Analysis Of Time Zero Reset Method for Virtual Slope Walking

Youbin Qiu and Mingguo Zhao

Abstract—In our previous work, we proposed a novel method Virtual Slope Walking (VSW) for biped locomotion. Mechanical energy is restored by actively extending stance leg in this method. VSW has two kinds of implementations, termed self-excited mode and parametric excited mode, respectively. In self-excited mode, stance leg extension is controlled according to its angle, but the measure of leg’s angle is difficult in practical implementation; while in parametric excited mode stance leg extension is controlled according to time. In this paper, we investigate a method for VSW using parametric excited mode named Time Zero Reset method, which controlling stance leg extension with relative time, time zero is reset at each heelstrike. The method generates stable period-1 and multi-period walking cycles which not seen in VSW before. The results of stability analysis show the method has strong tolerance to disturbances. Compared to previous parametric excited mode with open-loop control, it has wider speed range and is more robust to disturbances with low cost as only touchdown sensors are needed.

I. INTRODUCTION

McGeer’s ground breaking work demonstrates that biped walking is simple[1]. He proposed passive dynamic walking, shows that even without control, stable and nature-looking walking can achieve when walking down a gentle slope. His work has been extended by many researchers [2][3][4].

Based on passive dynamic principles, many researchers add actuation to robot as to realize level ground walking. To this end, energy must be complemented to compensate for mechanical energy lost. There are different ways to realize energy input: push-off at ankle[5], self-excited control at hip joint[6], actively swinging the leg[7] and so on. In particular, Asano et al. proposed parametric excitation in which energy can be restored by dynamically shifting the position of the center of leg’s mass[8]. They applied parametric excitation on swing leg, stance leg, knee joint and studied them systematically[9][10].

In our previous work, we imitated energy restoration point of view in passive dynamic walking and proposed a new approach named Virtual Slope Walking (VSW)[11]. By actively extending stance leg and shortening swing leg, mechanical energy is restored, and when it equals the energy that dissipates, the robot can achieve period walking. We have also built a planar biped robot named Stepper-2D to test VSW [12], which can achieved high-speed walking (top speed is 4.48 leg/s) with open-loop control.

VSW has two kinds of implementations, termed self-excited mode and parametric excited mode, respectively. In self-excited mode, stance leg extension is controlled according to its angle, while in parametric excited mode stance leg extension is controlled according to time. Center of Mass is actively changed in these two modes through variable leg length. We analysed self-excited mode in [12][13] and found robust gait can achieve. However, in practical implementation, the measure of leg’s angle is difficult. In parametric excited mode, open-loop method is analysed by our previous work [14]. The stance leg length changes according to rhythm signal just similar to CPG method, no sensor is needed. We first used this simple method to test our principle and succeeded. But there are both negative energy feedback resulting in stable walking and positive energy feedback may resulting in unstable walking in this method, so the time of leg extension should be carefully chosen. Open-loop method has two disturbance: initial angular velocity and initial time. If we resetting time zero at each heelstrike, then initial time disturbance will eliminate and the gait may become more robust, we term this method as Time Zero Reset method. In previous studies, [8] set each heelstrike as time zero but didn’t do any analysis, [15] demonstrate that phase resetting at each heelstrike has positive influence on gait stability against external perturbations for CPG method in biped locomotion. We will investigate Time Zero Reset method for VSW systematically to prepare for practical application.

In this paper, we first investigate the characteristics of our proposed method. We show period-1 and multi-period limit cycle using Poincaré map. Stability analysis is carried out by simulation using Jacobi Matrix eigenvalue and Basin of Attraction (BoA). Afterwards, we compare Time Zero Reset method with open-loop method on speed range and robustness to disturbance and show its advantages.

II. PROBLEM FORMULATION

A. Model of VSW

We believe that simple models may give greater insight into bipedal locomotion. So we consider a simple model as shown in Fig. 1 as well as our previous model in [12]. We aim to use this model to analyse the effect of CoM (Center of Mass) changes by extending stance leg. The model has the following characteristics: masses of the model concentrated at the hip, two straight massless legs are connected by frictionless hinge at the hip; Two legs are modeled as a telescoping actuator with a point foot, the stance leg will be extended from l_0 to l_1 , the swing leg will shortened from...
Fig. 1. A typical step of VSW. Dashed line represents swing leg, while solid line represents stance leg. $m$, the mass of the body, $l_0, l_1$, the length of the stance leg before and after leg extension; $\theta$: the angle of the stance with respect to the vertical; $\phi$: inter-leg angle; $g$: gravitational acceleration. $F$: force on the stance leg during extension.

$l_1$ to $l_0$ instantaneously, so the leg extension ratio is defined as $\beta = l_1 / l_0$.

Fig. 1 shows the $n_{th}$ walking cycle. A step starts at instant I when the new stance leg (solid line) has just made contact with the ground. The stance leg swing freely as an inverted pendulum with the length $l_0$ until it reach instant II$^-$. Then the stance leg extends to the length $l_1$ at instant II$^+$ instantaneously. After instant II$^+$, the stance leg swings freely as an inverted pendulum with the length $l_1$ until it reaches instant III. At instant III, the swing leg hits the ground, heelstrike happens.

To simplify the problem, we make three assumptions.

First, the stance leg is in contact with ground all the time, acting like a hinge, until the swing foot reaches heelstrike, so there is no flight phase. Second, the stance leg extends instantaneously. Any trajectory leg extension can be equivalent to instantaneous leg extension. The detailed reasons can be seen in [13]. The third assumption, the impact of the swing leg with the ground is assumed to be fully inelastic (no slip, no bounce), double support is assumed to occur instantaneously. Moreover, before and after heelstrike, the inter-leg angle $\phi$ is always constant during each step. We assume that the leg can be swung forward fast enough and keep there at the desired inter-leg angle $\phi_0$ we set.

B. Governing Equations

We select stance leg angle $\theta$ as generalized coordinate. The governing equations of the model are as follows.

1) From instance I to II$^-$, swing with leg length $l_0$: the equation is similar to that of a inverted pendulum:

$$\ddot{\theta}(t) - \frac{g}{l_0} \sin\theta(t) = 0$$

(1)

rescale the time by defining dimensionless time $\tau = \sqrt{l_0/g} t$, (1) can be rewritten as

$$\ddot{\theta}(\tau) - \sin\theta(\tau) = 0$$

(2)

We will refer to dimensionless time $\tau$ as the time variable, henceforward.

2) Transition from instant II$^-$ to II$^+$: since the stance leg extends instantaneously, the angular momentum of the robot about support point on the ground is conserved. Transition rules can be written as

$$\dot{\theta}^{+}_I = \frac{1}{\beta^2} \dot{\theta}^{-}_I$$

(3)

where subscripts ‘II$^-$’ and ‘II$^+$’ represents the instant II$^-$ and II$^+$ respectively.

3) From instance II$^+$ to III, swing with leg length $l_1$: the equation is similar to (2)

$$\dot{\theta} - \frac{1}{\beta} \sin(\theta) = 0$$

(4)

4) Heelstrike at instant III: according to geometric constrains, the stance angle after heelstrike, which is the initial angle of subsequent step is

$$\theta_I = \frac{\beta \cos\phi_0 - 1}{\beta \sin\phi_0}$$

(5)

where $\phi_0$ is the constant inter-leg angle at heelstrike.

From conservation of angular momentum, the relationship between angular velocity after heelstrike $\dot{\theta}^+$ and before heelstrike $\dot{\theta}^-$ can be written as:

$$\dot{\theta}^+ = \beta \cos\phi_0 \dot{\theta}^-$$

(6)

C. Two kinds of methods in parametric excited mode

Here we show two control methods in parametric excited mode described in Section I more detailedly. Fig. 2(a) shows open-loop method that adopts absolute time, while Fig. 2(b) shows Time Zero Reset method that the reference time is each heelstrike $T_{h,n}(n)$ is the moment when actual heelstrike happens at step $n$, we set each heelstrike as time zero; $T_e$ is a preset parameter, representing the interval relative to planned heelstrike in Fig. 2(a) while to each actual heelstrike in Fig. 2(b). In open-loop method, the planned gait period is $T_n T_{h,n}(n=1,2,...)$ is the planned moment of heelstrike, and the moment of leg extension can be written as $t_{ext}(n) = (n - 1)T + T_e, n = 1,2,3,...$; while in Time Zero Reset method, $t_{ext}(n)$ can be written as $t_{ext}(n) = T_e$.

Fig. 2. Two kinds of methods in parametric excited mode.
III. ANALYSIS OF THE MODEL

A. Poincaré Section and Limit Cycle

We employ Poincaré map to analyse the walking cycle. Our Poincaré section is just after heelstrike. One step is considered as a function or ‘mapping’ from a Poincaré section to the next Poincaré section. The one degree of freedom of our model has a two dimensional phase space with coordinate \((\theta, \dot{\theta})\). The Poincaré section can be written as:

\[
\Sigma = \left\{ \begin{array}{ll}
\theta = -\frac{\phi}{2}, & \dot{\theta} > 0 \text{ if } T_{h,s} \leq T_e \\
\theta = \theta_l, & \dot{\theta} > 0 \text{ if } T_{h,s} > T_e 
\end{array} \right.
\]

where \(\theta_l\) is given by (5). \(\theta = \theta_l\) means leg extension occurs during last step, so impact posture is asymmetric. \(\theta = -\frac{\phi}{2}\) means there is no leg extension during last step, so the length of two legs equals at heelstrike. As the moment of leg extension is fixed relative to heelstrike, so proper angular velocity will make leg extension happens at the moment \(T_e\), but overlarge initial angular velocity may cause no leg extension during this step, because the gait period is smaller than \(T_e\).

Fig.3 shows the phase diagram and section \(\Sigma\). Two heavy lines represent Poincaré section, the left represents \(\theta = -\frac{\phi}{2}\) while the right represents \(\theta = \theta_l\). Four successive steps are shown in Fig.3, starting from point 1. These four curves represent four types of phase trajectories. The heelstrike happens at point 1', 2', 3', 4' and the state jumps at heelstrike. When the initial angular velocity is sufficiently large, there is no leg extension, as shown 1'~1, 2'~2', after two collisions, the velocity slows down, leg extension occurs during 3'~3', 4'~4', the angular velocity jumps at the instant of leg extension.

To simplify the definition of the map and its interpretation, we define a new variable \(q = \dot{\theta}^2\), representing the kinetic energy of the model. So Poincaré map \(f(q)\) can be written as:

\[
f(q) = \begin{cases} 
f_1(q) & \text{if } T_{h,s} \leq T_e \\
f_2(q) & \text{if } T_{h,s} > T_e 
\end{cases}
\]

where

\[
f_1(q) = \cos^2\phi_0(q + 2(\cos\theta_0 - \cos\frac{\phi_l}{2}))
\]

\[
f_2(q) = \frac{\cos^2\phi_0[q + 2(\beta^2 - 1)\cos\theta_0 + 2\cos\theta_0]}{\beta^2} - 2\cos^2\beta\cos(\phi_0 + \theta_l)
\]

\[
\theta_e = \int_{0}^{T_e} \dot{\theta}(t) \, dt
\]

where \(f_1(q)\) is the stride function without leg extension while \(f_2(q)\) is the stride function with leg extension. \(\theta_0\) is the initial angle of a step, its value can be \(\phi_0/2\) or \(\theta_l\), alternatively. \(\theta_e\) represents angle of leg extension at instant \(T_e\).

As parameter \(\beta, \phi_0, T_e\) varies, there exist different types of limit cycle, Fig.4 shows four types of limit cycle, they are periodic-1, 2, 3 and 4 limit cycle. Actually, there are more than four types, higher period limit cycles may also exist. Trajectory in Fig.4(a) is sectionally continuous, trajectory \(I \sim II^-, II^- \sim II^+, II^+ \sim III, III \sim I\) corresponding to free swing with length \(L_0\), the instantaneous extension of stance leg, the free swing phase with length \(L_1\) and the collision respectively. If there exist period-1 limit cycle, it must be of this type, more specifically it must contain leg extension, otherwise there will be no energy input to complement energy dissipation. But for period-2 and higher period limit cycle, as we can see from Fig.4(b)~4(d), it contains step that doesn’t have leg extension. The gait of multi-period has interesting asymmetry, just like limping. Even missing leg extension at some step, there still be periodic walking.

B. Period-1 Gait Local Stability

We do linear analysis to study the local stability of the model as McGeer did for rimless wheel[1]. Substitute \(\theta_l\) with \(\theta_l\) in (8) and simplify the equation, we get stride function

\[f(q) = \frac{\cos^2\phi_0(q + 2(\cos\theta_0 - \cos\frac{\phi_l}{2}))}{\beta^2} - 2\cos^2\beta\cos(\phi_0 + \theta_l)
\]
with leg extension:

\[ f_2(q) = \frac{\cos^2 \phi_0 \beta^2 - q + \frac{2\cos^2 \phi_0 (\beta^3 - 1) \cos \theta_e}{\beta^2}}{\frac{2\cos^2 \phi_0 (1 - \beta^2) \cos \theta_e}{\beta^2}} \]  

(9)

for fixed point \( q^* \), it satisfies :

\[ f_2(q^*) = q^* \]  

(10)

We set the time of stance leg extension \( T_e \). Local stability is determined by eigenvalues of Jacobian at fixed point, which can be written as:

\[ \lambda = \left. \frac{df_2}{dq} \right|_{q=q^*} = -\frac{2\cos^2 \phi_0 (\beta^3 - 1) \sin \theta_e^* \partial \theta_e^*}{\beta^2} \left. \frac{\partial \theta_e}{\partial q} \right|_{q=q^*} + \frac{\cos^2 \phi_0}{\beta^2} \]  

(11)

where \( \theta_e^* \) is the angle of leg extension at fixed point.

Different from self-excited mode in which eigenvalues of Jacobian is constant(\( \cos^2 \phi_0 / \beta^2 \)), in our proposed method, the eigenvalue changes because \( \theta_e \) is a function of \( q \).

If \(|\lambda| < 1\), the gait cycle is stable. As seen from (11), \( \lambda \) is influenced by parameter \( \phi_0, T_e \). Here we fixed \( \phi_0 = \pi/4, \beta = 1.1 \), vary \( T_e \) from 0.4 to 1.55. Numerical eigenvalues are shown in Fig.5. All eigenvalues are within the unit circle, so all gait cycles are locally stable.

C. BoA Analysis

BoA of 1-period gait can be analysed by Fig.6. Due to the complexity of solving the Poincaré map in analytic way, we study the BoA from Poincaré map based on simulation. Fig.6 shows simulation result of Poincaré map \( f \). Given an initial state \( \theta_0 \), by Poincaré map, we can obtain state of the next step \( \theta_1 \). Line \( \theta_1 = \theta_0 \) means the state of next step equals the state in this step. Intersection of these two curves is fixed point. If the state deviates from fixed point, for example, at point B which is larger than fixed point, according to Poincaré map curve, the step sequences are as follows: \( B_1, B_2, B_3, \ldots \). The step sequences will converge to the fixed point asymptotically, without overshoot. For initial condition that is smaller than fixed point, taking point A as an example, the situation is similar. To sum up, for any initial condition, as long as it doesn’t exceed the bound(without falling back and missing leg extension), then it will converge to the fixed point.

BoA of multi-period fixed point is shown in Fig.7. Dotted lines represent fixed points. The minimum of \( \theta_0 \) is the initial angular velocity such that the walker reaches the zenith with angular velocity 0; the maximum of \( \theta_0 \) is the minimum initial angular that results in running, solid line represents all maximum value of \( \theta_0 \). As we see, when \( T_e \) is small, all fixed points are period-1, BoA of these fixed points are large. As \( T_e \) become larger, period-1 fixed points disappear, higher-period fixed points appear. It’s interesting that period-3 fixed points first appear than period-2 fixed points as \( T_e \) become larger although it contains narrow area. In period-2 area, as \( T_e \) increases, the BoA area of the corresponding fixed points decreases. For the same \( \theta_0 \), the larger \( T_e \), the more likely swinging stance leg without extension, so less energy will input which result in the walker can’t reach zenith and falling.

Fig. 5. Eigenvalue of Jacobian. \( \phi_0 = \pi/4, \beta = 1.1 \).

Fig. 6. State in the next step \( \theta_1 \) versus state in this step \( \theta_0 \). \( \beta = 1.1, T_e = 0.4824, \phi = \pi/4 \). The lower bound of \( \theta_0 \) is the minimum value that keeping leg extension occurs.

Fig. 7. Basin of Attraction(BoA), \( \beta = 1.25, \phi = \pi/4 \). The heavy solid line on the right of the figure represents the bound for running. Dotted lines represents fixed point. In the figure, area (1) represents BoA of period-1, area (2) represents BoA of period-3, area (3) represents BoA of period-2.
back. From Fig.7 we can conclude that the BoA of the gait is very large for period-1 and multi-period fixed points. Large BoA of multi-period demonstrate that even with delay on leg extension caused by inaccurate timing, the gait may also converge to stable walking.

BoA of open-loop method is shown in[14]. Compared with similar parameter in open-loop method, BoA of Time Zero Reset method is much larger. Here is the reason.

In open-loop method, there are two disturbances in the walking: \(\delta \omega_n\) is the disturbance of \(n^{th}\) step’s stance leg angular velocity, \(\delta t_n\) is the disturbance of \(n^{th}\) step’s initial time(when in steady walking, \(n^{th}\) step’s initial time is \(nT\), while not in steady walking, \(n^{th}\) step’s initial time is deviated from \(nT\), the deviation is \(\delta t_n\)). \(\delta \omega_n > 0\) means the stance leg swings faster than the planned gait, \(\delta t_n > 0\) means actual step’s initial time is before the planned time.

As shown in Fig.8(a), we study the situation that on the left half plane. In periodic walking, supposing the leg extension position is position 1. when \(\delta \omega_n > 0\), the stance leg will swing faster, so the leg extension position shifts to position 1’, the complementary potential energy will increase, the total energy after extension increases. So \(n+1\) step’s initial time disturbance \(\delta t_{n+1} < 0\); the stance leg swings faster result in \(\delta \omega_{n+1} > 0\); \(n+1\) step’s initial time is \(nT\). \(\delta \omega_{n+1} > 0\) means actual step’s initial time is after the planned time.\(nT\).

In this numerical simulation, we gradually change the robot’s locomotion speed. As we fix inter-leg angle, so each step length is constant. In order to change locomotion speed, the only way is to change gait period. We use gait period to represent locomotion speed. Here, we fix \(\beta = 1.1, \phi_0 = \pi/4\), vary \(T_e\) to change speed with proposed method and vary \(T_e\) in the case of open-loop method. During the steady walking, the same \(T_e\) of these two methods result in the same fixed point. So we investigate speed changing versus \(T_e\), as shown in Fig.9.

The gait period gets it minimum value at intermediate point, so if we want to realize high speed, the parameter \(T_e\) should not be set too large or too small. The curve with open-loop method overlap with part of the curve with our proposed method. So the speed ranges of open-loop method is much narrower than that of proposed Time Zero Reset method.

To give a fair comparison between different walkers, normalize walking speed is given by the Froude number \(Fr\), which is the actual average walking speed in stable
walking divided by the square root of gravity times leg length: $Fr = \bar{v}/\sqrt{gl_0}$. The average walking speed can be calculated by:

$$\bar{v} = \frac{L_{step}}{T} = \frac{l_0\sqrt{\beta^2 + 1 - 2\beta \cos \phi_0}}{T} \tag{12}$$

where $L_{step}$ is the step length, $T$ is the gait period in dimensionless time unit. The Froude number of Time Zero Reset method varies from 0.12−0.8, while the Froude number of open-loop method varies from 0.15−0.59.

B. Recovery from Disturbance

In order to investigate the recovery ability from disturbance, we add relative large disturbance on initial angular velocity. Before adding disturbance, the walker with both methods are in stable walking. We add disturbance 1 $\Delta s_1 = 0.05$ and disturbance 2 $\Delta s_2 = 0.1$ successively, see Fig.10. If the initial angular velocity of some steps is within 0.2% of fixed point, we consider it has been in steady walking. The less steps taken to recover from disturbance, the stronger ability of disturbance rejection. When adding disturbance 1, these two methods can both recover from the disturbance, but our Time Zero Reset method takes 3 steps without overshoot while the open-loop method takes 22 steps with overshoot. When adding disturbance 2, robot with open-loop method falls down while the robot with Time Zero Reset method takes 4 steps to recover from disturbance. The simulation results reveal that Time Zero Reset method is more robust to disturbance.

V. CONCLUSIONS AND FUTURE WORKS

We investigate Time Zero Reset method for parametric excited mode of VSW. Time zero of leg extension is reset each at heelstrike. Period-1 and multi-period stable limit cycle exist in this method. The interesting gait of multi-period is just like limping, which missing leg extension at some step. From BoA analysis by simulation, both period-1 and multi-period fixed points have large BoA which shows strong tolerance to disturbances. Different to open-loop method which control parameter $T_e$ should be carefully chose, the range for $T_e$ is broad for stable walking in Time Zero Reset method. Compared to open-loop method, Time Zero Reset method have larger speed range and shows more robust to disturbance. Leg extension position for stable walking with our method are wider than that with open-loop method. More important, the method is easy to implement as it only needs touchdown sensor.

We think that open-loop method and Time Zero Reset method for VSW are all promising methods. If we pursue for more robust gait with low cost, Time Zero Reset method will be better.

REFERENCES