dot{y}_r$, otherwise $\dot{y}_r$ is obtained numerically which would not be accurate due to sensitivity to noise. Fortunately in many applications and due to the smoothness of the required trajectory, an estimate of the analytic expression of $y_r$ can be made.
Hence in general, the input–output linearizing controller is efficient in output tracking. However, it still has two main drawbacks which are the difficulty to construct the nonlinear part \(g(z_c)\) in addition to a necessary exact knowledge of the system parameters. To circumvent this problem, we suggest here to construct a linear adaptive controller

\[
    u_{\text{lin}}(t) = \hat{K}(t)^T e(t),
\]

where \(\hat{K}(t)\) is adjusted adaptively such that in the limit we have

\[
    u_{\text{lin}}^*(t) = (\hat{K}^*)^T e(t) = u_{io}(t),
\]

the asterisk in \(\hat{K}^*\) denotes the optimal constant gain vector. Substituting \(u_{\text{lin}}\) in (3c) yields to

\[
\begin{align*}
    \dot{z}_{c3} &= f(z_c) + pw_{\text{lin}}, \\
    &= f(z_c) + pu_{io} - p(u_{io} - u_{\text{lin}}), \\
    &= \ddot{y}_r + K^T e - p(u_{io} - u_{\text{lin}}),
\end{align*}
\]

hence

\[
    \dot{e}_3 = \ddot{y}_r - z_{c3} = -K^T e + p(\hat{K}^* - \hat{K})^T e.
\]

If we define \(B_c = (0,0,p)^T\) then we obtain the following closed loop system

\[
    \dot{e} = A_c e + B_c (\hat{K}^* - \hat{K})^T e. 
\]

(6)

Note that since \(A_c\) is Hurwitz stable then for any positive definite matrix \(Q\) there exists a positive definite matrix \(P\) such that the following Lyapunov equation is satisfied:

\[
    A_c^T P + PA_c = -Q.
\]

Proposition 1. The linear adaptive controller

\[
    u_{\text{lin}} = \hat{K}(t)^T e(t)
\]

with

\[
    \hat{K}(t) = \gamma \int_0^t e(t)^T P B_c e(t) dt, \quad \gamma > 0,
\]

(7)

leads to output tracking i.e. \(\lim_{t \to \infty} x_{cl}(t) = \lim_{t \to \infty} \hat{z}_{c1}(t) = y(t)\)

Proof. Given system (6), let us choose a Lyapunov function candidate

\[
    V(e, \hat{K}) = e^T Pe + \frac{1}{\gamma} (\hat{K}^* - \hat{K})^T (\hat{K}^* - \hat{K}).
\]

The time derivative of \(V(e, \hat{K})\) along the trajectories of (6) is given by

\[
    \dot{V}(e, \hat{K}) = e^T (A_c^T P + PA_c)e \\
    + 2e^T P B_c (\hat{K}^* - \hat{K})^T e
\]

\[
    - \frac{2}{\gamma} (\hat{K}^* - \hat{K})^T \ddot{K}
\]

\[
    = -e^T Q e - 2(\hat{K}^* - \hat{K})^T \left( \frac{\ddot{K}}{\gamma} - e^T P B_c e \right).
\]

Using (7) it follows that

\[
    \dot{V}(e, \hat{K}) = -e^T Q e \leq -\lambda_{\text{min}}(Q) \|e\|^2,
\]

(8)

where \(\lambda_{\text{min}}(Q)\) is the smallest eigenvalue of \(Q\). We deduce that system (6) is Lyapunov stable, which in turn implies that \(e \in L_\infty\).

Integrating (8) we obtain

\[
    \int_0^t \|e\|^2 dt \leq \frac{V(0) - V(t)}{\lambda_{\text{min}}(Q)}.
\]

Since \(V(t) \in L_\infty\) and \(V(0)\) is finite, this implies that \(e \in L_2\). Also from (6) we obviously have \(\dot{e} \in L_\infty\) in addition to \(e \in L_\infty\) and \(e \in L_2\). Eventually by Barbalat’s lemma [Khalil, 1992] \(\lim_{t \to \infty} e(t) = 0\), particularly

\[
    \lim_{t \to \infty} x_{cl}(t) = \lim_{t \to \infty} \hat{z}_{c1}(t) = y(t).
\]

Therefore, output tracking is obtained. ■

Notice that \(u_{\text{lin}} = \hat{K}(t)^T e(t)\), however \(\hat{K}(t)\) depends on \(B_c\) which in turn depends on the system parameter \(p\). To show that \(p\) is unnecessary in the controller construction, we let \(P_n\) be the \(n\)th column of \(P\), then we can write

\[
    \hat{K}(t) = \gamma \int_0^t e(t)^T P_n e(t) dt, \quad p > 0, \quad \gamma > 0
\]

\[
    = \dot{\gamma} \int_0^t e(t)^T P_n e(t) dt, \quad \dot{\gamma} > 0
\]

Since positiveness is the only constraint on \(\gamma\) and \(\dot{\gamma}\) then robustness with respect to \(p\) is ensured.

2.1.2. Stabilization of the equilibrium points

In addition to output tracking the proposed adaptive controller \(u_{\text{lin}}\) also leads to stabilization of the equilibrium points. Indeed, it suffices to consider them as constant trajectories to be tracked. It is important to notice that using \(\Phi_1(x_c)\) the equilibrium points are transformed to \(E_0 = (0,0,0)\) and \(E_\pm = (\pm \sqrt{0.5}, 0, 0)\).
2.1.3. Synchronization of two cubic Chua’s systems

We can also show that the proposed linear adaptive controller can lead to synchronization of two cubic Chua’s systems. Let us consider system (1) as a master system and the controlled system (2) as the slave system. We consider \( y_r = x_{m1} \) and \( y = x_{c1} \), next we apply the state transformation \( \Phi_1 \) to the master system and we get:

\[
y_r = z_{m1}, \quad \dot{y}_r = z_{m2}, \quad \ddot{y}_r = z_{m3}.
\]

Using the proposed linear adaptive controller, in the limit we obtain \( e = 0 \) that is

\[
z_{c1} = y_r = z_{m1}, \quad z_{c2} = \dot{y}_r = z_{m2}, \quad z_{c3} = \ddot{y}_r = z_{m3}.
\]

Using the fact that \( \Phi_1 \) is a diffeomorphism, it follows that

\[
x_{c1} = x_{m1}, \quad x_{c2} = x_{m2}, \quad x_{c3} = x_{m3}.
\]

therefore synchronization is obtained.

2.2. Control design when \( y = x_{c2} \)

2.2.1. Output tracking of a smooth orbit

When \( y = x_{c2} \) the relative degree of the system (2) is \( \rho = 2 \), hence the system is not exact input–output linearizable. Therefore we choose the following state transformation:

\[
\Phi_2(x_e) = \begin{bmatrix}
z_{c1} \\
z_{c2} \\
z_{c3}
\end{bmatrix} = \begin{bmatrix}
x_{c2} \\
x_{c1} - x_{c2} + x_{c3} \\
x_{c1}
\end{bmatrix},
\]

thus in the new coordinates, system (2) is expressed by the following equations

\[
\begin{aligned}
\dot{z}_{c1} &= z_{c2}, \\
\dot{z}_{c2} &= (p - q)z_{c1} - z_{c2} - \frac{p}{t}(2z_{c3}^3 - z_{c3}) + u, \\
\dot{z}_{c3} &= pz_{c1} - \frac{p}{t}(2z_{c3}^3 - z_{c3}).
\end{aligned}
\]

In this case we define a two-dimensional tracking error:

\[
e_1 = y_r - z_{c1}, \quad e_2 = \dot{y}_r - z_{c2},
\]

and choose an input–output linearizing controller

\[
u_{io} = - \left( (p - q)z_{c1} - z_{c2} - \frac{p}{t}(2z_{c3}^3 - z_{c3}) \right) + \dot{y}_r + K^T e,
\]

\[
K^T = (k_1 \quad k_2).
\]

The resulting closed loop system becomes

\[
\begin{aligned}
\dot{e} &= A_e e, \\
\dot{z}_{c3} &= p(y_r - e_1) - \frac{p}{t}(2z_{c3}^3 - z_{c3}).
\end{aligned}
\]

with

\[
A_e = \begin{bmatrix}
0 & 1 \\
-k_1 & -k_2
\end{bmatrix}.
\]

We let \( K \) be a Hurwitz vector and using a similar analysis as in the previous section for the subsystem (10a), we construct a linear adaptive controller

\[
u_{lin} = \hat{K}(t)^T e(t)\]

that mimics \( u_{io} \) with

\[
\hat{K}(t) = \gamma \int_0^t e(t)^T PB_e e(t) dt,
\]

\[
\gamma > 0, \quad B_e[0,1]^T.
\]

We notice that using \( u_{lin} \) the subsystem (10b) is uncontrollable. Besides, \( u_{io} \) depends on \( z_{c3} \), hence we should have a bounded \( z_{c3} \) to obtain a bounded \( u_{io} \) and \( u_{lin} \) and equivalent bounded \( \hat{K} \).

**Proposition 2.** If \( A_e \) is exponentially stable and the desired trajectory is bounded \( |y_r| < \alpha_1 \) then

\[
\lim_{t \to \infty} |y_r(t)| = 0 \quad \text{and} \quad \lim_{t \to \infty} |z_{c3}| < \infty.
\]

**Proof.** Since \( A_e \) is exponentially stable then

\[
\lim_{t \to \infty} |e(t)| = 0 \quad \text{that is in the limit} \quad x_{c2} = z_{c1} = y_r.
\]

Next, from (9c) we have

\[
\dot{z}_{c3} = f(z_{c1}, z_{c3}) = pj_{c1} - \frac{p}{t}(2z_{c3}^3 - z_{c3}).
\]

Let us first consider the autonomous system

\[
\dot{z}_{c3} = f(0, z_{c3}) = -\frac{p}{t}(2z_{c3}^3 - z_{c3}).
\]

We choose the Lyapunov function candidate \( V(z_{c3}) = (1/2)z_{c3}^2 \); its derivative along the trajectories of (12) is \( (dV/dz_{c3})f(0, z_{c3}) = -(p/t)z_{c3}^2(2z_{c3}^2 - 1) \) thus for all \( |z_{c3}| > 1 \) we have

\[
\frac{dV}{dz_{c3}}f(0, z_{c3}) < -\frac{p}{t}z_{c3}^2.
\]

Moreover

\[
\frac{dV}{dz_{c3}} < \alpha_2|z_{c3}|, \quad \alpha_2 > 1.
\]

We now investigate the boundedness of \( z_{c3} \). The derivative of \( V(z_{c3}) \) along the trajectories of
The state transformation (11) is
\[
\dot{V} = \frac{dV}{dz_3}(f(z_{c1}, z_3) - f(0, z_3))
\]
\[
< -\frac{p}{t} z_3^2 + \frac{dV}{dz_3}(f(z_{c1}, z_3) - f(0, z_3))
\]
\[
< -\frac{p}{t} z_3^2 + \alpha_2 |z_3| \cdot p \cdot \alpha_1 ,
\]
thus
\[
\dot{V} < 0 \quad \text{for} \quad |z_3| \geq 7\alpha_1 \cdot \alpha_2 .
\]

Therefore, any trajectory \( z_3(t) \) starting at a finite value \( z_3(0) \) will eventually end inside an interval containing the origin of radius \( R = \max\{1, 7\alpha_1, \alpha_2\} \), this implies that \( |z_3| < \infty \). Finally output tracking is obtained using the linear adaptive control \( u_{\text{lin}} \) and
\[
limit_{t \to \infty} x_{c2} = \lim_{t \to \infty} z_3 = y_r .
\]

2.2.2. Stabilization of the equilibrium points

The state transformation \( \Phi_2(x_c) \) transforms the equilibrium points into \( E_0 = (0, 0, 0) \) and \( E_{\pm} = (0, 0, \pm \sqrt{0.5}) \) hence they are distinguished by the last component \( z_3 \). The application of \( u_{\text{lin}} \) makes the evolution of \( z_3 \) uncontrollable, consequently the equilibrium points are not controllable. However, by applying \( u_{\text{lin}} \) we can drive \( z_{c1} \) and \( z_{c2} \) to zero. As a result the evolution of \( z_3 \) is governed by the following equation:
\[
\dot{z}_3 = -\frac{p}{t} (2z_3^3 - z_3) .
\]

Consider the Lyapunov function candidate \( V(z_3) = z_3^2 \), its derivative along the trajectories of (15) is
\[
\dot{V}(z_3) = -\frac{2p}{t} z_3^2 (2z_3^2 - 1)
\]
\[
= -\frac{2p}{t} V(z_3)(2V(z_3) - 1)
\]
The derivative \( \dot{V}(z_3) \) is positive for \( V(z_3) < 1/2 \) and negative for \( V(z_3) > 1/2 \). Hence on the level interval \( V(z_3) = a_1 \) with \( 0 < a_1 < 1/2 \) all trajectories will be moving away from the origin, while on the level interval \( V(z_3) = a_2 \) with \( a_2 > 1/2 \) all trajectories will be moving towards the origin. This shows that the interval
\[
I = \{z_3 \in \mathbb{R} | a_1 \leq V(z_3) \leq a_2 \}
\]
is positively invariant. Since \( V(z_3) = z_3^2 \) it follows that \( I = I_- \cup I_+ \) with
\[
I_- = \{z_3 \in \mathbb{R} | -\sqrt{a_2} \leq z_3 \leq -\sqrt{a_1}\}
\]
\[
I_+ = \{z_3 \in \mathbb{R} | \sqrt{a_1} \leq z_3 \leq \sqrt{a_2}\}
\]

Furthermore, since the above argument is valid for any \( a_1 < 1/2 \) and any \( a_2 > 1/2 \), we can let \( a_1 \) and \( a_2 \) approach 1/2 so that the intervals \( I_- \) and \( I_+ \) shrink to the points \( \{z_3 = -\sqrt{0.5}\} \) and \( \{z_3 = \sqrt{0.5}\} \), respectively.

Finally, using LaSalle’s invariance theorem, we deduce that when applying \( u_{\text{lin}} \) to drive \( z_{c1} \) and \( z_{c2} \) to zero, the state \( z_3 \) will be attracted to \(-\sqrt{0.5}\) (respectively \( \sqrt{0.5} \)) if \( z_3(t) \) is in \( I_- \) (respectively \( I_+ \)) at the moment of application of the controller. The equilibrium point \( E_0 \) stays unstable and uncontrollable by the application of \( u_{\text{lin}} \) in this case.

2.2.3. Synchronization of two cubic Chua’s systems

Let the output of the master system be \( y_r = x_{m2} \), then using the state transformation \( \Phi_2 \) we get:
\[
y_r = z_{m1} , \quad \dot{y}_r = z_{m2} .
\]

If we apply the linear adaptive controller we obtain \( e = 0 \) that is
\[
z_{c1} = y_r = z_{m1} , \quad z_{c2} = \dot{y}_r = z_{m2} .
\]

To show that in the limit we also have \( z_{c3} = z_{m3} \) we define \( e_3 = z_{m3} - z_{c3} = x_{m1} - x_{c1} \). Subtracting \( f_1(x_m) - f_1(x_c) \) we get
\[
\dot{e}_3 = p(x_{m2} - x_{c2})
\]
\[
= -\frac{p}{t} e_3 (2(x^2_{m1} + x_{m1}x_{c1} + x^2_{c1}) - 1)
\]
but since \( x_{c2} = x_{m2} \) then
\[
\dot{e}_3 = -\frac{p}{t} e_3 (2(x^2_{m1} + x_{m1}x_{c1} + x^2_{c1}) - 1) .
\]

Evaluating the derivative of the Lyapunov function candidate \( V(e_3) = (1/2)e^2_3 \) along the trajectories of (16) yields
\[
\dot{V}(e_3) = -\frac{p}{t} e^3_3 (2(x^2_{m1} + x_{m1}x_{c1} + x^2_{c1}) - 1) .
\]

Therefore if
\[
(x^2_{m1} + x_{m1}x_{c1} + x^2_{c1}) > \frac{1}{2}
\]
it follows that \( \dot{V}(e_3) < 0 \) and \( x_{c1} = x_{m1} \). Furthermore, from \( z_{c2} = z_{m2} \) we obtain \( x_{c3} = x_{m3} \).
3. Numerical Simulations

In this section, we will show a set of numerical simulations corresponding to the results obtained in the foregoing section. In all the simulations initial conditions are \((x_{m1}, x_{m2}, x_{m3}) = (0.2, 0, 0)\) and \((x_{c1}, x_{c2}, x_{c3}) = (0.7, 0, -0.7)\). The designed linear adaptive control is applied to system (2) at time \(T_0\).

3.1. Simulation results when \(y = x_{c1}\)

In this case we fixed

\[
\dot{\gamma} = 7.5, \quad A_c = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
-1 & -3 & -3
\end{bmatrix},
\]

\[
P = \frac{1}{8} \begin{bmatrix}
37 & 31 & 8 \\
31 & 52 & 13 \\
8 & 13 & 7
\end{bmatrix}, \quad Q = \begin{bmatrix}
2 & 0 & 0 \\
0 & 2 & 0 \\
0 & 0 & 2
\end{bmatrix}.
\]

The desired trajectory to be tracked is \(y_r = x_{m1}\) generated by the master system with \((p, q) = (10.8, 100/7)\), with these parameters the master system is not chaotic but has a limit cycle. Figure 1 shows that \(x_{c1}\) (solid line) tracks \(x_{m1}\) (dotted line). The behavior of the controlled system is chaotic before control (attractor in dotted line) and becomes periodic after control (limit cycle in solid line). Figure 2 delineates the evolution of the adapted gain vector as well as the evolution of the adaptive controller. The stabilization of the equilibrium points and the stabilizing controllers are depicted in Figs. 3–5 for \(E_+, E_-\) and \(E_0\), respectively. Finally, synchronization of two cubic Chua’s systems using \(x_{m1}\) is shown in Fig. 6. The synchronizing controller is shown in Fig. 7.

3.2. Simulation results when \(y = x_{c2}\)

In this case we fixed

\[
\gamma = 5000, \quad A_c = \begin{bmatrix}
0 & 1 \\
-4 & -4
\end{bmatrix},
\]

\[
P = \frac{1}{16} \begin{bmatrix}
36 & 4 \\
4 & 5
\end{bmatrix}, \quad Q = \begin{bmatrix}
2 & 0 \\
0 & 2
\end{bmatrix}.
\]

The trajectory to be tracked is \(y_r = x_{m2}\) generated by the master system having a periodic behavior. Figure 8 delineates the correct tracking of the output signal and the behavior of the controlled system before and after the control application. Figure 9 sketches the designed adaptive controller. The stabilization of \(E_+\) is shown in Fig. 10, we see that the evolution of \(x_{c1}\) is around \(+\sqrt{0.5}\) before the application of the controller \((T_0 = 200)\). Figure 11 shows

![Fig. 1. Output tracking using adaptive controller \((T_0 = 200)\).](image)
Fig. 2. Evolution of the adaptive controller ($T_0 = 200$).

Fig. 3. Stabilization of $E_+$ ($T_0 = 200$).
Fig. 4. Stabilization of $E_\infty$ ($T_0 = 200$).

Fig. 5. Stabilization of $E_0$ ($T_0 = 200$).
Fig. 6. Synchronization using $x_{m1}$ ($T_0 = 200$).

Fig. 7. Controller leading to synchronization using $x_{m1}$ ($T_0 = 200$).
Fig. 8. Output tracking using adaptive controller ($T_0 = 200$).

Fig. 9. Evolution of the adaptive controller ($T_0 = 200$).
Fig. 10. Stabilization of $E_+ (T_0 = 200)$.

Fig. 11. Stabilization of $E_- (T_0 = 160)$. 
Fig. 12. Synchronization and no-synchronization regions.

Fig. 13. Evolution of $x_{m1}$ (dotted) and $x_{c1}$ (solid) and their error. Pulses denote intervals of no-synchronization ($T_0 = 200$).
Fig. 14. Synchronization of modified Chua’s systems using $x_{m2}$ ($T_0 = 200$).

Fig. 15. Evolution of the gain vector and the corresponding adaptive controller ($T_0 = 200$).
that the evolution of $x_{c1}$ is around $-\sqrt{0.5}$ at time $T_0 = 160$ hence the system was stabilized at $E_-$ when the controller is applied. It has been shown that synchronization is obtained if condition (17) is satisfied, Fig. 12 shows an ellipse where (17) is violated. Actually, for Chua’s systems the evolution of $x_{m1}$ is mainly outside the ellipse, then the error $e_3 = x_{m1} - x_{c1}$ will tend to zero. During short intervals of time, $x_{m1}$ crosses the ellipse along its short principal axis and the error slightly diverges. On the whole, the error tends to zero and synchronization occurs. We see in Fig. 13 the synchronization of $x_{m1}$ and $x_{c1}$. On a logarithmic scale the error evolution is delineated. In the figure the pulses denote the interval of times when $x_{m1}$ crosses the “No synchronization” region. We see that when $221 < t < 223$, $x_{m1}$ is inside the ellipse for a relatively long time and the error is slightly diverged. The synchronization of two cubic Chua’s systems using an adaptive control design and $x_{m2}$ as a synchronizing signal is depicted in Fig. 14 and the corresponding controller is sketched in Fig. 15.

4. Conclusion

In this paper we have proposed a novel method to control and synchronize the cubic Chua’s system using an adaptive linear controller. In our paper we show that an additive control to Chua’s system can lead to reference tracking of either of the capacitors voltages. The unstable equilibrium points can be stabilized using the same additive control. Moreover, synchronization of two Chua’s systems can also be obtained.

Despite this wide range of applications, the proposed controller construction is very simple, and does not depend on the model of the Chua’s system, hence with our proposed controller any modified version of Chua’s system (piecewise-linear, cubic, sinusoidal [Tang et al., 2001]) can be controlled, besides robustness is guaranteed. Our method of controlling the cubic Chua’s system is a generalization of the result presented in [Yassen, 2003] and outperforms other methods presented in [Hwang et al., 1996; Wu & Chen, 2002] in terms of the simplicity of construction and its wide range of application.

References


