Heuristic approach to the airline schedule disturbances problem

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When disturbances make it impossible to realise the planned flight schedule, the dispatcher at the airline operational centre defines a new flight schedule based on airline policy, in order to reduce the negative effects of these perturbations. Depending on airline policy, when designing the new flight schedule, the dispatcher delays or cancels some flights and reassigns some flights to available aircraft. In this paper, a decision support system (DSS) for solving the airline schedule disturbances problem is developed aiming to assist decision makers in handling disturbances in real-time. The system is based on a heuristic algorithm, which generates a list of different feasible schedules ordered according to the value of an objective function. The dispatcher can thus select and implement one of them. In this paper, the possibilities of DSS are illustrated by real numerical examples that concern JAT Airways’ flight schedule disturbances.

Keywords: airline schedule; airline disturbances; airline operational centre; decision support system; heuristic

1. Introduction

Airline flight schedules are designed to fulfil passenger demand, utilise available resources and satisfy different operational requirements. The greater the time buffer between two flights, the greater the probability of servicing the flight schedule according to plan. However, at the same time this means that the airline’s resources are less efficiently employed and its income is potentially reduced accordingly. The flight schedule design process aims to harmonise these two conflicting criteria (punctuality and utilisation), in order to secure a reliable timetable and employ resources efficiently.

The daily schedule is the final version of the flight schedule for any given day, which takes into account all the changes that were undertaken during the period from its publication (flight schedules are usually issued several months in advance and cover a period of one season, i.e. 6 months) until just before its realisation. The daily schedule, handed to the dispatchers at the airline operations control centre in charge of its implementation, is a set of aircraft and crew (flight and cabin) routings for a one-day time period. For each planned flight within that time interval there are defined departure and arrival times, departure and arrival airports, flight numbers,
aircraft type and its registration, which, according to the schedule, is to execute the
given flight.

As the flight schedule is realised, disturbances occur that cannot be predicted by
the airline in advance. These disturbances are caused by various factors, such as
meteorological conditions, aircraft failure, crew absence or delay, errors in estimation
of block or turnaround time at certain airports, airport congestion, etc. They can be
classified as internal, which means that the airline can influence their resolution, or
external, i.e. the airline cannot influence their resolution. The results of flight
schedule disturbance are flight delay and/or flight cancellation that leads to
additional costs, loss of passenger's loyalty, damage to the airline's reputation, etc.

When disturbances make it impossible to realise the planned flight schedule, the
dispatcher at the airline operations centre (AOC) defines a new daily operational
flight schedule. The main actions that can be taken by dispatchers are:

- **Delay flight** – which directly affects the passengers on that flight, and indirectly
  the passengers on following flights in the rotation of that particular aircraft.
- **Swap aircraft** – deploy a different aircraft to service the flight, which is not in
  its original rotation, if the capacity matches the number of passengers on the
  given flight.
- **Cancel flight** – an extreme option both for passengers and the airline, and it
  may cause serious disturbances.
- **Ferry flight** – fly an aircraft without passengers.
- **Introduce spare aircraft** – possible only if the airline has a spare aircraft
  resource.

The airline schedule disturbances problem (ASDP), which is faced by dispatchers at
the AOC day-to-day, is considered in this paper. During attempts to solve the ASDP
the dispatcher at the AOC considers airline policy and defines a new flight schedule
that will optimise an objective function and satisfy the corresponding constraints.

Depending on airline policy, when designing the new flight schedule, the
dispatcher delays or cancels some flights and reassigns flights to available aircraft
in order to reduce the negative effects of disturbance. Airline policy implies flight
cancellation, flight delay and aircraft swapping, without ferry flights and spare
aircraft.

The airline flight schedule redesign during the traffic monitoring process in the
case of disturbances is a very complex, combinatorial problem. This problem has
been addressed in the literature with several different models, objective functions,
constraints and assumptions, as well as different solution approaches. The goal of
every airline experiencing perturbations is to restore traffic in accordance with the
planned flight schedule as soon as possible. Another goal of every airline is to reduce
the impact of perturbations on income and/or to reduce the total or additional costs
induced by perturbations. These objectives are considered in the literature by many
authors.

For example, Stojković (1990) and Teodorović and Stojković (1990) developed
several models of airline schedule recovery problems and the corresponding exact
methods and heuristics for solving them. Stojković et al. (2002) developed a model
which can be solved to optimality in real-time, but only in cases of small disturbances
(airport congestion or unexpected head wind). Jarrah et al. (1993) considered flight
cancellations and delays in two separate network models. Rakshit et al. (1996)
presented a decision support system (DSS) incorporating the models considered by Jarrah et al. (1993). Yan and Yang (1996) developed a framework for handling schedule perturbations caused by the breakdown of aircraft in the case of a single fleet and non-stop flights. Yan and Tu (1997) continued to research this problem by extending research into multi-fleet routing and multi-stop scheduling during airline schedule perturbations. Yan and Lin (1997) used the same methods as Yan and Yang (1996) to solve schedule perturbations resulting from the temporary closure of airports. None of these papers, however, consider maintenance or crew constraints or passenger connection requests.

Luo and Yu (1997) considered the airline schedule perturbation problem caused by ground delay. The problem is modelled as an integer programming problem. Thengvall et al. (2000) presented a network model with side constraints to recover an aircraft schedule with minor deviations from the original aircraft routings, if the airline has a single fleet. Thengvall et al. (2003) continued research by presenting a bundle algorithm for solving a multi-commodity network model for determining a recovery plan when an airline has multiple fleets, during hub closure. Wu and Caves (2002) investigated how the scheduling of aircraft rotation influences flight schedule reliability. Golany et al. (2002) proposed goal programming for solving airline schedule recovery problems during disturbances.

Rosenberger et al. (2003) presented an optimisation model that reschedules legs and reroutes aircraft by minimising rerouting and cancellation costs. Andersson and Varbrand (2004) and Andersson (2006) suggested using heuristics and meta-heuristics for solving the flight perturbation problem. Bratu and Barnhart (2006) presented models and algorithms for airline schedule recovery which consider aircraft, crew and passengers simultaneously and decide which leg should be delayed and which one cancelled. Abdelghany et al. (2008) presented an integrated decision-support tool for airline schedule recovery during irregular operations, which includes a schedule simulation model and a resource assignment optimisation model.

Kalic and Pavkovic (2003, 2004) developed a special heuristic algorithm for solving the schedule disturbances problem. The model is based on two objective functions: minimisation of the number of cancelled flights and minimisation of the total time-delay of all flights. Nedeljkovic (2004) developed a model and heuristic algorithm, which offers the AOC dispatcher several solutions as a response to schedule disturbances. The objective function minimises additional costs caused by disturbances. Analysis of the reasons for flight schedule perturbations (delay sources and their duration) and possibilities for their eliminations were considered by Paskota and Babic (2007).

This paper presents a mathematical formalisation for the ASDP and develops a heuristic algorithm for generating new daily operational flight schedules. The corresponding DSS is presented to assist the dispatcher in handling disturbances in real-time. The mathematical model is defined in such a way that by changing the objective function, it can support different decision-making strategies (policies) of an airline. The proposed heuristic algorithm offers a list of different feasible solutions ordered according to the value of an objective function, thus the dispatcher is in position to select and implement one of them. During the decision-making process the dispatcher could define a new strategy, resolve a problem and select a solution.

The paper has five sections: the introduction and literature review are followed by the definition of the problem and its mathematical formalisation in Section 2. A
heuristic algorithm proposed for its solution is described in Section 3. In Sections 4 and 5, a numerical example is given and conclusions presented, including further research recommendations.

2. Problem definition and mathematical formalisation

The ASDP, considered in this paper, can be defined in the following way.

For a planned daily flight schedule of an airline under conditions when a disturbance has occurred, the dispatcher should design a new daily operational flight schedule as a response to schedule disruption, such that the total ‘profit’ of the airline is maximised and that the corresponding constraints are satisfied. More precisely, in the case when a disturbance has occurred at an airport or on an aircraft and consequently some flights can no longer be realised according to the planned daily schedule, a new departure time shall be defined and/or a new aircraft shall be assigned to such a flight, or it shall be cancelled, in such a way that the objective function is maximised. This function represents the profit of the airline, which can be expressed as the difference between passenger revenue and costs. The costs consist of six elements: the fixed direct operational cost and the additional costs caused by disturbances, i.e. the priority and the non-priority flight cancellation cost, the aircraft regular maintenance disturbance cost, the aircraft balance cost and the flight delay cost.

In order to overcome disturbances and find a new daily operational flight schedule, some assumptions are introduced:

- A disturbance may occur at an airport or on an aircraft. All disturbed airports and aircraft are identified and the corresponding starting time (time when the disturbance disappears) for a new flight schedule is defined.
- The flight schedule recovery period, for which a new flight schedule is to be designed, is the time period from the starting time, corresponding to the identified disturbance, to the last moment at which the timetable for the next day is not disturbed.
- A rotation is a sequence of flights by an aircraft where the first flight in the series departs from the base airport, and the last one arrives at the base airport.
- The airline has a fleet, which consists of different types of aircraft characterised by different seat capacities, where aircraft of the same type have the same seat capacity.
- The aircraft ground handling time depends on the aircraft type and the airport where the handling occurs.
- Aircraft can be swapped – bigger aircraft can service flights originally assigned to smaller ones, and smaller aircraft can service flights originally assigned to bigger ones, if the number of passengers is not greater than its seat capacity. The aircraft swapping cost is not considered – if an aircraft is swapped, it is assumed that there is no additional cost.
- There are no spare aircraft in the fleet.
- A set of priority flights is given (flights with the slot time, transfer passengers, etc.).
- The maximal allowed delay is defined for each flight.
- Ferry flights are not allowed.
- Priority flights can be realised according to the so-called ‘VIA principle’, which is illustrated in Figure 1. Flights $i$ and $j$ are planned flights, where $i$ is a priority flight from airport B to airport C, while $j$ is a non-priority one from airport A to airport C. If at airport B the aircraft assigned to flight $i$ has a failure, the aircraft assigned to flight $j$ can realise an additional unplanned flight $i'$ (from A to B), and then priority flight $i$. In this way that aircraft can service both flights $i$ and $j$, but only if its seat capacity is not smaller than the total number of passengers on these two flights. Therefore, for each planned priority flight (flight $i$) a set of additional non-planned flights (flights $i'$), which can be used for its realisation applying the VIA principle, is given in advance. Obviously, such an additional flight should have the destination airport identical to the origin airport of the priority flight. Also, there should exist at least one non-priority flight (flight $j$) with the origin and the destination airports identical to that of the origin airport of the additional flight and the destination airport of the priority flight, respectively.
- For each flight the average fare per passenger, the average delay cost per time unit and the cancellation cost are known in advance. The cancellation costs are different for priority and non-priority flights.
- Crew constraints are not considered.

Starting with the previous assumptions, the following constraints of the ASDP are specified as follows:

(a) Time constraints:
- each flight in a new flight schedule should not depart earlier than the known departure time planned before the disturbance occurrence;
- if the departure time of a flight in a new schedule is delayed with respect to the planned time, this delay should not be greater than the maximal delay allowed for this flight;
- the time period between two consecutive flights in the rotation of an aircraft should be sufficient for its ground handling;
- the take-off and landing of each flight in the new schedule should be serviced at the corresponding airport by the end of its working hours, i.e. by its closing time valid on the day for which this schedule is designed;
- in the case when the aircraft assigned to a flight has a detected disturbance, the departure time of this flight in the new schedule should not be sooner than the earliest possible departure time of the aircraft after the elimination of its disturbance; and

![Figure 1. Servicing a priority flight using the VIA principle.](image-url)
- if the origin or the destination airport of a flight has a detected disturbance, then the departure or landing time of this flight should not be sooner than the earliest possible time when its origin or destination airport can accept flights following elimination of the disturbance.

(b) Aircraft maintenance constraints:
- each aircraft, planned to have the regular technical maintenance at the end of the flight schedule recovery period, should finish its rotation at the defined airport where the maintenance is performed.

(c) Aircraft balance constraints:
- in order to successfully service the planned flights after the recovery period, it is necessary that at the end of this period certain types of aircraft are available at each airport in sufficient numbers (otherwise the disturbance will be extended into the following day). This condition can be satisfied by requiring that, for each of such types, a given number of aircraft with the seat capacity not smaller than the seat capacity characterising this type, should finish their rotation at the given airport.

(d) Capacity constraints:
- the number of passengers on each flight should not be greater than the seat capacity of the aircraft assigned to this flight; and
- the seat capacity of an aircraft, used to service a priority flight applying the VIA principle, should not be smaller than the total number of passengers on that flight and a non-priority flight that is indirectly realised in this way (see flights \(i\) and \(j\) in Figure 1).

In order to make the problem definition more precise and easier to understand, we will give a mathematical formalisation of the objective function and the main constraints of the ASDP. Let us introduce the following notation for its input parameters:

\(F\): set of flights, which should be serviced in the recovery period;
\(Pr\): set of priority flights, \(Pr \subseteq F\);
\(VIA(i)\), \(i \in Pr\): set of additional flights, which can be used in the realisation of priority flight \(i\) using the VIA principle, \(VIA(i) \cap F = \emptyset\);
\(id(i,i')\), \(i \in Pr, i' \in VIA(i)\): a non-priority flight, which can be indirectly realised by servicing flights \(i\) and \(i'\) using the VIA principle, \(id(i,i') \in F\);
\(F'\): set \(F\) enlarged with all additional flights which can be used for the VIA realisation of priority flights, i.e. \(F' = F \cup \bigcup_{i \in Pr} VIA(i)\);
\(AP\): set of airports;
\(o(i), d(i), i \in F\): the origin and the destination airport of flight \(i\), \(o(i), d(i) \in AP\);
\(TP(i), i \in F\): the departure time of flight \(i\) planned before the disturbance occurrence;
\(delay(i), i \in F\): the maximal allowed delay of flight \(i\);
\(pax(i), i \in F\): the number of passengers on flight \(i\);
\(fare(i), i \in F\): the average fare per passenger on flight \(i\);
\(k(i), i \in Pr\): the cancellation cost of priority flight \(i\);
\(k_2(i), i \in F \setminus Pr\): the cancellation cost for non-priority flight \(i\).
$k_1$: the delay cost of a flight per a time unit;
$AC$: set of aircraft available for servicing flights from $F$;
$TYPE$: set of aircraft types;
$type(j), j \in AC$: the type of aircraft $j$, $type(j) \in TYPE$;
$\text{cap}(l), l \in TYPE$: the seat capacity of an aircraft of type $l$;
$a(l,k), l \in TYPE, k \in AP$: the ground handling time for an aircraft of type $l$ at airport $k$;
$\text{dis}(j), j \in AC$: the airport where aircraft $j$ is located at the starting time of the recovery period, $\text{dis}(j) \in AP$;
$t(i,j), i \in F, j \in AC$: block time (the time between engine start at the airport of origin and engine stop at the airport of destination) or the duration of flight $i$ serviced by aircraft $j$;
$D(i,j), i \in F, j \in AC$: the direct operational cost when flight $i$ is serviced by aircraft $j$;
$MNT$: set of aircraft, which should finish its rotation at certain airports for the regular technical maintenance, $MNT \subset AC$;
$mnt(j), j \in MNT$: the airport where aircraft $j$ should finish its rotation, $mnt(j) \in AP$;
$k_2(j), j \in AC$: the penalty cost if aircraft $j$ is not present at airport $mnt(j)$ at the end of the recovery period;
$\text{clo}(k), k \in AP$: the closing time of airport $k$ valid on the day for which a new schedule is designed;
$PTYPE(k), k \in AP$: set of aircraft types, which should be available at airport $k$ at the end of the recovery period;
$\text{notype}(l,k), k \in AP, l \in PTYPE(k)$: the number of aircraft with the seat capacity not smaller than $\text{cap}(l)$, which should finish their rotation at airport $k$;
$\text{pen}(l,k), k \in AP, l \in PTYPE(k)$: the penalty cost per aircraft when the number of aircraft, with the seat capacity not smaller than $\text{cap}(l)$, which finish their rotation at airport $k$, is smaller than $\text{notype}(l,k)$;
$\text{DAP}$: set of airports with detected disturbances, $\text{DAP} \subset AP$;
$\text{TDAP}(k), k \in \text{DAP}$: the earliest possible time when airport $k$ can start operation after the disturbance elimination;
$\text{DAC}$: set of aircraft with detected disturbances, $\text{DAC} \subset AC$; and
$\text{TDAC}(j), j \in \text{DAC}$: the earliest possible departure time of aircraft $j$ from airport $\text{dis}(j)$ after the disturbance elimination.

The variables of the ASDP can be defined in the following formal manner:
$\text{rot}(l,j)$, $\text{rot}(2,j), \ldots$, $\text{rot}(l(j),j)$: the rotation of aircraft $j$, where $l(j)$ is the number of flights in this rotation, $\text{rot}(l,j) \in F$ for $l = 1, 2, \ldots, l(j)$ and $j \in AC$;
$\text{TR}(i), i \in F$: the earliest possible departure time for flight $i$;
$\text{X}(i,j), i \in F, j \in AC$: a binary variable equal to $1$ if aircraft $j$ is assigned to flight $i$, $0$ otherwise;
$\text{can}(i), i \in F$: a binary variable equal to $1$ if flight $i$ is cancelled, $0$ otherwise;
$\text{ment}(j), j \in MNT$: a binary variable equal to $1$ if aircraft $j$, requiring the regular technical maintenance, has not finished its rotation at airport $mnt(j)$, $0$ otherwise; and
$\text{sat}(s,l,j,k), k \in AP, j \in AC, l \in PTYPE(k), s \in \{1, 2, \ldots, \text{notype}(l,k)\}$: a binary variable equal to $1$ if aircraft $j$, finishing its rotation at airport $k$, satisfies the $s$-th necessity of this airport for aircraft type $l$, $0$ otherwise.
Using the previously introduced notations, the objective function of the ASDP can be formally expressed as:

\[
\max F = \sum_{i \in F} \text{fare (i)} pax (i) (1 - \text{can (i)}) - \left( \sum_{i \in F} \sum_{j \in AC} D (i, j) X (i, j) \right. \\
+ \sum_{i \in F} k (i) \text{can (i)} + k_1 \sum_{i \in F} (TR(i) - TP(i))(1 - \text{can (i)}) \\
+ \sum_{i \in F} k_2(i) \text{can (i)} + \sum_{j \in MNT} k_3(j) \text{ment (j)} \\
\left. + \sum_{k \in AP} \sum_{l \in PTYPE (k)} \text{pen (l, k) (notype (l, k))} - \sum_{s=1} \sum_{j \in AC} \text{sat (s, l, j, k)} \right)
\]  

(2.1)

The first term in (2.1) represents passenger ticket revenue, while the second term (given in brackets) is the cost consisting of the direct operational flight cost, the priority-flight cancellation cost, the flight-delay cost, the non-priority flight cancellation cost, the aircraft maintenance disturbance cost and the balance disturbance cost. Let us note that, defining the objective function in the form (2.1), we in fact relax the aircraft maintenance and the aircraft balance constraints, introducing the total penalised violation of these constraints as a part of its cost. In this way, although the constraints can be unsatisfied, the maximisation of the objective function tends to minimise their violation.

The main constraints of the ASDP can be formally defined by:

\[ TR(i) \geq TP(i), \quad \text{for } i \in F, \]  

(2.2)

\[ TR(i) - TP(i) \leq \text{delay (i)}, \quad \text{for } i \in F, \]  

(2.3)

\[ TR(\text{rot (l, j)}) + t(\text{rot (l, j), j}) + d(\text{otype (j), j}) \leq TR(\text{rot (l + 1, j)}), \quad \text{for } l = 1, 2, ..., (j) - 1, \quad j \in AC, \]  

(2.4)

\[ d(\text{rot (l, j)}) = o(\text{rot (l + 1, j)}), \quad \text{for } l = 1, 2, ..., (j) - 1, \quad j \in AC, \]  

(2.5)

\[ o(\text{rot (1, j)}) = \text{dis (j)}, \quad \text{for } j \in AC, \]  

(2.6)

\[ TR(i) \leq \text{clo (o(i))}, \quad \text{for } i \in F', \]  

(2.7)

\[ TR(i) + t (i, j) \leq \text{clo (d (i))}, \quad \text{for } i \in F', j \in AC, X(i, j) = 1, \]  

(2.8)

\[ TR(i) \geq \text{TDAC (j)}, \quad \text{for } j \in DAC, X(i, j) = 1, \]  

(2.9)

\[ TR(i) \geq \text{TDAP (k (j))}, \quad \text{for } o(i) \in DAP, \]  

(2.10)

\[ TR(i) \geq \text{TDAP (d (i))} - t(i, j), \quad \text{for } d(i) \in DAP, X(i, j) = 1, \]  

(2.11)

The conditions (2.2)–(2.11) express the time constraints of the problem: (2.2) and (2.3) mean that each flight departs not earlier than the planned departure time, while its delay is not greater than the maximal allowed value. Constraints (2.4) indicate that the following flight in the rotation of an aircraft cannot take-off before the previous flight has landed and the aircraft has been ground-handled. Equalities (2.5) and (2.6) provide that in the rotation the destination airport of a flight and the origin airport of the following flight are identical, as well as the first flight begins from the airport where the aircraft is located stays at the starting time of the recovery period. Inequalities (2.7) and (2.8) express that a flight should take-off before the closing of its origin airport and it should not land after the closing of its destination airport.
Also, a flight cannot take-off before repairing a breakdown of the aircraft assigned to it (constraints (2.9)). In the case when its origin airport has a detected disturbance, a flight cannot take-off before the airport is reopened (constraints (2.10)), while if its destination airport is disturbed, it cannot land before the disturbance has been eliminated (constraints (2.11)).

\[
cap(atype(j)) - cap(l) \geq 0, \quad \text{for } sat(s, l, j, k) = 1, \\
k \in AP, j \in AC, \ l \in PTYPE(k), \ s \in \{1, 2, \ldots, \ notype(l, k)\}
\] (2.12)

Inequalities (2.12) are related to the aircraft balance constraints and provide that each aircraft, which satisfies the need for a certain aircraft type at an airport to have the seat capacity not smaller than the seat capacity characterising this type.

\[
cap(atype(j)) \geq pax(i), \quad \text{for } i \in F, \ j \in AC, \ X(i, j) = 1,
\] (2.13)

\[
cap(atype(j)) \geq pax(id(i, i)) + pax(i), \quad \text{for } i \in Pr, \ j \in VIA(i), \ X(i, j) = 1
\] (2.14)

Conditions (2.13) and (2.14) are the capacity constraints: (2.13) means that the number of passengers on each flight is not greater than the seat capacity of the assigned aircraft, while (2.14) expresses that the total number of passengers on that flight and an indirectly realised non-priority flight is not greater than the seat capacity of the assigned aircraft, when a priority flight is realised using the VIA principle.

The mathematical modelling of ASDP, defined by (2.1)–(2.14), is only partial, hence many constraints, which are assumed to be satisfied, are not formalised. For example, the objective function (2.1) is correct only if we assume that an aircraft cannot be assigned to a cancelled flight, i.e. \(can(i) = 1\), if and only if \(X(i, j) = 0\) for each \(j \in AC\). But this mathematical formalisation could be a basis for developing more sophisticated mathematical models of the ASDP, such as a mixed integer programming or a constraint programming model. As this problem is known to be NP-hard and it should be solved in real-time, instead of making a great effort to find such an appropriate model and try to solve it using an exact method, we focus on a heuristic approach to the problem. Therefore, in the next section we propose a special heuristic technique for determining a list of feasible ‘satisfactory’ (sub-optimal) new daily flight schedules, among which the dispatcher can select and implement the most convenient one.

### 3. Heuristic algorithm for airline schedule disturbances problem (ASDP)

Before presenting the steps of a proposed heuristic algorithm for ASDP, we will describe in more detail some of the basic notions introduced in Section 2.

A rotation is a series of flights by an aircraft where the first flight in the series departs from the base airport, and the last one arrives at the base airport. The rotation can be defined as a one-day or multi-day rotation. In this paper, rotation is defined as a one-day rotation or part of multi-day rotation during a considered day. Rotations consist of mini rotations and simple rotation segments. A mini rotation is a series of flights attached to each other where the departure airport of the first flight is the same as the arrival airport of the last flight in the series (for example, Belgrade–Prague–Belgrade). A simple segment of the rotation is the series of flights attached to each other where the departure airport of the first flight and the arrival airport of the last flight in the series are different airports (for example, Belgrade–Beirut–Dubai).
The results of disturbances are flight delay and/or cancellation. The flight can be cancelled temporarily or permanently. A non-priority flight is temporarily cancelled if its delay is greater than the maximal allowed one or if this delay causes a new earliest possible departure time/arrival time of the flight within the closure period of its origin/destination airport. If such a flight belongs to a mini rotation, this mini rotation is temporarily cancelled. If such a flight is a part of a simple segment of a rotation, all following flights in this segment until the end of the considered day are temporarily cancelled. A mini rotation can also be temporarily cancelled if it precedes a mini rotation/simple segment of the same rotation with a priority flight whose delay is greater than the maximal allowed one. In this case, the priority flight and its mini rotation/simple segment of the rotation can be possibly realised without any delay or within the maximal allowed one, since some of the preceding mini rotations are temporarily cancelled.

A temporarily cancelled flight can be realised by adding (if it is possible) its cancelled mini rotation/simple segment to the rotation of some of the other aircraft in one of the following three ways: before the first flight in the rotation (Figure 2), between flights in the rotation (Figure 3), or after the last flight in the rotation (Figure 4). The temporarily cancelled flights and its cancelled mini rotations/simple segments are permanently cancelled if there is no possibility to add them to the rotation of one of the other aircraft by any of the ways previously described. Also, a priority flight and its mini rotation, or all following flights of its simple segment by the end of the considered day, are permanently cancelled if there is no possible way to realise it using the VIA principle.

In order to reduce the total delay and/or the balance disturbance cost, the delayed flights can be crossed. Crossing delayed flights can be achieved by two operations: removing part of one and adding it to the other rotation, and interchanging parts of two rotations. Removing part of one and adding to the other rotation refers to the delayed flights whose delay is not directly caused by the airport disturbance. Shifting those flights to the other rotation is an attempt to reduce its delay, i.e. to decrease the value of the objective function (2.1).

Interchanging parts of two rotations (Figure 5) is possible only if it leads to a delay reduction. In order to interchange some parts of two rotations, it is necessary that these parts depart from the same airport within the time period from the planned departure time to the time caused by the maximal allowed delay of the considered flights. Let us note that not only parts with the delayed flights can be interchanged, but also with delayed and no delayed flights if the total delay is reduced in this way.

Starting from the previous considerations we will present a special heuristic algorithm which consists of the following steps:

**Step 1:** Designing the basic feasible schedule

![Figure 2](image.png)

Figure 2. Adding a temporarily cancelled mini rotation at the beginning of the rotation.
In order to create a new feasible daily operational flight schedule, all operating aircraft are considered, both aircraft that have landed at their arrival airports and aircraft that are in flight at the moment of the disturbance.

For each disturbed aircraft whose rotation does not contain priority flights, a new feasible schedule is designed as follows (Figure 6):

- If the delay is less than the maximal allowed, the basic feasible solution is designed by shifting the delayed flight by the delay time. The following flight departs either on time or after completing the previous flight (the flight is completed at the moment when aircraft is ready to commence another flight). The procedure is repeated until the end of the aircraft’s rotation.
- If the flight delay is greater than the allowed maximum, the basic feasible solution is designed so that the mini rotation containing the delayed flight is temporarily cancelled (Figure 6). If the other flights in that rotation do not have a delay greater than the allowed maximum, they are realised with or without delay. If there are flights whose delay will be greater than allowed after the temporary cancellation of the mini rotation, those flights are temporarily cancelled according to the given procedure. The procedure is repeated as long as there are flights that need to be temporarily cancelled. Mini rotations/simple segments of rotation with allowed delay are also temporarily cancelled if the constraint related to airport working hours is violated.
If the flight belongs to the simple segment of rotation, and delay is greater than the allowed maximum, all flights until the end of that rotation are temporarily cancelled.

For each disturbed aircraft whose rotation contains a priority flight, a new feasible schedule is designed as follows (Figure 6):

- If delays of the priority flight and the previous flights in the rotation (if they exist) are not greater than the maximum allowed ones, then these flights are realised with this delay, while a following flight in the rotation departs either on time or it is shifted to start immediately after the completion of the previous flight.
- If the delay of the priority flight is greater than the maximal allowed one and there is a mini rotation(s) which precedes this flight, then this mini rotation(s) is temporarily cancelled, until the priority flight is serviced with the allowed delay.
If, after the temporary cancellation of a mini rotation, the delay of the priority flight is still greater than the maximum allowed one, then Procedure A (when this flight is a part of a simple segment of the rotation) or Procedure B (when this flight is a part of a mini rotation) is applied (Figure 6).

For Procedure A (Figure 7) – where the priority flight is a part of a simple segment:

- Find an aircraft (of the same type or a different one with the corresponding capacity) which can realise the priority flight using the VIA principle and whose rotation does not contain any other priority flight. If there are more than one such aircraft, choose among them the aircraft whose VIA rerouting causes the minimal number of temporarily cancelled flights in its rotation. If there are several aircraft that can be VIA rerouted with the same minimal number of temporarily cancelled flights, choose the aircraft with the minimal total delay. And if the delays are equal, choose the aircraft found first.

- If the rotations of all aircraft, which can realise the considered priority flight using the VIA principle, contain other priority flights, choose among them an aircraft (of the same type or a different one with the corresponding capacity) whose rotation has unrealised priority flights such that after its VIA rerouting all these flights can be realised within their maximum allowed delays.

- When several aircraft satisfy the previous conditions, the selection of an aircraft for the VIA principle is done in the same way as in the case of the rotations without priority flights.

- If there are no aircraft, which can realise the considered priority flight using the VIA principle, cancel permanently this flight and all following flights of its simple segment until the end of the day.

**Procedure A**

(Priority flight in the simple segment)

- VIA (aircraft without priority flight)
  - Yes
  - VIA (aircraft with priority flight)
    - No
    - Permanently cancel the flight

The end of the Procedure A

Figure 7. The scheme for Procedure A.
For Procedure B (Figure 8) – where the priority flight is part of a mini rotation:

- Assign the mini rotation of the considered priority flight to other aircraft (of the same type or a different one with the appropriate capacity) whose rotation does not contain any other priority flight and which can realise this mini rotation within the maximum allowed delay. If there are several such aircraft, choose the aircraft whose rerouting causes the minimal number of temporarily cancelled flights. If there is more than one aircraft that can be rerouted with the same minimal number of temporarily cancelled flights, choose the aircraft with the minimal total delay. And if the delays are equal, choose the aircraft found first.

- If there are no aircraft without priority flights in their rotation which can realise the mini rotation of the considered priority flight within allowed delays, try to find an aircraft whose rotation also does not contain any other priority flight, but which can realise the considered flight using the VIA principle. This selection is performed in the same way as in the case when the priority flight belongs to a simple segment (see Procedure A).

- If the previous selection fails, try to find an aircraft (of the same type or a different one with the corresponding capacity) whose rotation contains priority flights and which can realise the mini rotation of the considered priority flight within the maximum allowed delays. This selection is

![Figure 8. The scheme for Procedure B.](image)
performed in the same way as in the case of aircraft without priority flights in its rotation. If there is no such aircraft, try to find an aircraft with priority flights in their rotation, which can realise the considered flight using the VIA principle. (This selection is the same as in Procedure A).

- If the considered priority flight cannot be realised by any of the previously described ways, cancel permanently this flight and its mini rotation.
- If the considered aircraft has more than one priority flight in its rotation, repeat the previous procedure for each of them.

Now the complete basic feasible schedule is designed by repeating the entire Step 1, presented in Figure 6 for each aircraft that is influenced by the disturbance, and the corresponding value of the objective function (2.1) is calculated.

If there are no temporarily cancelled flights in the designed complete basic schedule, the heuristic algorithm goes to Step 3. Otherwise it goes to Step 2.

**Step 2: Adding temporarily cancelled flights.**

All temporarily cancelled flights are sorted in a list according to the following procedure:

1. Partition the set of all temporarily cancelled flights into the cancelled mini rotations and simple sections and sort them into the following four groups:
   - mini rotations that begin and end at the base airport of their rotations;
   - mini rotations that begin and end at some other airport;
   - simple segments that begin or end at the base airport of their rotation; and
   - simple segments that neither begin nor end at the base airport of their rotations.

2. Sort elements (mini rotations or simple segments) of each group according to the decreasing total number of flights and decreasing total delay.

Starting from the top of the list of the temporarily cancelled flights, the heuristic algorithm passes through all its elements and tries to add each of them to the basic schedule, determined in Step 1. The element (a mini rotation or a simple segment) should be added to the schedule within the time period between the planned departure time and the maximal allowed delay.

![Figure 9. Adding a temporarily cancelled mini rotation.](image-url)
time and the latest possible departure time (according to the maximal allowed delay or airport working hours) of the first flight in this element (Figure 9). The element is added to the rotation of an aircraft of the same type as the aircraft that was originally supposed to realise it. If such an aircraft does not exist, the algorithm tries to add the element to the rotation of an aircraft of a different type, but with a sufficient capacity.

Try to assign a temporarily cancelled flight to the first available aircraft (the aircraft is at the appropriate airport and can depart at the time which is closest to the planned departure time), then to the second available aircraft, etc. If adding flights is successful, calculate the objective function value. Stop the procedure if flights are not delayed after adding them and the other flights from the rotation do not suffer additional delay. Accept this solution and put it in first place in the solution list (Figure 11).

Temporarily cancelled flights are permanently cancelled if they are not assigned to any aircraft. Consider the next group of flights from the solution list using the same procedure. Adding temporarily cancelled flights can cause the temporary cancellation of following flights (Figure 10). These new temporarily cancelled flights are put in the appropriate place on the list of temporary cancelled flights. It is necessary to emphasise that, in this case, the total number of temporary cancelled flights does not increase (number of temporary cancelled flights, which are added were not lower than the number of newly temporary cancelled flights). When the list of temporary cancelled flights is exhausted, the algorithm goes to Step 3 if there are delayed flights in the current schedule. Otherwise, it stops (Figure 11).

**Step 3: Rotation crossing.**

All mini rotations and simple segments of the schedule, obtained by Step 2, which contain delayed flights, are sorted into the list of delayed flights according to decreasing total delay. (The flights delayed because of external factors (meteorological conditions or slots) are not considered in this step, as their delays cannot be decreased.)

Rotation crossing refers to removing parts of the rotation and adding them to some other rotation (the first part of Step 3, Figure 12) and interchanging parts of two rotations (the second part of Step 3, Figure 13) in order to reduce the total delay.

Take the first group of flights from the list. Try to add them to another rotation. Consider aircraft of the same type and the rotations where the departure of the first flight in the group of flights is the earliest (but not earlier than the planed departure

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**Figure 10.** Temporary cancellation of flights from the basic schedule.
time and not later than the latest departure time (Figure 10)). After that consider aircraft of different type if capacity is not lower than the number of passengers on the considered flight. The flights are added to each rotation (firstly to the aircraft of the same type, and then to the aircraft of a different type) to all possible places of the considered rotation where flights are added.

For each successfully added group of flight to the rotation, calculate the objective function value and put the obtained solution at the appropriate position on the solution list. If the new obtained solution is top ranked on the $n$ solution list, then this solution becomes the initial solution and a new delayed flight list should be created. After that, the procedure abovementioned is repeated.

Figure 11. Step 2 of the heuristic algorithm.
If the delayed flights list is exhausted and if there are delayed flights from different rotations, continue interchanging rotations parts (Figure 13), otherwise go to Step 4. Make the delayed flights list in relation to the delays of the first solution from the solution list, the same way as in the first part of Step 3. Take the first group of flights from the delayed flight list (obtained at the end of the first part of Step 3) and try to interchange it with any following group from the list. For successful interchange, calculate the objective function value and place the obtained solution on the appropriate position on the solution list.

If the new flight schedule is put on the first position on the $n$ solution list, then this schedule becomes the initial solution and a new delayed flight list should be created. Next the abovementioned procedure is repeated. The procedure (interchanging parts of two rotations) ends when the delayed flights list is exhausted. As a result of Step 3 the final $n$ solutions list is offered to the dispatcher, sorted in descending order of the objective function value. Then go to Step 4.

Figure 12. The first part of Step 3.
Step 4: The end of the algorithm.

4. Numerical example

As a result of the DSS’s application, at least one feasible solution is provided. Each solution generated by the DSS is presented as a graphical representation of the daily schedule and table which includes planned and obtained data on the flight schedule (flight delay, flight cancellation, swapping registration or type of aircraft, etc.). In the graphical representation, the aircraft assigned to scheduled rotations within the given day are presented on the left hand side, while the dynamic time axis is shown at the top of the screen, from left to right, as a red vertical line, which represents the time horizon within which the system currently is observed. The status of flights at the time horizon is defined by their colour on the screen:

- green-coloured flights are those that have already been realised at the given moment of time;
- blue-coloured flights are operating flights at the given moment;
yellow-coloured flights are those that should be realised, according to the daily rotations schedule; and
red-coloured flights are those that will be affected by the disturbance.

The time axis moves at five-minute intervals, and then the status of flights automatically changes, as does their colour on the screen. Changes to the status of flights can also be made manually, by activating a corresponding option in the window, which shows up if a flight in the rotation chart is selected, or by inputting corresponding changes into the model's database.

The algorithm is illustrated with reference to *JAT Airways* timetable. The chosen values of penalties were defined in cooperation with *JAT Airways* representatives and in our numerical examples they are as follows:

- \[ k(i) = 20,000 \text{ units}, \text{penalty for priority flight cancellation}; \]
- \[ k_2(i) = 10,000 \text{ units}, \text{penalty for non-priority flight cancellation}; \]
- \[ k_3(j) = 5000 \text{ units}, \text{penalty for regular maintenance disturbance}; \]
- \[ pen(l,k) = 3000 \text{ units}, \text{penalty for aircraft balance disturbance}; \]
- \[ k_1 = 1 \text{ unit}, \text{penalty for flight delay, per minute}. \]

In the examples considered, the following values for delay are accepted:

- maximum allowed delay for domestic flight, \( delay(i) = 360 \text{ minutes} \); and
- maximum allowed delay for international flight, \( delay(i) = 180 \text{ minutes} \).

For a given day, operations were executed with nine aircraft (four B733’s, three B734’s and two AT7’s), assigned to 47 flights. Because there is no data for aircraft maintenance within the technical maintenance system for this day, it was assumed that all aircraft introduced into the realisation of scheduled operations were available until the end of the given day for making service changes if a disturbance occurred, and if the constraint related to the technical base it was not considered.

The list offered to the dispatchers consists of a maximum of 10 solutions.

For illustrative purposes, consider the situation where an aircraft becomes inoperable. Here aircraft AT7-YUALO fails and is being repaired from 10:50 to 18:00. The DSS software offered three solutions, as follows:

**Solution 1:** The disturbance directly affects four flights (JAT680, JAT681, JAT420 and JAT421) in rotation without priority flight (Figure 14). In the first step of the algorithm, those flights are temporarily cancelled (two mini rotations). The second step attempts to add temporarily cancelled flights to other aircraft. In this step mini rotation BEG–TIV–BEG (JAT680, JAT681) is added to aircraft 733-YUANF, where it is realised with a delay. Because of adding those flights, mini rotation BEG–TIV–BEG (JAT678, JAT679) and flight JAT412 will be realised with delays (160, 150 and 110 minutes, respectively). Flights JAT680 and JAT681 will also both be realised with 270 minutes delay. The mini rotation BEG–IST–BEG (JAT420, JAT421) is cancelled permanently. In the third step, in the rotations’ crossing, the total delay is reduced. The delayed mini rotation BEG–TIV–BEG (JAT678, JAT679) is added to aircraft 734-YUAOR (realised with delay) and the other flights are served without delay. The value of the objective function is 180,588 units (Table 1 – as is data about flights influenced by the disturbance). It can be observed in Table 1 that there are two cancelled and four delayed flights, and that four flights are assigned to other aircraft.
Solution 2: The first and second steps are the same as in Solution 1. The second solution is obtained in the third step. Mini rotation \( \text{BEG} \rightarrow \text{TIV} \rightarrow \text{BEG} \) (JAT678, JAT679) has a delay caused by moving mini rotation \( \text{BEG} \rightarrow \text{TIV} \rightarrow \text{BEG} \) (JAT680, JAT681) from aircraft AT7-YUALO to 733-YUANF. In the third step, in the rotations’ crossing, the delayed flights are added to another aircraft in order to reduce total delay. According to this, mini rotation \( \text{BEG} \rightarrow \text{TIV} \rightarrow \text{BEG} \) (JAT678, JAT679) is moved from aircraft 733-YUANF to 734-YUAOO and causes delay to the flight \( \text{BEG} \rightarrow \text{TGD} \) (JAT668). The value of the objective function is 180,508 units (Table 2 – as well as data about flights influenced by the disturbance).

Solution 3: The third solution is obtained after implementation of the first and second steps. The value of the objective function is 179,890 units (Table 3 – as well as data about flights influenced by the disturbance). It can be observed in Table 3 that there are two cancelled and five delayed flights, and that two flights are assigned to other aircraft.

Table 1. Data on flights influenced by disturbance (Solution 1).

<table>
<thead>
<tr>
<th>Flight</th>
<th>Departure airport</th>
<th>Arrival airport</th>
<th>Planned departure time</th>
<th>Real departure time</th>
<th>Delay (min)</th>
<th>A/C planned to realise the flight</th>
<th>A/C which actually realised the flight</th>
<th>Objective function value</th>
</tr>
</thead>
<tbody>
<tr>
<td>JAT420</td>
<td>BEG</td>
<td>IST</td>
<td>14:20</td>
<td>Cancelled</td>
<td></td>
<td>YUALO</td>
<td>–</td>
<td>180,588</td>
</tr>
<tr>
<td>JAT421</td>
<td></td>
<td></td>
<td>17:20</td>
<td>Cancelled</td>
<td></td>
<td>YUALO</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>JAT680</td>
<td></td>
<td></td>
<td>10:50</td>
<td></td>
<td></td>
<td>YUALO</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>JAT681</td>
<td></td>
<td></td>
<td>12:30</td>
<td></td>
<td></td>
<td>YUALO</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>JAT678</td>
<td></td>
<td></td>
<td>16:00</td>
<td></td>
<td></td>
<td>YUANF</td>
<td>–</td>
<td>180,588</td>
</tr>
<tr>
<td>JAT679</td>
<td></td>
<td></td>
<td>17:40</td>
<td></td>
<td></td>
<td>YUANF</td>
<td>–</td>
<td>180,588</td>
</tr>
</tbody>
</table>

Figure 14. Graphic presentation of flight schedule at the moment of disturbance.
From the above presented solutions, it can be seen that the second and third steps of the algorithm improve the basic solution by increasing the value of the objective function. Also, solutions are sorted by decreasing value of the objective function. This DSS is installed on a PC Pentium IV and yields a set of solutions in less than 10 seconds.

5. Conclusions and future directions

The ASDP is a problem that airlines face daily. Because of disturbances, airlines incur additional costs, passengers can be dissatisfied and the reputation of an airline can be reduced. In order to minimise disturbance effects, dispatchers in an AOC
need to perceive the consequences of different solutions from all aspects, in real-time. DSS that can offer several feasible solutions based on airline policy in real-time are needed in order to simplify the decision-making process.

This paper has presented a mathematical formalisation of the airline schedule disturbance problem, a special heuristic algorithm and appropriate software with user-friendly interface, i.e. DSS, for designing a new daily operational flight schedule due to disturbances. This heuristic algorithm can be used in real-time to generate several new daily schedules, which are sorted by decreasing value of the objective function. Any of these flight schedules could be accepted by dispatchers. The proposed model is supported by the corresponding software, which was illustrated using a JAT Airways numerical example.

Based on the review of the developed DSS (i.e. its mathematical formalisation, the special heuristic algorithm, developed software with user-friendly interface) and numerical example given in this paper, a number of conclusions can be reached. First, the model yields as a result a list of feasible solutions in real-time, so the decision-maker can choose and apply any of the solutions from the list, taking into consideration, if necessary, criteria not included into the model. Second, delaying, cancellation and resource (aircraft) substitution are suggested as the main actions for solving disturbance problems. Using spare resources and ferry flights are not foreseen by the developed model. Third, costs are not presented by real values, but rather as penalties. The dispatcher can change these penalties according to experience, where an instantaneous traffic situation has to be solved, or airline policy. New strategies can be defined by changing the values of these penalties (costs), therefore, the DSS can accommodate different airline policies. Fourth, models are usually made to meet specific airline needs. A great advantage of the model presented in this paper is the possibility of changing penalty values, which can adapt the model for different airline needs (this model is installed and in use at the JAT Airways AOC and has also been successfully tested at the AOC of a European middle-size airline). Fifth, the developed DSS is easy to use for less experienced dispatchers and is a potentially useful tool in dispatcher training. Finally, the use of the developed DSS in the AOC could make the dispatchers’ work simpler and faster, as well as enable them to foresee the effects of any applied solution.

Furthermore, the model presented in this paper gives solutions that are feasible from the aspect of aircraft availability, while crew availability is not considered despite the close link between aircraft and crew. In further research these two resources could be merged, so that the obtained solution is feasible from the aspect of both aircraft and crew. This will facilitate the dispatchers’ work, because they could reallocate all resources running a single DSS software application. This could also additionally reduce the time needed for disturbance problem solving. Passenger costs are not considered in this model, which can also be a possibility for new research. Some airlines have several priority flight categories. Therefore, priority flight categories could be divided into subcategories, with different penalties attached to them. Ferry flights can also be introduced as well as the use of spare aircraft.

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References


