CHAPTER 11

CIE COLOUR APPEARANCE MODELS AND ASSOCIATED COLOUR SPACES

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Introduction

In 1931, the Commission Internationale de l’Eclairage (CIE) recommended a colour specification system\(^1\). After various additions over the years it now includes a series of colorimetric measures\(^2\) such as the tristimulus values (\(XYZ\)), chromaticity coordinates, dominant wavelength, and excitation purity for colour specification and colour matching, CIELAB and CIELUV colour spaces for presenting colour relationships, and CIELAB, CIELUV, and more recently the CIEDE2000\(^3\) formulae, for evaluating colour differences (see chapters 3 and 4).

While the CIE system has been successfully applied for over 70 years, it can only be used under quite limited viewing conditions, e.g., daylight illuminant, high luminance level, and some standardised viewing/illuminating geometries. However, with recent demands on cross-media colour reproduction, e.g. to match the appearance of a colour or an image on a display to that on hard copy paper, conventional colorimetry is becoming insufficient. It requires a colour appearance model capable of predicting colour appearance across a wide range of viewing conditions.

A great deal of research has been carried out to understand colour appearance phenomena and to model colour appearance. In 1997, the CIE recommended a colour appearance model designated CIECAM97s\(^4\),\(^5\), in which the ‘s’ represents a simple version and the ‘97’ means the model was considered as an interim model with the expectation that it would be revised as more data and better theoretical understanding became available. Since then, the model has been extensively evaluated by not only academic researchers but also industrial engineers in the imaging and graphic arts industries. Some shortcomings were identified and the original model was revised. In 2002, a new model: CIECAM02\(^6\),\(^7\) was recommended, which is simpler and has a better accuracy than CIECAM97s. Both CIE colour appearance models CIECAM97s and CIECAM02 are introduced.
Colorimetry includes three major topics: colour specification, colour difference evaluation and colour appearance measurement. In the past, they have been separately studied. The only attempt to unify these functions into one model was the LLAB model developed by Luo et al.\textsuperscript{8} In this chapter, it will be shown that in addition to quantifying colour appearance, the CIECAM02 model can be extended to accurately predict colour differences and hence become a universal colorimetric tool.

**Viewing conditions**

Various aspects of the viewing field impact on the colour appearance of a stimulus. Hence accurate definitions and descriptions of the components of the viewing field as shown in Figure 11 - 1 are necessary for the development and correct use of a colour appearance model. Here we follow the definitions given by Hunt \textsuperscript{9,10} and Fairchild.\textsuperscript{11}

![Figure 11 - 1: Illustration of specification of components of viewing field.](image)

*Stimulus*

A stimulus is a colour element for which a measure of colour appearance is required. Typically, the stimulus is taken to be a uniform patch of about 2° angular subtense.

*Proximal Field*

A proximal field is the immediate environment of the colour element considered, extending typically for about 2° from the edge of that colour element in all or most directions.

*Background*

The background is defined as the environment of the colour element considered, extending typically for about 10° from the edge of the proximal field in all, or most
directions. When the proximal field is the same colour as the background, the latter is regarded as extending from the edge of the colour element considered.

*Surround*

A surround is a field outside the background. In practical situations, the surround can be considered to be the entire room or the environment in which the image is viewed. For example, printed images are usually viewed in an illuminated (average) surround, projected slides in a dark surround, and domestic television displays in a dim surround.

*Adapting Field*

An adapting field is the total environment of the colour element considered, including the proximal field, the background, and the surround, and extending to the limit of vision in all directions.

**Colour appearance data sets**

Colour appearance models based on colour vision theories have been developed to fit various experimental data sets, which were carefully generated to study particular colour appearance phenomena. Over the years, a number of experimental data sets were accumulated to test and develop various colour appearance models. Data sets investigated by CIE TC 1-52 *Chromatic Adaptation Transforms* include: Mori *et al.*[12] from the Color Science Association of Japan, McCann *et al.*[13] and Breneman[14] using a haploscopic matching technique; Helson *et al.*[15], Lam and Rigg[16] and Braun and Fairchild[17] using the memory matching technique; and Luo *et al.*[18,19] and Kuo and Luo[20] using the magnitude estimation method. These data sets, however, do not include visual saturation correlates. Hence, Juan and Luo[21,22] investigated a data set of saturation correlates using the magnitude estimation method. The data accumulated played an important role in the evaluation of the performance of different colour appearance models and the development of the CIECAM97s and CIECAM02.

**Chromatic adaptation transforms**

Chromatic adaptation can be considered as the most important colour appearance phenomena and has long been extensively studied. A Chromatic Adaptation Transform (CAT) is capable of predicting corresponding colours, which are defined as pairs of colours that look alike when one is viewed under one illuminant (for example D65°) and the other is under a different illuminant (for example A). The following is divided into two parts: light and chromatic adaptation, and the development of CAT02.

*Light and chromatic adaptation*

Adaptation can be divided into two: light and chromatic. The former is the adaptation due to the change of light levels. It can be further divided into two: light adaptation and dark adaptation. Light adaptation is the decrease in visual sensitivity upon increase in the overall

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a In this chapter we will use for simplified terms “D65” and “A” instead of the complete official CIE terms: “CIE standard illuminant D65” and “CIE standard illuminant A”.
level of illumination. An example occurs when entering a bright room from a dark cinema. Dark adaptation is opposite to light adaptation and occurs, for example, when entering a dark cinema from a well-lit room.

Physiological mechanisms

The physiology associated with adaptation mainly includes rod-cone transition, pupil size (dilation and constriction), receptor gain and offset. It is known that there are two kinds of receptor: cones and rods. The former are less sensitive and respond to high (photopic) levels of illumination (above approximately 10 cd/m²) while the latter are more sensitive and respond to low (scotopic) levels of illumination (below approximately 0.01 cd/m²). From the example given earlier, when we enter a cinema from a well-lit room, the rods respond to the scotopic level in the cinema and gradually take over from the cones to provide vision. Conversely, when moving from the cinema to the well-lit room, the cone responses take over from the rods. Both adaptation processes will take a finite period of time, sometimes quite substantial time, to stabilize. In some cases, both rods and cones are functioning in the so-called mesopic region (approximately 0.01 cd/m² to 10 cd/m²). An example might be when driving along a (poorly) lit road at night.

The pupil size plays an important role in adjusting the amount of light that enters the eye by dilating or constricting the pupil: it is able to adjust the light by a maximum factor of 5. During dark viewing conditions, the pupil size is the largest. Each of the three cones responds to light in a nonlinear manner and is controlled by the gain and inhibitory mechanisms.

Chromatic adaptation

Light and dark adaptations only consider the change of light level, not the difference of colour between two light sources (up to the question of Purkinje shift due to the difference in the spectral sensitivity of the rods and cones). Under photopic adaptation conditions the difference between the colour of two light sources produces chromatic adaptation. This is responsible for the colour appearance of objects, and leads to the effect known as colour constancy (see also Chapter 8, Colour rendering, where colour appearance changes of samples is discussed, that can occur if the illumination colour is unchanged, only the spectrum of the two lamps is different). The effect can also be divided into two stages: a ‘chromatic shift’ and an ‘adaptive shift’. Consider for example, what happens when entering a room lit by a tungsten light from outdoor daylight. We experience that all colours in the room instantly become reddish reflecting the relative hue of the tungsten source. This is known as the ‘colorimetric shift’ and it is due to the operation of the sensory mechanisms of colour vision, which occur because of the changes in the spectral power distribution of the light sources in question. After a certain short adaptation period, the colour appearances of the objects become more normal. This is caused by the fact that most of coloured objects in the real world are more or less colour constant (they do not change their colour appearance under different illuminants). The most obvious example is white paper always appears white regardless of which illuminant it is viewed under. The second stage is called the ‘adaptive shift’ and it is caused by physiological changes and by a cognitive mechanism, which is based upon an observer’s knowledge of the colours in the scene content in the viewing field. Judd 23 stated that ‘the processes by means of which an observer adapts to the illuminant or discounts most of the effect of non-daylight illumination are complicated; they are known to be partly retinal and partly cortical’.
CHROMATIC ADAPTATION TRANSFORMS

The von Kries coefficient law is widely used to quantify chromatic adaptation. In 1902, von Kries\(^b\) assumed that, although the responses of the three cone types (RGB) are affected differently by chromatic adaptation, the spectral sensitivities of each of the three cone mechanisms remain unchanged. Hence, chromatic adaptation can be considered as a reduction of sensitivity by a constant factor for each of the three cone mechanisms. The magnitude of each factor depends upon the colour of the stimulus to which the observer is adapted. The relationship, given in Equation (11 - 1), is known as the von Kries coefficient law.

\[
\begin{align*}
R_c &= \alpha \cdot R \\
G_c &= \beta \cdot G \\
B_c &= \gamma \cdot B
\end{align*}
\]  

(11 - 1)

where \( R_c, G_c, B_c \) and \( R, G, B \) are the cone responses of the same observer, but viewed under test and reference illuminants respectively. \( \alpha, \beta \) and \( \gamma \) are the von Kries coefficients corresponding to the reduction in sensitivity of the three cone mechanisms due to chromatic adaptation. These can be calculated using Equation (11 - 2).

\[
\begin{align*}
\alpha &= \left( \frac{R_{\text{rw}}}{R_w} \right) \\
\beta &= \left( \frac{G_{\text{rw}}}{G_w} \right) \\
\gamma &= \left( \frac{B_{\text{rw}}}{B_w} \right)
\end{align*}
\]  

(11 - 2)

where

\[
\begin{align*}
\frac{R}{R_w} &= \frac{R_c}{R_{\text{rw}}} \\
\frac{G}{G_w} &= \frac{G_c}{G_{\text{rw}}} \\
\frac{B}{B_w} &= \frac{B_c}{B_{\text{rw}}}
\end{align*}
\]  

(11 - 3)

Here \( R_{\text{rw}}, G_{\text{rw}}, B_{\text{rw}} \), and \( R_w, G_w, B_w \) are the cone responses for the reference white under the reference and test illuminants, respectively. Over the years, various CATs have been developed but most are based on the von Kries coefficient law.

Development of the CAT02 used in CIECAM02

In 1997, Luo and Hunt\(^2\) modified the best available CAT at that time, the Bradford transform\(^1\) derived by Lam & Rigg. The transform named CMCCAT97 was then recommended by the Colour Measurement Committee (CMC) of the Society of Dyers and Colourists (SDC). This transform is included in the CIECAM97s\(^4\) for describing colour appearance under different viewing conditions. CMCCAT97 was originally derived by fitting only one data set, Lam & Rigg\(^1\). Although it gave a reasonably good fit to many other data sets, it predicted badly the McCann data set\(^1\). In addition, CMCCAT97 includes an exponent

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\(^b\) In this chapter the RGB symbols will be used for the cone fundamentals, in other chapters the reader will find the LMS symbols. The use of RGB here should not be confused with the RGB primaries used in visual colour matching.
p for calculating the blue corresponding spectral response (hence, it can be considered as a modification of the von Kries type of transform). This causes uncertainty in reversibility and complexity in the reverse mode. Li et al. 26 addressed this problem and provided a solution by including an iterative approximation using the Newton method. However, this is unsatisfactory in imaging applications where the calculations need to be repeated for each pixel. Li et al. 27 gave a linearisation version by optimising the transform to fit all the available data sets, rather than just the Lam & Rigg set16. The new transform, named CMCCAT2000, not only overcomes all the problems with respect to reversibility discussed above, but also gives a more accurate prediction than other transforms of almost all the available data sets.

At a later stage, CIE TC 8-01 Colour Appearance Modelling for Colour Management Systems had to choose a linear chromatic transform for CIECAM02. Multiple candidates such as CMCCAT2000,27 the sharp chromatic transform developed by Finlayson et al., and CAT026,7 were proposed for use as a von Kries type transform. All had similar levels of performance with respect to the accuracy of predicting various combinations of previously derived sets of corresponding colours. The main difference between these CATs is in the transform from the tristimulus values to the cone responses. Figures 11 - 2, 11 - 3, and 11 - 4 show the spectral sensitivity functions of CMCCAT97, CMCCAT2000, and Finlayson et al. corresponding to the blue, green, and red channels respectively. In addition, the Hunt-Pointer-Estevez (HPE)29 spectral sensitivity functions are also plotted, which provide a widely used transform based on the study carried out by Estevez30. It can be clearly seen that there are small differences between the functions in the blue channel. However, there are large variations between all the other functions and that of the HPE functions for the red and green channels, i.e. all the other functions are much sharper and have negative values compared with the HPE functions. Their peak wavelengths are also very similar and correspond to Thornton’s prime-colour wavelengths at 448 nm, 537 nm and 612 nm31,32, they provide the least degree of metamerism if they are used as light sources.

![Figure 11 - 2: Blue spectral sensitivity functions for the HPE, CAT02, Finlayson et al., CMCCAT2000, CMCCAT97.](image-url)
Figure 11 - 3: Green spectral sensitivity functions for the HPE, CAT02, Finlayson et al., CMCCAT2000, CMCCAT97.

Figure 11 - 4: Red spectral sensitivity functions for the HPE, CAT02, Finlayson et al., CMCCAT2000, CMCCAT97.
In addition to the sharpening of the spectral sensitivity functions, considerations used to select the CIE transform included the degree of backward compatibility with CIECAM97s and error propagation properties by combining the forward and inverse linear chromatic adaptation transforms, and the data sets which were used during the optimisation process. Finally, CAT02 was selected because it is compatible with CMCCAT97 and was optimised using all available data sets except the McCann et al. set, which includes a very chromatic adapting illuminant. It is interesting to note that the primaries in CAT02 are sharper than those used in CMCCAT97. However, the sharper primaries are less backwards compatible with CIECAM97s, which was optimised using the Lam and Rigg data set. The full forward and reverse equations for CAT02 are given in Appendix A.

CIE Colour appearance models

As mentioned earlier, CIE has recommended two colour appearance models, CIECAM97s and CIECAM02. A simple schematic diagram is given in Figure 11-5 to illustrate the input and output parameters of these models.

The input to the model are the CIE XYZ values of the stimulus (see definition in Section “Viewing conditions”) together with the viewing parameters as shown in the shaded areas: \( X_w, Y_w, Z_w \) are the tristimulus values of the reference white under the test illuminant; \( L_A \) specifies the luminance of the adapting field; \( Y_b \) defines the luminance factor of background; the surround (see definition in Section “Viewing conditions”) is described by ‘average’, ‘dim’ and ‘dark’ conditions, which roughly correspond to viewing reflection samples in a viewing cabinet, viewing TV with dim ambient lighting, and watching movie in a cinema, respectively.

There are many output parameters from the model: Lightness (\( J \)), Brightness (\( Q \)), Redness-Greenness (\( a \)), Yellowness-Blueness (\( b \)), Colourfulness (\( M \)), Chroma (\( C \)), Saturation (\( s \)), Hue composition (\( H \)) and Hue angle (\( h \)). These attributes, defined in the Glossary of Terms section, can be combined to form various spaces according to different applications. They can be divided into two types for evaluating colour appearance and colour difference respectively. For example, \( JCh \) and \( JCH \) spaces are typically used by the colour and imaging industries. The hue angle (\( h \)), ranges from 0 to 360 degree in the a and b plane, and is based on the concept of equal perceived difference, and the hue composition (\( H \)) describes colour appearance in terms of four unitary hues, ranged from 0 (pure red), 100 (pure yellow), 200 (pure green), 300 (pure blue) and back to pure red at 400. Note that the 0, 90, 180, 270 and 360 degrees in the \( JCh \) space do not correspond to pure hue perceptions of red, yellow, green, blue and red perceptions respectively.
CIE CAM97s

The CIE held an expert symposium on ‘Colour Standards for Image Technology’ in 1996. A decision was made to develop a CIE colour appearance model based on the 12 principles outlined by Hunt:

1. The model should be as comprehensive as possible, so that it can be used in a variety of applications; but at this stage, only static states of adaptation should be included, because of the great complexity of dynamic effects.

2. The model should cover a wide range of stimulus intensities, from very dark object colours to very bright self-luminous colours. This means that the dynamic response function must have a maximum, and cannot be a simple logarithmic or power function.

3. The model should cover a wide range of adapting intensities, from very low scotopic levels, such as occur in starlight, to very high photopic levels, such as occur in sunlight. This means that rod vision should be included in the model; but because many applications will be such that rod vision is negligible, the model should be usable in a mode that does not include rod vision.

4. The model should cover a wide range of viewing conditions including backgrounds of different luminance factors, and dark, dim, and average surrounds. It is necessary to cover the different surrounds because of their widespread use in projected and self-luminous displays.

5. For ease of use, the spectral sensitivities of the cones should be a linear transform of the CIE 1931 or 1964 standard colorimetric observer, and the $V'(\lambda)$ function should
be used for the spectral sensitivity of the rods. Because scotopic photometric data is often unknown, methods of providing approximate scotopic values should be provided.

6. The model should be able to provide for any degree of adaptation between complete and none, for cognitive factors, and for the Helson-Judd effect, as options.

7. The model should give predictions of the perceptual correlates in terms of hue angle, hue composition, brightness, lightness, saturation, chroma, and colourfulness.

8. The model should be capable of being operated in a reverse mode.

9. The model should be no more complicated than is necessary to meet the above requirements.

10. Any simplified version of the model, intended for particular applications, should give the same predictions as the complete model for some specified set of conditions.

11. The model should give predictions of colour appearance that are not appreciably worse than those given by the model that is best in each application.

12. A version of the model should be available for application to unrelated colours such as those seen in dark surrounds in isolation from other colours.

Four colour appearance models were considered to be most advanced at that time: Hunt, Nayatani, RLAB, and LLAB. An agreement was achieved that CIE TC1-34 Testing Colour Appearance Models should examine the existing colour appearance models and combine their best features into a high performance model for general use, and the model should adequately predict all available data sets. At the meeting held in Kyoto in 1997, CIE TC1-34 agreed to adopt a simplified model, which was named CIECAM97s. The comprehensive version was never formulated due to an apparent lack of demand, and a lack of suitable data to aid its formulation.

CIECAM02

Soon after the recommendation of CIECAM97s, CIE TC8-01, Colour Appearance Modelling for Colour Management Systems, was formed to evaluate CIECAM97s for its predictions of colour appearance, and its appropriateness for engineering and implementation requirements for open colour management systems. Various trials were conducted and some problems were identified as summarised below:

1. To simplify and improve CMCCAT97 transform by adopting CAT02 as described in Section “Development of the CAT02 used in CIECAM02”.

2. To correct the error that the lightness \( J \) was not equal to zero for a stimulus having a \( Y \) tristimulus value of zero, as reported by Li et al.

3. To ensure that the sizes of the gamut volumes from the colour appearance model rank from the largest to smallest in the order of average, dim, and dark surround conditions as addressed by Moroney and Li et al.
4. To improve the prediction of chroma for near neutral colours: Newman and Pirrotta\(^3\) had reported that the predictions given by CIECAM97s for colourfulness and chroma are too high for colours close to the neutral axis.

5. To improve the fit to the saturation results accumulated by Juan and Luo\(^{21,22}\), which are the only available saturation data to test the colour appearance model.

Various methods\(^{26,39,40,41}\) were proposed for overcoming the above identified shortcomings in the CIECAM97s model and in 2002, CIE TC8-01 recommended a new model: CIECAM02\(^6,7\). It is not only a refinement of CIECAM97s, removing many shortcomings, but also an improvement giving equivalent or better predictions of colour appearance data sets\(^{42,43}\). A typical example is given here. Figure 11 - 6 plots the Munsell chroma data against the chroma predictions from (a) CIECAM97s, (b) CIELAB, and (c) CIECAM02. The results show that the CIECAM02 model outperforms the other two models, i.e., it gives the smallest scattering of the data and converging to zero for neutral colours. The full forward and reverse modes of the CIECAM02 model are given in Appendix B. These are different from those given in the CIE publication\(^7\) in some of the computational steps, in that all computations that depend only on the test illuminant and the surround conditions are grouped together as Step 0. Since they do not depend on the samples, they only need to be computed once. This is very useful for image processing applications.

**Colour appearance phenomena**

This section describes a number of colour appearance phenomena studied by various researchers. Examples are given to illustrate how the CIECAM02 model predicts these effects.

*Chromatic adaptation*

Chromatic adaptation has been extensively investigated by many researchers. In fact, most of the data described in the previous section were accumulated to study this effect. The results are formulated in the form of corresponding colours

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**Figure 11 - 6**: The predictions from (a) CIECAM97s, (b) CIELAB, and (c) CIECAM02 are plotted against the Munsell Chroma data. Both the 45° line and the best-fit line are plotted. For perfect results, these lines should overlap.
for which each pair of colours represents the same colour appearance when viewed under different illuminants.

Figure 11 - 7 illustrates fifty-two pairs of corresponding colours predicted by CIECAM02 (or its chromatic adaptation transform, CAT02) from illuminant A (open circles of vectors) to $S_E$ (open ends of vectors) plotted in the CIE $u'v'$ chromaticity diagram for the $2^\circ$ observer. The open circle colours have a value of $L^*$ equal to 50 according to CIELAB under illuminant A. These were then transformed by the model to the corresponding colours under illuminant $S_E$ (the equi-energy illuminant). Thus, the ends of each vector represent a pair of corresponding colours under the two illuminants. The input parameters are (the luminance of adapting field) $L_A = 63.7 \text{ cd/m}^2$ and average surround. The parameters are defined in the end of Appendix B.

The results show that there is a systematic pattern, i.e., for colours below $v'$ equal to 0.48 under illuminant A, the vectors are predicted towards the blue direction under the illuminant $S_E$. For colours outside the above region, the appearance change is in a counter-clockwise direction, i.e. red colours shift to yellow, yellow to green, and green to cyan as the illuminant changes from A to $S_E$.

**Hunt effect**

Hunt\textsuperscript{44} studied the effect of light and dark adaptation on colour perception and collected data for corresponding colours via a visual colorimeter using the haploscopic matching technique, in which each eye was adapted to different viewing conditions and matches were made between stimuli presented in each eye.
CIE COLOUR APPEARANCE MODELS

Figure 11 - 8 illustrates this effect as successfully modelled by the CIECAM02 model. Five colours were selected, having a constant $L^*$ (CIELAB lightness) of 50 and hue angle of $2^\circ$ (red) with $C^*$ (CIELAB chroma) varying from 0 (neutral colour) to 80 (a high chroma colour) under illuminant $S_E$. These colours were predicted by CIECAM02 under 9 illuminance levels ranging from 0.01 to 1,000,000 lux.

![Figure 11 - 8: The Hunt effect predicted by the CIECAM02 model. The colourfulness ($M$) predictions for five samples of varying CIELAB chroma ($C^*$) values are plotted against nine illuminance levels on a log$_{10}$ scale.](image)

Each (nearly) horizontal curve represents the change of colourfulness appearance for a particular sample. Each vertical line expresses the degree of colourfulness contrast under a particular illuminance level. The results clearly demonstrate the Hunt effect, i.e. each sample represented by each curve increases its colourfulness ($M$) (except for the neutral colours) when the illuminance of the reference white increase until reaching a value of about 1,000,000 lux. In addition, the colourfulness contrast increases from dark to bright illuminance levels as shown by the lengths of the vertical lines between the dark and bright levels.

Stevens effect

Stevens and Stevens$^{45}$ asked observers to make magnitude estimations of the brightness of stimuli across various adaptation conditions. The results showed that the perceived brightness contrast increased with an increase in the adapting luminance level according to a power relationship.
Five neutral samples having $L^*$ values of 0.01, 20, 40, 60 and 80 under illuminant $S_E$ were selected to demonstrate the Stevens effect as predicted by CIECAM02. Figure 11 - 9 illustrates the Stevens effect by plotting brightness ($Q$) against the base 10 logarithm of $E_w$ (the illuminance of the light source in lux, see the end of the Appendix B) of the test illuminant, i.e. an increase of brightness contrast with an increase of the illuminance. The incremental increase in brightness is very marked for the lighter samples (above $L^*$ of 20) but the effect is almost zero for the darkest sample ($L^*$ = 0.01). This leads to an increase of brightness contrast, i.e. the lighter samples appear much brighter.

**Figure 11 - 9**: The Stevens effect predicted by the CIECAM02 model. The brightness ($Q$) predictions for five neutral sample of varying CIELAB lightness $L^*$ are plotted against nine illuminance levels in log$_{10}$ scale.

**Surround effect**

Bartleson and Breneman\textsuperscript{46} found that the perceived contrast in colourfulness and brightness increased with increasing illuminance level from dark surround, dim surround to average surround. This is an important colour appearance phenomenon to be modelled, especially for the imaging and graphic arts industries where, on many occasions, it is required to reproduce images on different media under quite distinct viewing conditions.

Two figures are used to illustrate the surround effect: the colourfulness ($M$) and lightness ($J$) predicted by CIECAM02 under the average, dim and dark surrounds. These are plotted in Figure 11 - 10 and Figure 11 - 11, respectively. Figure 11 - 10 shows the
colourfulness \((M)\), with different surrounds, of samples with CIELAB \(C^*\) values of 0, 20, 40, 60 and 80 with constant \(L^*\) of 50 and \(h\) (the CIELAB hue angle) of 2° (red) under the illuminant \(S_E\). (The other parameters were set to: \(L_A = 63.7\, \text{cd/m}^2\), \(Y_b = 20\), and constants \(F\), \(c\), and \(N_c\) were chosen according to Table 11-A1 in Appendix B). Figure 11 - 11 shows the lightness \((J)\), with different surrounds, of neutral samples with CIELAB \(L^*\) of 0.001 to 80 with the same model parameters as used for Figure 11 - 10. Note that constant \(F\) is a factor for degree of adaptation, \(c\) the impact of surround, and \(N_c\) the chromatic surround induction factor (see Appendix B).

Figure 11 - 10 shows that for each of the five test colours having \(C^*\) values of 0, 20, 40, 60 and 80, there is a slight decease of colourfulness from average, through dim to dark surround conditions except for \(C^*\) of zero. This leads to a reduction of colourfulness contrast from average to dark surround conditions. Figure 11 - 11 shows that for each of the five neutral test colours having \(L^*\) of 0.001, 20, 40, 60 and 80, there is a decrease of lightness contrast from average, through dim to dark surround conditions.

![Figure 11 - 10: The surround effect predicted by the colourfulness \((M)\) scale of CIECAM02. The colourfulness \((M)\) predictions for the five samples varying in CIELAB chroma \(C^*\) values are plotted against the 'average', 'dim', and 'dark' surround conditions.](image)

**Lightness contrast effect**

The lightness contrast effect\(^{47}\) reflects that the perceived lightness increases when colours are viewed against a darker background and vice versa. It is a type of simultaneous contrast effect considering the change of colour appearance due to different coloured backgrounds. This effect has been widely studied and it is well known that a change in the background colour has a large impact on the perception of lightness and hue. There is some effect on colourfulness, but this is much smaller than the effect on lightness and hue\(^{47}\).
Figure 11 - 11: The surround effect predicted by the lightness ($J$) scale of the CIECAM02. The lightness ($J$) predictions for the five neutral samples varying in CIELAB lightness $L^*$ values are plotted against the ‘average’, ‘dim’, and ‘dark’ surround conditions.

The lightness contrast effect predicted by the CIECAM02 model is illustrated in Figure 11–12 by plotting the lightness ($J$) predicted by CIECAM02 against the luminance factor of the backgrounds ($Y_b$) for five neutral test colours having $L^*$ values of 0.001, 20, 40, 60 and 80 under the illuminant $S_E$. It can be seen from Figure 11 - 12 that for all test colours, their lightness reduces when the background becomes lighter.

Figure 11 - 12: The lightness contrast effect predicted by the CIECAM02 model. The lightness ($J$) predictions for the five neutral samples varying in CIELAB lightness $L^*$ values are plotted against neutral background having different luminance factors.
Helmholtz-Kohlrausch effect

The Helmholtz-Kohlrausch effect refers to a change in the brightness of colour produced by increasing the purity of a colour stimulus while keeping its luminance constant within the range of photopic vision. This effect is quite small compared with others and is not modelled by CIECAM02.

Helson-Judd effect

When a grey scale is illuminated by a light source, the lighter neutral stimuli will exhibit a certain amount of the hue of the light source and the darker stimuli will show its complementary hue, which is known as the Helson-Judd effect. Thus for tungsten light, which is much yellower than daylight, the lighter stimuli will appear yellowish, and the darker stimuli bluish. This effect is not modelled by CIECAM02.

Uniform Colour Spaces based on CIECAM02

As mentioned in the previous section, CIECAM02 gives accurate prediction of all the available colour appearance data described. Attempts have been made by the authors to extend CIECAM02 for predicting available colour discrimination data sets, which include two types, for Large and Small magnitude Colour Differences, designated by LCD and SCD respectively. The former includes six data sets: Zhu and Luo, OSA, Guan and Luo, BADB-Textile, Pointer, and Munsell. They have 144, 128, 292, 238, 1308 and 844 pairs respectively, having an average 10 $\Delta E^*$ units over all the sets. The SCD data, having an average 2.5 $\Delta E^*$ units, are a combined data set used to develop the CIE 2000 colour difference formula: CIEDE2000.

CIECAM02 based colour spaces

CIECAM02 includes three attributes in relation to the chromatic content: chroma ($C$), colourfulness ($M$) and saturation ($s$). These attributes together with lightness ($J$) and hue angle ($h$) can form three colour spaces: $CCbaJ$, $MMbaJ$, and $ssbaJ$, where

\begin{align*}
    a_c &= C \cdot \cos(h) \\
    b_c &= C \cdot \sin(h) \\
    a_M &= M \cdot \cos(h) \\
    b_M &= M \cdot \sin(h) \\
    a_s &= s \cdot \cos(h) \\
    b_s &= s \cdot \sin(h)
\end{align*}

Li et al. found that a colour space derived using $J, a_M, b_M$ gave the most uniform result when analysed using the large and small colour difference data sets. Hence, various attempts were made to modify this version of CIECAM02 to fit all available data sets. Finally, a simple, generic form, Equation (11 - 4) was found that adequately fitted all available data.

\begin{align*}
    J' &= \frac{(1+100 \cdot c_1) \cdot J}{1+c_1 \cdot J} \\
    M' &= (1/c_2) \cdot \ln(1+c_2 \cdot M)
\end{align*}

where $c_1$ and $c_2$ are constants given in Table 11 - 1.
The corresponding colour space is \( J', a'_M, b'_M \) where \( a'_M = M' \cdot \cos(h) \) and \( b'_M = M' \cdot \sin(h) \). The colour difference between two samples can be calculated in \( J', a'_M, b'_M \) space using equation (11 - 5).

\[
\Delta E' = \sqrt{(\Delta J'/K_L)^2 + (\Delta a'_M)^2 + (\Delta b'_M)^2}
\]  

(11 - 5)

where \( \Delta J' \), \( \Delta a'_M \) and \( \Delta b'_M \) are the differences of \( J' \), \( a'_M \) and \( b'_M \) between the ‘standard’ and ‘sample’ in a pair. Here \( K_L \) is a lightness parameter and is given in Table 11 – 1.

Three colour spaces named CAM02–LCD, CAM02–SCD, and CAM02–UCS were developed for large, small and combined large and small differences, respectively. The corresponding parameters in Equation (11 - 4) and Equation (11 - 5) are listed in Table 11 – 1.

<table>
<thead>
<tr>
<th>Versions</th>
<th>CAM02 -LCD</th>
<th>CAM02-SCD</th>
<th>CAM02-UCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_L )</td>
<td>0.77</td>
<td>1.24</td>
<td>1.00</td>
</tr>
<tr>
<td>( c_1 )</td>
<td>0.007</td>
<td>0.007</td>
<td>0.007</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>0.0053</td>
<td>0.0363</td>
<td>0.0228</td>
</tr>
</tbody>
</table>

**Comparing the performance of the new UCSs with some selected colour models**

The three new CIECAM02 based colour spaces, CAM02–LCD, CAM02–SCD, and CAM02–UCS, together with the best available colour difference formulae including CIEDE2000\(^3\) and DIN99d\(^59\), and uniform colour spaces such as CIELAB\(^2\), IPT\(^60\), OSA\(^51\) and GLAB\(^52\) were also tested by Luo et al.\(^58\) using the available small and large colour difference data sets. It was found that CAM02–LCD and CAM02–SCD performed either better than or equal to the other best available colour spaces for the LCD and SCD data respectively. The performance results are summarised in Table 11 - 2 in terms of PF/3 measure\(^61\). For a perfect agreement between the visual results and a formula’s or space's predictions, PF/3 should equal zero. A larger PF/3 value means a larger prediction error. A PF/3 of 30 can roughly be considered as 30% disagreement between the visual data and a formula prediction. It was also very encouraging that CAM02–UCS, developed to fit both the large and small colour difference data sets, also gave excellent performance in predicting the data sets. When selecting one UCS to evaluate colour differences across a wide range, CAM02–UCS can be considered a suitable candidate.

The experimental colour discrimination ellipses used in the previous studies\(^62,63\) were also used for comparing different colour spaces. Figures 11 - 13, and 11 - 14 show the ellipses plotted in CIELAB and CAM02–UCS spaces, respectively. The size of the ellipse was adjusted by a single factor in each space to ease visual comparison. For perfect agreement
between the experimental results and a uniform colour space, all ellipses should be constant radius circles. Overall, it can be seen that the ellipses in CIELAB (Figure 11 - 13) are smaller in the neutral region and gradually increase in size as chroma increases. In addition, the ellipses are orientated approximately towards the origin except for those in the blue region in CIELAB space. All ellipses in CAM02–UCS (Figure 11 14) are approximately equal-sized circles. In other words, the newly-developed CAM02–UCS is much more uniform than CIELAB.

Table 11 - 2: Testing uniform colour spaces and colour difference formulae using the combined LCD and SCD data sets

<table>
<thead>
<tr>
<th>Combined LCD data set</th>
<th>PF/3</th>
<th>Combined SCD data set</th>
<th>PF/3</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIELAB</td>
<td>26</td>
<td>CIELAB</td>
<td>52</td>
</tr>
<tr>
<td>IPT</td>
<td>26</td>
<td>IPT</td>
<td>52</td>
</tr>
<tr>
<td>OSA</td>
<td>24</td>
<td>CIEDE2000</td>
<td>33</td>
</tr>
<tr>
<td>GLAB</td>
<td>24</td>
<td>DIN99d</td>
<td>35</td>
</tr>
<tr>
<td>CIECAM02</td>
<td>25</td>
<td>CIECAM02</td>
<td>47</td>
</tr>
<tr>
<td>CAM02-LCD</td>
<td>23</td>
<td>CAM02-LCD</td>
<td>41</td>
</tr>
<tr>
<td>CAM02-SCD</td>
<td>27</td>
<td>CAM02-SCD</td>
<td>34</td>
</tr>
<tr>
<td>CAM02-UCS</td>
<td>25</td>
<td>CAM02-UCS</td>
<td>35</td>
</tr>
</tbody>
</table>

Figure 11–13: Experimental chromatic discrimination ellipses plotted in CIELAB.
Conclusions

This chapter has described the development of the CIE colour appearance models, CIECAM97s and CIECAM02. The viewing condition parameters are clearly defined. The CAT02 chromatic adaptation transform and CIECAM02 are given in the Appendices. The colour appearance phenomena predicted by the model are also introduced. Finally, three new extensions were developed to form new uniform colour spaces for predicting colour differences. A space designated CAM02–UCS can predict colour differences over a large range with reasonable accuracy and should be recommended for future evaluation.

Overall, the CIECAM02 is capable of accurately predicting colour appearance under a wide range of viewing conditions. It has been proved to achieve successfully cross-media colour reproduction (for example the reproduction of an image on a display, on a projection screen, or as hardcopy) and is adopted by the Microsoft Company in their latest colour management system, Window Color System (WCS). It can also be applied to quantify the degree of colour inconstancy of a single specimen, to evaluate the metamerism of a pair of samples, and to estimate the colour rendering properties of light sources. Furthermore, it can be used to specify the colour appearance of each stimulus in terms of a comprehensive set of clearly defined colour appearance attributes. With the extension to include new colour spaces as described in the previous section, it can accurately evaluate colour differences under various viewing conditions. (In contrast, all existing colour difference equations can only be
used under daylight illuminants.) Hence, CIECAM02 performed satisfactorily for all three major colorimetric tasks: colour specification, colour difference evaluation and colour appearance prediction. It can be considered a universal colour model.

REFERENCES


5. CIE (1998), The CIE 1997 Interim Colour Appearance Model (Simple Version), CIECAM97s. CIE Publication 131, CIE Central Bureau, Vienna, Austria.


CIE COLOUR APPEARANCE MODEL AND ASSOCIATED COLOUR SPACES


REFERENCES


Appendix A: Chromatic Adaptation Transform: CAT02

Part 1: Forward Mode

Input data:
Sample in test illuminant: $X, Y, Z$
Adopted white in test illuminant: $X_w, Y_w, Z_w$
Reference white in reference illuminant: $X_{wr}, Y_{wr}, Z_{wr}$
Luminance of test and reference adapting fields (cd/m²): $L_A$
(Note that for the calculation of $L_A$ see the note in the end of Appendix B.) In addition, when applying chromatic adaptation transform, the other viewing conditions such as surround, luminance factor of background, luminance level of reference and test fields should be fixed.

Transformed data to be obtained
Sample corresponding colour in reference illuminant: $X_c, Y_c, Z_c$
Step 1: Calculate cone responses

\[
\begin{pmatrix}
R \\
G \\
B
\end{pmatrix}
= M_{\text{CAT02}} \cdot \begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix},
\begin{pmatrix}
R_w \\
G_w \\
B_w
\end{pmatrix}
= M_{\text{CAT02}} \cdot \begin{pmatrix}
X_w \\
Y_w \\
Z_w
\end{pmatrix},
\begin{pmatrix}
R_{wt} \\
G_{wt} \\
B_{wt}
\end{pmatrix}
= M_{\text{CAT02}} \cdot \begin{pmatrix}
X_{wt} \\
Y_{wt} \\
Z_{wt}
\end{pmatrix}
\]

where

\[
M_{\text{CAT02}} = \begin{pmatrix}
0.7328 & 0.4296 & -0.1624 \\
-0.7036 & 1.6975 & 0.0061 \\
0.0030 & 0.0136 & 0.9834
\end{pmatrix}
\]

Step 2: Calculate the degree of adaptation, \( D \)

\[
D = F \cdot \left[ 1 - \left( \frac{1}{3.6} \right) \cdot e^{\left( \frac{-L_A - 42}{92} \right)} \right]
\]

where \( F \) equals 1.0, 0.9, and 0.8 for average, dim, and dark surround viewing conditions respectively, and where \( L_A \) is the luminance of adapting field (reference and testing). If \( D \) is greater than one or less than zero, set it to one or zero accordingly. Note that for the selection of surround parameters please see the note in the end of Appendix B.

Step 3: Calculate the corresponding response

\[
R_c = R \cdot \left( \alpha \cdot \frac{R_{wt}}{R_w} + 1 - D \right)
\]

\[
G_c = G \cdot \left( \alpha \cdot \frac{G_{wt}}{G_w} + 1 - D \right),
\]

\[
B_c = B \cdot \left( \alpha \cdot \frac{B_{wt}}{B_w} + 1 - D \right)
\]

where

\[
\alpha = D \cdot \frac{Y_w}{Y_{wt}}.
\]

Step 4: Calculate the corresponding tristimulus values

\[
\begin{pmatrix}
X_c \\
Y_c \\
Z_c
\end{pmatrix}
= M_{\text{CAT02}}^{-1} \cdot \begin{pmatrix}
R_c \\
G_c \\
B_c
\end{pmatrix}
\]

Note that for the coefficients in the inverse matrix are given in the note at the end of the Appendix B.
Part 2: Reverse Mode

Input data:
Corresponding colour in reference illuminant: \( X_c, Y_c, Z_c \)
Others are the same as the forward.

Output data:
Sample colour in test illuminant: \( X, Y, Z \)

Step 1: Calculate cone responses

\[
\begin{pmatrix}
R_w \\
G_w \\
B_w 
\end{pmatrix} = M_{CAT02} \cdot \begin{pmatrix}
X_w \\
Y_w \\
Z_w 
\end{pmatrix},
\begin{pmatrix}
R_{wt} \\
G_{wt} \\
B_{wt} 
\end{pmatrix} = M_{CAT02} \cdot \begin{pmatrix}
X_{wt} \\
Y_{wt} \\
Z_{wt} 
\end{pmatrix},
\begin{pmatrix}
R_c \\
G_c \\
B_c 
\end{pmatrix} = M_{CAT02} \cdot \begin{pmatrix}
X_c \\
Y_c \\
Z_c 
\end{pmatrix}
\]

Step 2: Calculate the \( D \) using Step 2 of the forward mode.

Step 3: Calculate the cone response

\[
R = \frac{R_c}{\alpha \cdot \frac{R_{wt}}{R_w} + 1 - D},
\]
\[
G = \frac{G_c}{\alpha \cdot \frac{G_{wt}}{G_w} + 1 - D},
\]
\[
B = \frac{B_c}{\alpha \cdot \frac{B_{wt}}{B_w} + 1 - D}
\]

where
\[
\alpha = D \cdot \frac{Y_w}{Y_{wt}}.
\]
APPENDIX A: CHROMATIC ADAPTATION TRANSFORM: CAT02

Step 4: Calculate the original tristimulus values

\[
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix} = M_{\text{CAT02}}^{-1} \begin{pmatrix}
R \\
G \\
B
\end{pmatrix}
\]

Note that for the coefficients in the inverse matrix are given in the note at the end of the Appendix B.

Appendix B: CIE Colour Appearance Model: CIECAM02

Part 1: The Forward Mode

**Input:** \( X \), \( Y \), \( Z \) (under test illuminant \( X_w \), \( Y_w \), \( Z_w \))

**Output:** Correlates of lightness \( J \), chroma \( C \), hue composition \( H \), hue angle \( h \), colourfulness \( M \), saturation \( s \), and brightness \( Q \)

**Illuminants, viewing surrounds set up and background parameters**

(See the note at the end of this Appendix for determining all parameters)

Adopted white in test illuminant: \( X_w \), \( Y_w \), \( Z_w \)

Background in test conditions: \( Y_b \)

(Reference white in reference illuminant: \( X_{wr} = Y_{wr} = Z_{wr} = 100 \), which are fixed in the model)

Luminance of test adapting field (\( \text{cd/m}^2 \)): \( L_A \)

All surround parameters are given in Table 11 - A1 below

<table>
<thead>
<tr>
<th>Surplus Parameters</th>
<th>( F )</th>
<th>( c )</th>
<th>( N_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>1.0</td>
<td>0.69</td>
<td>1.0</td>
</tr>
<tr>
<td>Dim</td>
<td>0.9</td>
<td>0.59</td>
<td>0.9</td>
</tr>
<tr>
<td>Dark</td>
<td>0.8</td>
<td>0.535</td>
<td>0.8</td>
</tr>
</tbody>
</table>
Note that for determining the surround conditions see the note at the end of this Appendix. $N_c$ and $F$ are modelled as a function of $c$, and can be linearly interpolated as shown in the Figure 11 - A1 below, using the above points.

**Figure 11 - A1**: $N_c$ and $F$ varies with $c$.

**Step 0**: Calculate all values/parameters which are independent of input samples

\[
\begin{pmatrix}
R_w \\
G_w \\
B_w
\end{pmatrix} = M_{\text{CAT02}} \begin{pmatrix}
X_w \\
Y_w \\
Z_w
\end{pmatrix}, \quad D = F \cdot \left[ 1 - \left( \frac{1}{3.6} \right) \cdot e^{\left(-\frac{L_A-42}{92}\right)} \right]
\]

Note if $D$ is greater than one or less than zero, set it to one or zero respectively.

\[
D_R = D \cdot \frac{Y_w}{R_w} + 1 - D, \quad D_G = D \cdot \frac{Y_w}{G_w} + 1 - D, \quad D_B = D \cdot \frac{Y_w}{B_w} + 1 - D
\]

\[
F_L = 0.2 \cdot k^4 \cdot (5L_A) + 0.1 \cdot (1-k^2)^2 \cdot (5L_A)^{1/3}
\]

where \( k = \frac{1}{5 \cdot L_A + 1} \).

\[
n = \frac{Y_b}{Y_w}, \quad z = 1.48 + \sqrt{n}, \quad N_{bb} = 0.725 \cdot \left( \frac{1}{n} \right)^{0.2}, \quad N_{cb} = N_{bb}
\]
Appendix B: CIE Colour Appearance Model: CIECAM02

\[
\begin{pmatrix}
R_{wc} \\
G_{wc} \\
B_{wc}
\end{pmatrix} =
\begin{pmatrix}
D_R \cdot R_w \\
D_G \cdot G_w \\
D_B \cdot B_w
\end{pmatrix}
\begin{pmatrix}
R' \\
G' \\
B'
\end{pmatrix} =
M_{HPE} \cdot M_{CAT02}^{-1}
\begin{pmatrix}
R_{wc} \\
G_{wc} \\
B_{wc}
\end{pmatrix}
\]

\[
M_{HPE} =
\begin{pmatrix}
0.38971 & 0.68898 & -0.07868 \\
-0.22981 & 1.18340 & 0.04641 \\
0.00000 & 0.00000 & 1.00000
\end{pmatrix}
\]

\[
R'_{aw} = 400 \cdot \left( \frac{(F_{L} \cdot R_w')^{0.42}}{100} + 27.13 \right) + 0.1
\]

\[
G'_{aw} = 400 \cdot \left( \frac{(F_{L} \cdot G_w')^{0.42}}{100} + 27.13 \right) + 0.1
\]

\[
B'_{aw} = 400 \cdot \left( \frac{(F_{L} \cdot B_w')^{0.42}}{100} + 27.13 \right) + 0.1
\]

\[
A_w = [2 \cdot R'_{aw} + G'_{aw} + \frac{B'_{aw}}{20} - 0.305] \cdot N_{bb}
\]

Note that all parameters computed in this step are needed for the following calculations. However, they depend only on surround and viewing conditions, hence when processing pixels of image, they are computed once for all. The following computing steps are sample dependent.

**Step 1:** Calculate (sharpened) cone responses (transfer colour matching functions to sharper sensors)

\[
\begin{pmatrix}
R \\
G \\
B
\end{pmatrix} = M_{CAT02} \cdot
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix}
\]

**Step 2:** Calculate the corresponding (sharpened) cone response (considering various luminance level and surround conditions included in \( D \), hence in \( D_R, D_G, \) and \( D_B ))
CIE COLOUR APPEARANCE MODEL AND ASSOCIATED COLOUR SPACES

\[
\begin{pmatrix}
R_c \\
G_c \\
B_c
\end{pmatrix} =
\begin{pmatrix}
D_R \cdot R \\
D_G \cdot G \\
D_B \cdot B
\end{pmatrix},
\]

**Step 3:** Calculate the Hunt-Pointer-Estevez response

\[
\begin{pmatrix}
R' \\
G' \\
B'
\end{pmatrix} = M_{HPE} \cdot M_{CAT02}^{-1} \cdot \begin{pmatrix}
R_c \\
G_c \\
B_c
\end{pmatrix},
\]

**Step 4:** Calculate the post-adaptation cone response (resulting in dynamic range compression)

\[
R'_a = 400 \cdot \left( \frac{F_L \cdot R'}{100} \right)^{0.42} + 0.1
\]

If \( R' \) is negative, then

\[
R'_a = -400 \cdot \left( \frac{-F_L \cdot R'}{100} \right)^{0.42} + 0.1
\]

and similarly for the computations of \( G'_a \) and \( B'_a \) respectively.

**Step 5:** Calculate Redness – Greenness (\( a \)) , Yellowness – Blueness (\( b \)) components , and hue angle (\( h \)):

\[
a = \frac{R'_a}{11} - \frac{12 \cdot G'_a + B'_a}{11}
\]

\[
b = \frac{(R'_a + G'_a - 2 \cdot B'_a)}{9}
\]

\[
h = \tan^{-1} \left( \frac{b}{a} \right)
\]

make sure \( h \) between 0 and 360 degree.

**Step 6:** Calculate eccentricity (\( e_i \)) and hue composition (\( H \)), using the unique hue data given in Table 11 - A2; set \( h' = h + 360 \) if \( h < h_1 \), otherwise \( h' = h \). Choose a proper \( i \) (\( i = 1, 2, 3, \) or \( 4 \)) so that \( h_i \leq h' < h_{i+1} \). Calculate

\[
e_i = \frac{1}{4} \left[ \cos \left( \frac{h' \cdot \pi}{180} + 2 \right) + 3.8 \right]
\]

which is close to, but not exactly the same as, the eccentricity factor given in Table 11 - A2.
Appendix B: CIE Colour Appearance Model: CIECAM02

Table 11 - A2: Unique hue data for calculation of hue quadrature

<table>
<thead>
<tr>
<th>$i$</th>
<th>Red</th>
<th>Yellow</th>
<th>Green</th>
<th>Blue</th>
<th>Red</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_i$</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>$e_i$</td>
<td>0.8</td>
<td>0.7</td>
<td>1.0</td>
<td>1.2</td>
<td>0.8</td>
</tr>
<tr>
<td>$H_i$</td>
<td>0.0</td>
<td>100.0</td>
<td>200.0</td>
<td>300.0</td>
<td>400.0</td>
</tr>
</tbody>
</table>

$$H = H_i + \frac{100 \cdot (h' - h_i)}{h' - h_i + h_{i+1} - h'} \frac{e_i}{e_i}$$

**Step 7:** Calculate achromatic response $A$

$$A = (2 \cdot R_a + G_a + \frac{B_a}{20} - 0.305) \cdot N_{bb}$$

**Step 8:** Calculate the correlate of lightness

$$J = 100 \cdot \left(\frac{A}{A_w}\right)^{c-z}$$

**Step 9:** Calculate the correlate of brightness

$$Q = \left(\frac{4}{c}\right) \cdot \left(\frac{J}{100}\right)^{0.5} \cdot (A_w + 4) \cdot F_L^{0.25}$$

**Step 10:** Calculate the correlates of chroma ($C$), colourfulness ($M$) and saturation ($s$)

$$t = \left(\frac{500000}{13} \cdot N_c \cdot N_{eb}\right) \cdot e_i \cdot \left(a^2 + b^2\right)^{1/2}$$

$$R_a' + G_a' + \left(\frac{21}{20}\right) \cdot B_a'$$

$$C = t^{0.9} \cdot \left(\frac{J}{100}\right)^{0.5} \cdot (1.64 - 0.29^n)^{0.73}$$

$$M = C \cdot F_L^{0.25}$$

$$s = 100 \cdot \left(\frac{M}{Q}\right)^{0.5}$$
Part 2: The Reverse Mode

**Input:** $J$ or $Q$; $C$, $M$, or $s$; $H$ or $h$

**Output:** $X, Y, Z$ (under test illuminant $X_w, Y_w, Z_w$)

Illuminants, viewing surrounds, and background parameters are the same as those given in the forward mode. See notes at the end of this Appendix calculating/defining the luminance of the adapting field and surround conditions.

**Step 0:** Calculate viewing parameters

Compute all $F_L, n, z, N_{bb} = N_{bc}, R_w, G_w, B_w, D, D_R, D_G, D_B, R_{wc}, G_{wc}, B_{wc}, R_w', G_w', B_w', R_{wc}', G_{wc}', B_{wc}'$, and $A_w$ using the same formulae as in Step 0 of the Forward model. They are needed in the following steps. Note that all data computed in this step can be used for all samples (for example all pixels for an image) under the viewing conditions. Hence, they are computed once for all. The following computing steps are sample dependant.

**Step 1:** Obtain $J$, $C$ and $h$ from $H$, $Q$, $M$, $s$

The entering data can be in different combination of perceived correlates, i.e., $J$ or $Q$; $C$, $M$, or $s$; and $H$ or $h$. Hence the followings are needed to convert the others to $J$, $C$, and $h$.

**Step 1-1:** Compute $J$ from $Q$ (if start from $Q$)

$$J = 6.25 \cdot \left[ \frac{c \cdot Q}{(A_w + 4) \cdot F_L^{0.25}} \right]^2$$

**Step 1-2:** Calculate $C$ from $M$ or $s$

$$C = \frac{M}{F_L^{0.25}} \quad \text{(if start from } M \text{)}$$

$$Q = \left( \frac{4}{c} \right) \cdot \left( \frac{J}{100} \right)^{0.5} \cdot (A_w + 4.0) \cdot F_L^{0.25}$$

and

$$C = \left( \frac{s}{100} \right)^2 \cdot \left( \frac{Q}{F_L^{0.25}} \right) \quad \text{(if start from } s \text{)}$$

**Step 1-3:** Calculate $h$ from $H$ (if start from $H$)

The correlate of hue ($h$) can be computed by using data in Table 11 - A2 in the Forward mode.
Appendix B: CIE Colour Appearance Model: CIECAM02

Choose a proper \( i \) (\( i = 1, 2, 3, \) or \( 4 \)) so that \( H_i \leq H < H_{i+1} \).

\[
h' = \frac{(H - H_i) \cdot (e_{i+1} - e_i) - 100 \cdot h \cdot e_{i+1}}{(H - H_i) \cdot (e_{i+1} - e_i) - 100 \cdot e_{i+1}}
\]

Set \( h = h' - 360 \) if \( h' > 360 \), otherwise \( h = h' \).

**Step 2:** Calculate \( t \), \( e_i \), \( p_1 \), \( p_2 \), and \( p_3 \)

\[
t = \left[ \frac{J}{100} \cdot (1.64 - 0.29^n)^{0.73} \right]^{0.9}
\]

\[
e_i = \frac{1}{4} \left[ \cos \left( h \cdot \frac{\pi}{180} + 2 \right) + 3.8 \right]
\]

\[
A = A_w \cdot \left( \frac{J}{100} \right)^{1/3}e^{-z}
\]

\[
p_1 = \left( \frac{50000}{13} \cdot N_{e} \cdot N_{eb} \right) \cdot e_i \cdot \left( \frac{1}{t} \right), \quad \text{if } t \neq 0
\]

\[
p_2 = \frac{A}{N_{bb}} + 0.305
\]

\[
p_3 = \frac{21}{20}
\]

**Step 3:** Calculate \( a \) and \( b \)

If \( t = 0 \), then \( a = b = 0 \) and **go to Step 4**

(\text{be sure transferring } h \text{ from degree to radian before calculating } \sin(h) \text{ and } \cos(h))

If \( \left| \sin(h) \right| \geq \left| \cos(h) \right| \) then

\[
p_4 = \frac{p_1}{\sin(h)}
\]

\[
b = \frac{p_2 \cdot (2 + p_3) \cdot \left( \frac{460}{1403} \right)}{p_4 + (2 + p_3) \cdot \left( \frac{220}{1403} \right) \cdot \left( \frac{\cos(h)}{\sin(h)} \right) - \left( \frac{27}{1403} \right) + p_3 \cdot \left( \frac{6300}{1403} \right)}
\]

\[
a = b \cdot \left( \frac{\cos(h)}{\sin(h)} \right)
\]
If $|\cos(h)| \gg |\sin(h)|$, then

$$p_5 = \frac{p_1}{\cos(h)}$$

$$a = \frac{p_2 \cdot (2 + p_3) \cdot \left(\frac{460}{1403}\right)}{p_5 + (2 + p_3) \cdot \left(\frac{220}{1403}\right) - \left[\left(\frac{27}{1403}\right) - p_3 \cdot \left(\frac{6300}{1403}\right)\right] \cdot \left(\frac{\sin(h)}{\cos(h)}\right)}$$

$$b = a \cdot \left(\frac{\sin(h)}{\cos(h)}\right)$$

**Step 4:** Calculate $R'_a$, $G'_a$, and $B'_a$

$$R'_a = \frac{460}{1403} \cdot p_2 + \frac{451}{1403} \cdot a + \frac{288}{1403} \cdot b$$

$$G'_a = \frac{460}{1403} \cdot p_2 - \frac{891}{1403} \cdot a - \frac{261}{1403} \cdot b$$

$$B'_a = \frac{460}{1403} \cdot p_2 - \frac{220}{1403} \cdot a - \frac{6300}{1403} \cdot b$$

**Step 5:** Calculate $R'$, $G'$, and $B'$

$$R' = \text{sign}(R'_a - 0.1) \cdot \frac{100}{F_L} \cdot \left[\frac{27.13 \cdot |R'_a - 0.1|}{400 - |R'_a - 0.1|}\right]^{0.42}$$

Here $\text{sign}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$, and similarly computing $G'$, and $B'$ from $G'_a$, and $B'_a$.

**Step 6:** Calculate $R_C$, $G_C$, and $B_C$ (for the inverse matrix, see the note at the end of the Appendix)

$$\begin{pmatrix} R_c \\ G_c \\ B_c \end{pmatrix} = M_{CAT02} \cdot M_{HPE}^{-1} \cdot \begin{pmatrix} R' \\ G' \\ B' \end{pmatrix}$$

**Step 7:** Calculate $R$, $G$, and $B$
Appendix B: CIE Colour Appearance Model: CIECAM02

\[
\begin{pmatrix}
R \\
G \\
B
\end{pmatrix} = \begin{pmatrix}
\frac{R_c}{D_R} \\
\frac{G_c}{D_G} \\
\frac{B_c}{D_B}
\end{pmatrix}
\]

**Step 8:** Calculate \(X\), \(Y\), and \(Z\) (for the coefficients of the inverse matrix, see the note at the end of the Appendix)

\[
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix} = M_{\text{CAT02}}^{-1} \cdot \begin{pmatrix}
R \\
G \\
B
\end{pmatrix}
\]

---

**Notes to Appendices A and B**

1. It is recommended to use the matrix coefficients given below for the inverse matrix \(M_{\text{CAT02}}^{-1}\) and \(M_{\text{HPE}}^{-1}\):

\[
M_{\text{CAT02}}^{-1} = \begin{pmatrix}
1.096124 & -0.278869 & 0.182745 \\
0.454369 & 0.473533 & 0.072098 \\
-0.009628 & -0.005698 & 1.015326
\end{pmatrix}
\]

\[
M_{\text{HPE}}^{-1} = \begin{pmatrix}
1.910197 & -1.112124 & 0.201908 \\
0.370950 & 0.629054 & -0.000008 \\
0.000000 & 0.000000 & 1.000000
\end{pmatrix}
\]

2. For implementing the CIECAM02, the testing data and the corresponding results from the Forward and Reverse modes can be found from reference 7.

3. The \(L_A\) is computed using equation (11 - A1)

\[
L_A = \frac{E_w}{\pi} \cdot \frac{Y_b}{Y_w} = \frac{L_w \cdot Y_b}{Y_w},
\]

where \(E_w = \pi \cdot L_w\) is the illuminance of reference white in lux unit; \(L_w\) the luminance of reference white in cd/m² unit, \(Y_b\) the luminance factor of the background and \(Y_w\) the luminance factor of the reference white.
4. Surround conditions (average, dim, and dark) are determined by the surround ratio $S_r$ given by equation (11-A2):

$$S_r = \frac{L_{SW}}{L_{DW}}$$

(11–A2)

where $L_{SW}$ is the luminance of the reference white measured in the surround field and $L_{DW}$ is the luminance of the reference white measured in the display area. If $S_r$ is 0, then the surround condition is ‘dark’; if $0 < S_r < 0.2$, then the surround is ‘dim’; and if $S_r \geq 0.2$, then the surround is ‘average’.