Numerical simulation of wind turbulence and buffeting analysis of long-span bridges

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Abstract

A consistent approach for buffeting analysis of long-span bridges in time domain is presented. As the input for the approach, time histories of wind turbulence are numerically generated by the ARMA method. Validity of simulated turbulence is improved since the spatial correlation between different wind velocity components are fully accounted. A new buffeting analysis scheme is then presented. By complex modal analysis via Mode Tracing method, actual modal characteristics of the system in the presence of aeroelastic phenomena at a certain wind speed can be obtained. Coupled responses are thus effectively captured. Numerical example is made for the Akashi-Kaikyo bridge. By using the scaled version of turbulence generated in wind tunnel, the analytical results agree very well with those of experiment. Some effects on the results due to differences between characteristics of turbulence simulated in wind tunnel and turbulence characteristics proposed from literature are thus clearly pointed out. © 1999 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Recent need of stretching longer spans has made the dynamic behaviors of cable-supported bridges extremely complicated under wind actions. As for buffeting response, the full-model test of the Akashi-Kaikyo bridge in wind tunnel exhibited a strongly coupled three-dimensional vibration under gusty wind [1]. To predict the buffeting response, frequency domain approach is conventional. However, for a very...
long-span bridge, where highly nonlinear and coupled responses due to significant wind-structure interaction are encountered, the time-domain approach is a competitive alternative.

As the key element for a time-domain approach, the numerical simulation of the wind loads or, more primarily, wind velocity components as functions of time and space is needed. There exist some wind turbulence simulation works by different methods. However, each component of wind velocity has been generated separately [2], so that the relation between them could not be incorporated. This may lead to errors in estimating buffeting response as noted by some researchers [3]. In this study, the numerical simulation of wind turbulence is made by auto regressive-moving average (ARMA) method. Two velocity components of wind turbulence, along-wind ($u$) and vertical ($w$), are generated simultaneously and spatially, so that the spatial correlation between these components can be fully taken into account.

So far there have been many proposed methods to predict buffeting response of long-span bridges [2-4]. However, almost all methods, either in frequency domain or time domain, are based on the assumption that the modal characteristics of the structure do not change as the wind speed varies. The mechanical eigenvectors at zero wind speed were thus widely used for modal decomposition. This assumption appears not true for a very long-span bridge. Considerable evolution of modal characteristics due to the change of wind speed has been successfully traced out by the mode tracing method for flutter prediction [5]. In this study, an approach in time domain for buffeting analysis of long-span bridges based on this method is presented. Direct complex modal analysis of the three-dimensional model of a long-span bridge in the presence of aeroelastic phenomena is performed. The modal characteristics with the integration of aerodynamic effects at certain wind speed therefore, can be accurately obtained.

The Akashi–Kaikyo bridge’s buffeting response has been the point of interest of many analytical works [2]. However, whereas the torsional and vertical responses were predicted fairly well, the horizontal response was greatly over-estimated when compared to those of the full-model test of the bridge [1]. Using the proposed approach, this study effectively predicts the response and identifies reasons for the error. It is found that the overestimation of horizontal response was mainly caused by the use of the conventional Davenport’s spatial coherence function, which is much higher than that of the turbulence simulated in wind tunnel, especially in the low-frequency range.

2. ARMA model and application to wind field

Among many methods, the recursive digital filtering ARMA method is proved to be most efficient in terms of saving computational efforts and computer resource demand. The expression of ARMA model for an $m$-variate stationary process is [6]

$$Y_r = \sum_{i=1}^{q} A_i Y_{r-i} + \sum_{j=0}^{q} B_j X_{r-j},$$  (1)
where \( Y_r \) is the simulated \( m \)-variate time series, \( X_r \) is the \( m \)-variate Gaussian white noise series, \( q \) is the order. The ARMA coefficient matrices \( A_i \) and \( B_j \) are determined from a prescribed correlation function matrix \( C \) in the least-square sense so that the generated time series will have the prescribed statistical characteristics. In this study, the two-stage matching method formulated by Samaras et al. [6] is used to determine \( A_i \) and \( B_j \). The simulated \( m \)-variate time series \( Y_r \) can then be recursively generated for any required length by Eq. (1). The method is applied for the simulation of wind field, which can be modeled as consisting of a steady part \( U \) (mean wind speed) and fluctuating wind velocity parts \( u(t) \), \( v(t) \) and \( w(t) \). For bridge analysis, the lateral \( v(t) \) component is neglected, and hence the horizontal \( u(t) \) and vertical \( w(t) \) components are involved in the simulation. To take into account their correlation, \( u \) and \( w \) need to be generated simultaneously. Hence, \( u \) and \( w \) at \( n \) points are generated as an \( m \)-variate process. Each variate represents a velocity component at a point, then \( m = 2n \). The prescribed correlation matrix \( C(2n \times 2n) \) at a time lag \( \tau \) is formed as follows:

\[
C(\tau) = \begin{bmatrix}
R_{u1 u1}(\tau) & \ldots & R_{u1 un}(\tau) & R_{u1 w1}(\tau) & \ldots & R_{u1 wn}(\tau) \\
\vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
R_{un u1}(\tau) & \ldots & R_{un un}(\tau) & R_{un w1}(\tau) & \ldots & R_{un wn}(\tau) \\
R_{w1 u1}(\tau) & \ldots & R_{w1 un}(\tau) & R_{w1 w1}(\tau) & \ldots & R_{w1 wn}(\tau) \\
\vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
R_{wn u1}(\tau) & \ldots & R_{wn un}(\tau) & R_{wn w1}(\tau) & \ldots & R_{wn wn}(\tau)
\end{bmatrix}, \tag{2}
\]

where a correlation function \( R_{ab}(\tau) \) of two time series \( a \) and \( b \) can be obtained from its corresponding cross-spectrum \( S_{ab}(f) \) by inverse Fourier transform. Therefore, to construct the correlation function matrix, knowledge of turbulence spectra, spatial cross-spectra or spatial coherence functions are necessary.

3. Turbulence characteristics

3.1. Turbulence characteristics proposed from literature for natural wind

After Kaimal et al. [7], the auto-spectra \( S_u, S_w \), and the cospectrum \( C_{uw} \) of natural turbulence can be expressed as

\[
\frac{f S_u(f)}{u^*} = \frac{105 f_r}{(1 + 33 f_r)^{5/3}}, \quad \frac{f S_w(f)}{u^*} = \frac{2 f_r}{(1 + 5.3 f_r)^{5/3}}, \quad \frac{f C_{uw}(f)}{u^*} = \frac{-14 f_r}{(1 + 9.6 f_r)^{2/3}}, \tag{3}
\]

where \( f \) is frequency, \( f_r \) is the reduced frequency, and \( u^* \) is the friction velocity. No quantitative assessment has yet been made for quadrature spectrum, which appears to be negligible however. The cross-spectrum thus can be supposed to take the value of the cospectrum. A spatial cross-spectrum then can be calculated via a corresponding
coherence function. The conventional square-root coherence function of the same velocity component at two points \( P_1(x_1, y_1) \) and \( P_2(x_2, y_2) \) is given by Davenport (see Ref. [4]),

\[
\sqrt{\text{Coh}(f)} = e^{-f} \quad \text{where} \quad f = \frac{\sqrt{C_x^2(x_1 - x_2)^2 + C_y^2(y_1 - y_2)^2}}{0.5(U_{y_1} + U_{y_2})},
\]

\[ C_x = 10, \quad C_y = 16. \quad (4) \]

For the coherence function of \( u \) and \( w \) at different points, there is no information in literature. An approximation of this coherence function is proposed in the next section.

### 3.2. Characteristics of turbulence generated in the wind tunnel

In the full model test of the Akashi–Kaikyo bridge [8], the buffeting response of the full model was obtained under the action of the turbulent flow generated in the wind tunnel. Therefore, the analytical results will agree better with the experimental results if the wind-tunnel turbulence is used as the input. For that idea, records of wind-tunnel turbulence of the full model test [8] are analyzed to extract its statistical characteristics, which will in turn serve as the target input to the numerical simulation. The scaled spectra and point cospectrum of the turbulence can be expressed as

\[
\frac{f S_u(f)}{u^2} = \frac{5.11f_t}{(1 + 7.05f_t)^{5/3}}, \quad \frac{f S_w(f)}{u^2} = \frac{6.15f_t}{(1 + 3.7f_t)^{5/3}}, \quad \frac{f \text{Co}_{uw}(f)}{u^2} = \frac{-1.5f_t}{(1 + 2.94f_t)^{2.4}}. \quad (5)
\]

The spatial coherence \( \text{Coh}_u(f) \) and \( \text{Coh}_w(f) \), however, do not approach unity at zero frequency. This aspect, which is also noted in Ref. [1], is remarkably different from Davenport’s form (Eq. (4)). For these coherences, we propose the following modified forms.

\[
\text{Coh}_{u1u2}(f) = (1 - 0.001 \, dx - 0.0003 \, dx^2) \exp(-f \, C_x^u \, dx/U) \quad (6)
\]

where \( C_x^u = 12, \)

\[
\text{Coh}_{w1w2}(f) = (1 - 0.03 \, dx + 0.0002 \, dx^2) \exp(-f \, C_x^w \, dx/U) \quad (7)
\]

where \( C_x^w = 8, \)

where \( dx \) is spanwise distance. To complete the turbulence model for the input, we propose an approximation of the spatial coherence between \( u \) and \( w \) as follows:

\[
\text{Coh}_{u1w2}(f) = \frac{1}{2} \frac{\text{Co}_{uw}^2(f)}{S_u(f)S_w(f)} [\text{Coh}_{u1u2}(f) + \text{Coh}_{w1w2}(f)]. \quad (8)
\]

This approximate coherence is very well justified by the analysis of wind-tunnel turbulence as typically shown in Fig. 4.
4. Results of wind turbulence simulation and checks

The simulation is made at various values of time step. The correlation function matrix is constructed from the turbulence characteristics proposed in literature for natural wind \([4,7]\). The time histories of the simulated turbulence are statistically analyzed to compare with the target input. The spectra and cospectra of the simulated turbulence agree very well with the input target spectra at small values of time step, e.g. at \(h = 0.05\) s in Figs. 1–3. In Fig. 5, a smaller value of time step gives a better agreement of simulated results with empirical values of Reynolds stress from Ref. \([11]\).

\[
A = \frac{\bar{u}w}{\sqrt{\bar{u}^2 \bar{w}^2}} = -0.26 \sim -0.45. \tag{9}
\]

Therefore, if the time step \(h\) is smaller, resulting in a higher cut-off frequency \(f_c = \frac{1}{2h}\), the simulated turbulence has a better validity in terms of energy contained in the turbulence, and conversely, errors are more pronounced, especially on cospectrum \(C_{uw}\). From sensitive checks, this simulation gives good results if the time step ranges from 0.05 to 0.2 s, which is very small compared to those of existing works in literature \([2]\). By introducing the proposed \(uw\)-coherence function as expressed in Eq. (8), a very good performance of spatial simulation is obtained. Fig. 6 shows a typical comparison between coherence of the simulated turbulence and that of target input. A typical justification of the proposed \(uw\)-coherence function by the turbulence records in wind tunnel for full-model test of the Akashi–Kaikyo bridge is shown in Fig. 4. In addition, the simulation can be well performed at a very low order of ARMA model. The computational effort and computer resource demands are thus greatly reduced.

5. Buffeting analysis scheme by complex modes

5.1. General formulation

The equation of motion of a full-model bridge in the presence of aeroelastic phenomena can be written as \([9]\)

\[
Mu + Cu + Ku = F_{ae} + F_b, \tag{10}
\]

where \(M, K\) are mass and stiffness matrices formed by discrete technique of finite element method, \(u\) is the displacement vector, \(F_{ae}\) is the complex self-excited force depending on reduced frequency \(k = \omega b/U\) (where \(\omega\) is the circular frequency and \(U\) is the mean wind speed), and \(F_b\) is the buffeting force. Assume harmonic oscillation, the local self-excited force \(f_{ae}\) on each element of a bridge deck can be expressed via a set of unsteady coefficients as \([9]\)

\[
f_{ae} = \begin{bmatrix} L_{ae} \\ D_{ae} \\ M_{ae}/B \end{bmatrix} = -\pi \rho B^2 \begin{bmatrix} L_{yR} + iL_{yI} & L_{zR} + iL_{zI} & L_{zR} + iL_{zI} \\ D_{yR} + iD_{yI} & D_{zR} + iD_{zI} & D_{zR} + iD_{zI} \\ M_{yR} + iM_{yI} & M_{zR} + iM_{zI} & M_{zR} + iM_{zI} \end{bmatrix} \begin{bmatrix} \ddot{y} \\ \ddot{z} \\ \ddot{B} \end{bmatrix}
\]

\[
= -F_w \ddot{u}, \tag{11}
\]
where $B = 2b$; $L_{ae}$, $D_{ae}$, $M_{ae}$ are the aeroelastic lift, drag and moment, respectively, $F_w$ contains the set of unsteady coefficients, which are also equivalently known as Flutter derivatives. Other notations are depicted on Fig. 7. The global $F_{ae}$ then can be assembled by the FEM scheme. Neglecting damping and merging the self-excited
force to the left-hand side, Eq. (10) can be rewritten as [9]

\[ M_F \ddot{u} + Ku = F_b. \]  \hspace{1cm} (12)

The self-excited force is embedded in \( M_F \). Thus \( M_F \) is a complex function of reduced frequency \( k \), whereas the buffeting force \( F_b \) is a function of time. This difference makes Eq. (12) impossible to be solved by a normal direct reduction to the modal space. Different frequencies of multi-mode system indicate that at a certain wind speed, each mode of structure has its own reduced frequency. However, also based on this formulation, the mode tracing method [5] provides an alternative. This method targets one mode at a time, and then step by step increases the mean wind speed to find the complex eigen value at each wind speed by an iterative method. The starting point is at \( U = 0 \) m/s, and the tracing process can be made and stopped at any desired wind speed. The eigenvalues of many modes therefore can be determined at any prefixed wind speed. The modal decomposition for Eq. (12) at a certain wind speed
then can be performed by using the corresponding eigenvectors. Since $M_F$ is complex and not symmetric, two biorthogonal sets of eigenvectors, left one $v_L$ and right one $v_R$, exist for the modal decomposition. The left eigenvectors decide the contribution of
external forces to each mode, whereas the right eigenvectors express the mode shapes. The uncoupled equation of motion in generalized coordinate $r$ can be written as

$$\left(v_L^T \cdot M_F \cdot v_R\right)\ddot{r} + \left(v_L^T \cdot K \cdot v_R\right)r = v_L^T \cdot F_b,$$  \hspace{1cm} (13)

where $u=r$. However, this equation is in complex form, which makes it difficult to be solved by a direct integration method. The more convenient and explicit form is

$$\ddot{r}_i + 2\zeta_i\omega_i\dot{r}_i + \omega_i^2 r = \frac{1}{m_i} F_{bi},$$  \hspace{1cm} (14)

where $m_i, F_{bi}$ are, respectively, modal mass and modal buffeting force, obtained as defined in Eq. (13). Modal aerodynamic damping ratio $\zeta_i$ and modal frequency $\omega_i$ here are real values as follows:

$$\omega_i = \sqrt{\lambda_{Ri}^2 + \lambda_{Fi}^2}, \quad \zeta_i = \lambda_{Fi} / \sqrt{\lambda_{Ri}^2 + \lambda_{Fi}^2 + \zeta_{si}}$$  \hspace{1cm} (15)

in which $i = \sqrt{-1}$; $\lambda_{Ri} = \lambda_{Fi} + i\zeta_{si}$ is square root of the corresponding complex eigenvalue; $\zeta_{si}$ is modal structural damping ratio. At this stage, the aeroelastic effects have been integrated into the modal characteristics. This aspect is the unique attitude which makes the present approach more effective and accurate than other existing methods, especially in terms of analyzing strongly coupled responses. Moreover, since actual values of all modal characteristics of the system at the calculating wind speed are known, it is very easy to make judgments and then to select significant modes to include. The computational effort is thus greatly reduced for the investigation.
5.2. Buffeting force

Using the simulated turbulence \( u(t) \) and \( w(t) \), the buffeting force \( F_b \) can be computed by the quasi-steady assumption as follows:

\[
F_b = \begin{pmatrix} L_b \\ D_b \\ M_b/B \end{pmatrix} = \frac{1}{2} \rho U B \begin{bmatrix} 2C_L & (C_L + C_D) \\ 2C_D & C_D \\ 2C_M & C_M \end{bmatrix} \begin{pmatrix} u(t) \\ w(t) \end{pmatrix} \bigg|_{z = z_0},
\]

(16)

where \( L_b, D_b, M_b \) are the buffeting lift, drag and moment, respectively. \( C_L, C_D, C_M \) are the static coefficients for lift, drag and moment, respectively. The prime (') denotes their derivatives with respect to the angle of attack of the mean wind speed on the section model (Fig. 7). Values of these coefficients at the mean angle of attack \( \alpha_0 \) is used. Once the buffeting force is known, the buffeting analysis can be carried out by direct integration method for Eq. (14). Time histories of displacement response, including vertical, horizontal and torsional components, then can be obtained at any nodal points.

In fact, the buffeting force must be modified by an aerodynamic admittance function to account for the imperfect correlation of wind pressure around the deck’s section. The function would approach unity at low-frequency range as suggested by Davenport and Sears’ functions for a flat plate (see Ref. [2]). In this study, this function is assumed to be 1 following quasi-static assumption. This may be a fair assumption since frequencies of significant modes of a very long-span bridge are usually very small. Another note is that the steady part of buffeting force due to mean wind speed is excluded in Eq. (16), and therefore only dynamic responses due to fluctuating parts (zero mean) are the values of interest. This is a fair simplification. The present method is for a linear system. Excluding the mean wind-induced response, which is known to be very large for a long-span bridge [1], the dynamic response is too small for the linear analysis scheme to be valid. For the mean wind-induced response, a treatment by geometric nonlinear analysis was reported in Ref. [10].

6. Numerical example: the Akashi–Kaikyo bridge

The Akashi–Kaikyo bridge, completed in 1998, which is the longest suspension bridge in the world consists of a main span of 1990 m and two side spans of 960 m each. Buffeting response of the bridge has been obtained experimentally by a full model test in wind tunnel [1,8]. In this study, the finite element 3-D frame model of the bridge (Fig. 8) is used for the numerical example of the present method. The buffeting response is calculated at four levels of mean wind speed: \( U = 30, 54, 70 \) and 80 m/s. The set of unsteady coefficients (or flutter derivatives) and the static coefficients for the bridge at \( \alpha = 0 \) are used. Number of modes to be included is 32. Approximate structural logarithmic decrement for each mode is 0.03. The time step for turbulence simulation and direct integration, is 0.1 s. The duration to obtain response is 150 min. This duration is equivalent to the 15 min response of the full
Fig. 10. Spectrum $S_u$.

model test, for which the experimental results [8] are available for comparison. Turbulence intensity $I_u = 10\%$, $I_w = 6\%$.

A comparative analysis is made here. Two cases of turbulence, named (a) and (b), are simulated and then used for the buffeting calculation of the bridge. As previously presented, case (a) is turbulence with characteristics proposed from literature (hereafter called ‘literature turbulence’), and case (b) is turbulence generated in wind tunnel for the full-model test. Root mean square (RMS) and ensemble average maximum amplitude of the response’s time histories are then evaluated for comparison with experimental results [8]. The results of the central node of the main span for two cases of turbulence and those of experiment are shown in Fig. 9. The analytical torsional and vertical RMS responses agree very well with the experimental results in both cases of turbulence. The horizontal RMS response, which is highly overestimated for case (a), is much smaller in case (b) and thus well agrees with experimental result. The comparisons of the ensemble average maximum amplitude show the same improvement tendency in results for case (b). These results prove the correctness of the idea of using the wind-tunnel turbulence as input for a better match with the condition in the wind tunnel test [8] of the bridge. In addition, Fig. 9 also includes the results of response’s RMS from Ref. [2] by the relative velocity method in time domain. The comparison proves the accuracy of the present approach.

A closer look reveals that the horizontal response is governed mainly by the first symmetric sway-dominant mode, which has a very low frequency of around 0.038 Hz. At this frequency, the conventional Davenport’s coherence function gives very high coherence of turbulence along the bridge deck, but the wind-tunnel turbulence has much smaller values of the coherence as shown in Fig. 11. This is the main cause of around the 3-time overestimation for horizontal response when using the literature turbulence [4,7] as input. Moreover, higher values of $S_u$ spectrum of literature turbulence than those of wind-tunnel turbulence at low-frequency range due to the lack of large turbulent scales in wind tunnel [8] as seen in Fig. 10 contribute some more errors to a smaller extent. A sensitivity check indicates that 75% of the error is due to differences of the spatial coherence functions, and 25% of the error is due to differences in the turbulent spectra.
Fig. 11. Coherence function of $u$. Symbol and line: wind-tunnel's and Eq. (6). Dash line: Davenport's by Eq. (4).

Fig. 12. Evolution of mode shape and modal frequency of mode $d_{10}$.

Fig. 12 shows the evolution of the mode shape of mode $d_{10}$ at $U = 0$ and 70 m/s. Remarkable differences, especially in the contribution of vertical component are obtained. The mode shape at $U = 70$ m/s therefore exhibits the evidence of strongly three-dimensional coupled response, which apparently does not appear in the mode shape at $U = 0$ m/s. The frequency of this mode also notably changes from 0.147 to 0.138 Hz. The coupled response at high wind speeds also can be seen by the response spectrum of the middle of the main span as shown in Fig. 13. All three components of the response give spectral peaks of almost the same order of magnitude at $f = 0.138$ Hz. These response spectra are very similar to the experimental ones [8]. From these results, it can be concluded that strongly three-dimensional coupled response at this frequency is clearly and effectively predicted by using complex modes.

Fig. 14 shows the RMS of response along the bridge's deck at $U = 30, 54$ and 80 m/s. The shape of these along-deck RMS at $U = 80$ m/s reflects very well the dynamic behavior of the bridge at this wind speed, where mode $d_{10}$ becomes the dominant mode of the total response. This mode is indeed very prone to flutter instability at higher wind speeds. Another check on significant modes indicates that the response is governed by only 5 modes: $d_{1}$ (1st horizontal), $d_{2}$ (1st vertical), $d_{8}$ (3rd vertical), $d_{9}$ and $d_{10}$ (3-component coupled modes). Results from these modes are less than 5% than the results from 32 modes. This check thus proves again the effectiveness of using the complex mode shapes.

Since the $uw$-correlation is included in the turbulence simulation, its effects on the results can be checked. When the correlation is neglected, the vertical response is
overestimated by around 10%, the torsional response is underestimated by around 7%, and the horizontal response is almost unchanged, compared with the results when the correlation is included. These results are in accordance with the negativeness of the correlation and the coupled behaviors as indicated by the complex eigenvectors.

7. Concluding remarks

Fluctuating wind velocity components have been spatially simulated at very small time steps with good accuracy and validity. The correlation between different velocity components has been successfully incorporated. The results are very useful for buffeting analysis of long-span bridges and other spatial structures as well in time domain. Small time-step turbulence facilitates highly nonlinear and complex analysis, and good performance of low-order ARMA permits the spatial consideration for a large structure.
The new proposed method for buffeting analysis is effective and practical. By using complex modes, the aerodynamic effects on the system are accurately incorporated. Coupled responses are thus accurately captured. The buffeting scheme here together with mode tracing method for flutter analysis effectively suggests a comprehensive treatment for aeroelastic phenomena in long-span bridges [10].

In practice, the approach presented in this study has effectively provided a consistent framework for performing and checking many aspects related to buffeting response. Numerical calculation for the Akashi–Kaikyo bridge has given a good example, from which some interesting findings have been obtained. The agreement between analytical and experimental results when using simulated wind tunnel turbulence as input is indeed a milestone for the authors’ further research.

References