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Time-bound product returns and optimal order quantities for mass merchandisers

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The return guidelines for a mass merchandiser usually entail a grace period, a markdown on the original price and the condition of the returned items. This research utilises eight scenarios formed from the variation of possible return guidelines to model the cost functions of single-product categories for a typical mass merchandiser. Models for the eight scenarios are developed and solved with the objective of maximising the expected profit so as to obtain closed form solutions for the associated optimal order quantity. An illustrative example and sensitivity analysis are provided to demonstrate the applicability of the model. Our results show that merchandisers who allow for returns within a time window, albeit with a penalty cost imposed and the returned products being recoverable, should plan for larger order amounts as such products do not affect the business. Similarly, the merchandisers who allow for returns beyond a grace period and without any penalty charges, but where the returned products are irrecoverable, should manage their stocks in this category more judiciously by ordering as little as possible so as to limit the number of returns and carefully consider the effects of their customer satisfaction-guaranteed policies, if any.

Keywords: return policy; mass merchandiser; single-period product; retail

1. Introduction

As consumer satisfaction is a key consideration of the modern retail trade, satisfaction-guaranteed policies are commonplace in the retail sector (Westbrook 1981; Schmidt and Kernan 1985). For instance, retailers offer various policies to ‘guarantee’ customer satisfaction. So far, studies (Schmidt and Kernan 1985; Davis, Gerstner, and Hagerty 1995; Shieh 1996; Tsay 2001; Pan and Zinkhan 2006) suggest that policies such as the return of unsatisfactory items, money-back guarantees, lowest price and one-for-one replacements are practised widely, albeit to different degrees and in different forms.

Among these consumer-oriented policies, the return of sale items after purchase is frequently practised (Davis, Hagerty, and Gerstner 1998). In some cases, such policies allow customers to return products after purchase with little or no restrictions. In addition, full or partial refunds can be rendered depending on the condition of the returned item and the associated charges incurred, such as sales tax and gift wrapping cost. More details on this can be found on the websites of well-known global retailers such as Walmart (http://www.walmart.com/returns#37441, accessed on 23 November 2009).

Generally, products are returned to the retailer for a variety of reasons. First, wrong products in terms of size, shape, colour, quantity or content are sent by the retailers. This type of mistake entitles the customers to unconditionally return the products. Second, consumers mistakenly order unwanted products due to erroneous information or a lack of related knowledge, which may be due to misleading advertisements. Third, products are found to be defective when opened due to poor manufacturing quality or improper handling during transit. Finally, consumers change their minds after the purchase.

Traditional grocery stores have evolved various types of retail formats. Supermarket retailers such as Kroger, Safeway and Albertson dominated grocery shopping in the 1970s. With the advent of one-stop shopping, the number of product categories handled by retailers has mushroomed. Since the 1980s, mass merchandisers such as Wal-Mart, Kmart, Target and even the online shopping giant Amazon have become major retailers for consumer shopping (Fox, Montgomery, and Lodish 2004).

Retailers now carry a large assortment of products in order to offer a variety of seeking and buying flexibility (Kahn and Lehmann 1991; Kahn 1995).

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The number of items and product categories offered by a supermarket vary according to the size and operation strategy of the store. van Donselaar, van Woensel, Broekmeulen, and Franco (2006) suggest that different product categories in a supermarket present various supply chain issues and thus require different inventory management approaches.

According to the Food Marketing Institute Research (2007), the average number of stock keeping units (SKUs) at a supermarket is 45,000. For a typical mass merchandiser, the number of SKUs has now grown dramatically to more than 100,000. At Wal-Mart, for example, an average discount store occupies 107,000 sq ft and offers 120,000 items while a super centre occupies an average of 187,000 sq ft and offers 142,000 items (Wal-Mart Facts 2007). A Wal-Mart discount store features general merchandise products including family apparel, health and beauty aids, home furnishings, electronics, hardware and toys. In addition to general merchandise, a super centre features bakery goods, delicatessen foods, meat and dairy foods, fresh produce and specialty shops such as vision centres, tyre and lube services, fast food restaurants and even photo centres, hair salons and banks.

The ever increasing number of SKUs has rendered the handling of return products more challenging. Among the assorted product categories, many are considered short-lived or single-period products as such products are mainly perishable items, namely farm produce, meat, seafood, fresh flowers and bakery goods. Besides, cutting-edge innovations and advanced manufacturing technology permit more new product introductions, hence shortening their life cycles. Likewise, consumers now switch their product preferences more frequently and continue to seek goods that can satisfy their buying appetite.

Although product returns can take place in almost every type of transaction, the return of short-lived or perishable products has been well studied (Pasternack 1985; Lau and Lau 1999; Mantrala and Raman 1999; Lau, Lau, and Willett 2000; Vlachos and Dekker 2003; Mostard and Teunter 2006; Mostard and Teunter 2005). The study of the short-lived product, often referred to as the classical single-period product or the newsboy problem, involves finding optimum order quantities that maximise the expected profit in a single period under probabilistic demand. Khouja (1999) conducted a review and classified the related papers into 11 categories. Among them are extensions to different transaction policies as well as various degrees of system complexity.

Pasternack (1985) studied a manufacturer’s pricing and return policies for a single-period commodity sale to retailers. The retailer’s unsold products can either be returned with a full refund or no return at all to the manufacturer. The relationships among the manufacturing cost, selling price, return premium, salvage value and retail price are discussed. Paternack suggests that the adoption of product returns from retailers with a partial refund is best to achieve channel coordination. Emmons and Gilbert (1998) focus on the role of return policies for catalogue styled goods businesses. A predetermined price-dependent probability distribution of the demand is assumed to demonstrate a multiplicative demand model by applying a constant coefficient of variation. Their study suggests that coordinating the return policies between manufacturers and retailers is beneficial for maximising the total profit.

Mantrala and Raman (1999) further pursue return policies for central buyers of style goods who operate through multiple store outlets. The individual probability distribution of the demand for each store outlet is assumed and correlations among them are introduced. The results show that the central buyer, unlike the manufacturers, prefers an order quantity of style goods at the lowest possible wholesale price when buyback is not allowed.

Vlachos and Dekker (2003) claim that returns can serve as a secondary supply to satisfy a single-period demand. Defective returned products may also be recovered and resold before the selling season ends. This is especially true for style goods sold through e-commerce or direct mail retailers. Six models are developed representing scenarios constituting return, reuse, recovery and fixed recovery cost. Mostard et al. (2005) apply a distribution-free approach to analyse the effects of the various return rates. The relationship of these return rates with the various coefficients of the variation of the total demand is also inspected. Mostard and Teunter (2006) relaxed Vlachos and Dekker’s (2003) assumption by allowing returned products that have been resold more than once and adopting total demand estimators.

While previous studies on resalable returns for the newsboy problem have assumed that products can be returned any time before the selling period ends, many mass merchandisers such as Wal-Mart or Costco require consumers to return the purchased products within a specified review period or time window. When products are returned after the grace period, retailers may either reject the return or accept the return with only a partial refund. As a common practice, not-so-seriously flawed returns are also recovered and resold before the end of the selling season.

In this article, various time-bound product return policies implemented by mass merchandisers are analysed. Owing to the different nature of the product categories, various combinations of return policies are possible. The return policies deviate by the presence or
absence of a review grace period, penalty on return and the recoverability of the returned items. As such, eight scenarios are investigated and closed form solutions for the respective optimal order quantities are developed.

The remainder of this article is organised as follows. Section 2 illustrates the practices of mass merchandisers on the various return policies. Section 3 develops the mathematical models for the eight regimes. Closed form solutions for the respective optimal order quantities are derived. Illustrative examples used to demonstrate the usability of the models are given in Section 4. Finally, Section 5 concludes the article.

2. Depictions of time-bound return policies

Return policies or guidelines are usually posted at the stores’ bulletins or on the companies’ websites. For most stores, the factors determining whether and how to accept the return of a purchased item are: the time after purchase, product category and product condition at the time of return. This section presents the time-bound return policies for mass merchandisers and practical examples are given to demonstrate each of the eight scenarios, as shown in Table 1, to represent return policies applicable to the different handling options.

In Wal-Mart, for example, most purchased items accompanied with the invoice, original packaging and accessories can be returned to a local store within 90 days. However, electronic products such as computer hardware, cell phones and digital music players are limited to be returned for refund within 15 days and with the receipt of purchase. Camcorders and digital cameras must be returned within 30 days. Computer components and accessories, on the other hand, can be extended to 45 days.

Other guidelines regarding specific product categories are also stated in the return policies. For apparel, shoes and accessories, the products must be unworn when returned. Music CDs, DVDs, audiotapes, videotapes and video games have to remain unopened. Books must be returned unmarked and unused. Perishable goods such as dairy products, meat, fishery products, bakery products and flowers are prone to non-returns due to health codes. However, refunds may be rendered if defective items are sold. Pharmacy departments usually do not accept the return of medication for safety reasons. Most retailers also do not accept the return of gift certificates.

Upon receipt of the returned products, full refund policies are usually applied to guarantee customer satisfaction. However, some retailers provide partial refunds when the returns do not conform to their return guidelines. For instance, the online mass merchandiser Amazon offers 80% of the item price if any unopened music CD, DVD, computer software or video game is returned beyond 30 days. Once the return product has been unpacked, the refund reduces to 50% of its original price. Damaged items or products with missing parts may also be returned with up to 50% refund.

Some retailers may accept the return products if they can be recovered in their original condition. In this case, a penalty such as a restocking or repacking fee will be charged to compensate the retailer’s recovery cost. For instance, unwrapped CDs or DVDs in an intact condition can be recovered to their initial form with the help of a wrapping or packaging machine. The recovered items will be sold in a secondary market at the same store location at discounted prices.

This article investigates the effects of various return policies for a typical mass merchandiser. The mass merchandiser sells goods throughout its retail outlets until the end of the selling period or when the inventory is depleted. It is assumed that all the retail stores follow uniform pricing and return policies. Although customer satisfaction is emphasised, the mass merchandiser is assumed to establish the ordering decisions with an aim of maximising profit.

3. Model development

The following notations are used for the quantitative models for the eight return handling scenarios.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>unit purchase cost paid to the supplier</td>
</tr>
<tr>
<td>$C_i$</td>
<td>unit inventory cost for unsold items</td>
</tr>
<tr>
<td>$C_j$</td>
<td>unit inventory cost for returned items</td>
</tr>
<tr>
<td>$C_l$</td>
<td>cost of lost sales</td>
</tr>
<tr>
<td>$C_h$</td>
<td>unit cost for return handling</td>
</tr>
<tr>
<td>$C_r$</td>
<td>variable recovery cost</td>
</tr>
<tr>
<td>$C_f$</td>
<td>fixed recovery cost</td>
</tr>
<tr>
<td>$p_r$</td>
<td>unit penalty for returned items</td>
</tr>
<tr>
<td>$P$</td>
<td>unit selling price</td>
</tr>
</tbody>
</table>
3.1. Model description and assumptions

The models developed for the various scenarios of return policy based on the grace period, return penalty and product recoverability assume that the ordering decisions for the related product categories are made before the onset of the selling season. Any unwanted or dissatisfied customer order will be returned to the supplier. The quantity of demand, \( x \), has the density function, \( f(x) \) and cumulative distribution function \( F(x) \). We assume that the demand for all product categories obeys a normal distribution so that the associated means and variances can be estimated for further mathematical analysis. Also, demand is assumed to be significantly larger than the quantity of total returns during the selling season. Under certain return guidelines, product returns are allowed within a time window. Customers may either get a full refund or be charged with a certain amount of penalty \( P_r \). The returned products are handled and processed via the return channel with a unit return handling cost, \( C_h \).

Some retailers allow for returns beyond the grace period, with either a full or partial refund. A fixed probability \( r_w \) is assumed for each sold product to be returned within a defined time window and \( r_b \) is assumed if it is returned beyond that time limit.

For products that have been stored and shelved at the regional distribution centre and local outlets, inventory costs are incurred. The unsold products remain at the store until the end of the selling season while the returned items are transported and stored at the regional distribution centre awaiting disposal. The inventory carrying costs for the unsold and returned items are denoted by \( C_l \) and \( C_j \), respectively.

Among the returned orders, some intact products can be resold to fill new orders while others may undergo minor rework so that they can be recovered to the original condition. It is assumed that the recoverability of the returned products is based on the nature of the product sold. If the returned product is recoverable, a fixed recovery cost, \( C_r \), is incurred due to the utilisation of the facility for remanufacturing or repackaging. A variable cost, \( C_r \), is also assumed to denote the cost of processing recovery activities.

Initially, the unit purchase cost paid to the supplier for the ordered products is \( C \). The products are retailed at unit price, \( P \). The unsold products can be sold at the end of the selling season at a discounted salvage value, \( S \). Lost sales occur if the quantity ordered cannot meet the demand during the period. The unit cost of lost sales is denoted by \( C_l \), which is usually proportional to the retail price, \( P \).

Unlike the assumption in previous research (Vlachos and Dekker 2003; Mostard and Teunter 2006) that the returned products can be resold at a fixed serviceable rate, \( k \), this research suggests that all the recoverable returned products can be resold in an alternate discount market with discounted prices denoted by \( R \). Irrecoverable returned items, on the other hand, will be scrapped or returned to the merchandiser’s suppliers.

3.2. Model development

The optimal order quantities for each scenario are derived by solving the first-order condition of the expected profit functions, which is obtained by summing the revenue from the initial sales, salvage value, sales of recovered products and penalty for violation of return guidelines less the cost of purchasing, inventory, lost sales, return handling and recovering process of the items.

The total expected initial revenue for the mass merchandiser is given by

\[
ER = \int_0^Q P(1-r_w)xf(x)dx + \int_Q^\infty P(1-r_w)Qf(x)dx,
\]

where the first term on the right-hand side represents the expected revenue for the mass merchandiser when the demand \( x \) is less than the order quantity \( Q \), while the second term is the expected revenue when \( x \) is greater than \( Q \).

As the unsold items can be bought back by the suppliers at the end of the selling season, the total salvage value paid by the suppliers is

\[
SR = \int_0^Q S(Q-x)f(x)dx.
\]

If the returned items are irrecoverable, they will accompany the unsold items to be scrapped or returned to the suppliers at the end of the selling season. The total salvage value is thus increased to

\[
SR' = \int_0^Q S[Q-(1-r_w)x]f(x)dx.
\]

The returned items can be resold at a discounted price \( R \), yielding a revenue of

\[
RR = \int_0^Q R r_w xf(x)dx + \int_Q^\infty R r_w Qf(x)dx.
\]
When return guidelines are violated, product return may be allowed through a penalty $C_p$, which is paid by the customers and is considered as the revenue for the mass merchandiser and is given by

$$\text{PR} = \int_0^Q p_r x f(x) \, dx + \int_Q^\infty p_r Q f(x) \, dx.$$  \hfill (4)

The cost of purchase for the product is given by

$$\text{PC} = C Q.$$  \hfill (5)

This research assumes a uniform unit inventory cost. The total inventory holding costs can be computed as

$$\text{IC} = \int_0^Q C(Q - x)f(x) \, dx + \int_0^Q C_j r_w x f(x) \, dx$$

$$+ \int_Q^\infty C_j r_w Q f(x) \, dx,$$  \hfill (6)

where the first term on the right-hand side is the total inventory cost for unsold items when the demand $x$ is less than $Q$, while the second and third terms denote the holding cost for the returned items with respect to a different demand range.

When the demand $x$ is greater than $Q$, sales are lost and the service level deteriorates. The cost of the lost sales is given by

$$\text{LC} = \int_Q^\infty C_l (x - Q) f(x) \, dx.$$  \hfill (7)

The returning process is handled by the mass merchandiser’s customer service department, where return guidelines are reviewed, paperwork completed and the packaging and shipping for transporting the returned items to the regional packing and repair centre is ensured. The total handling cost can be given by

$$\text{HC} = \int_0^Q C_h r_w x f(x) \, dx + \int_Q^\infty C_h r_w Q f(x) \, dx.$$  \hfill (8)

The returned items undergo further repair and recovery work so that they can be resold. The total recovery cost is given by

$$\text{RC} = C_f + \int_0^Q C_r r_w x f(x) \, dx + \int_Q^\infty C_r r_w Q f(x) \, dx.$$  \hfill (9)

If returns after the grace period are permitted, the revenue and cost terms need to be adjusted and $r_h$ is inserted. Thus the total expected initial revenue becomes

$$\text{ER'} = \int_0^Q P(1 - r_w - r_h) x f(x) \, dx$$

$$+ \int_Q^\infty P(1 - r_w - r_h) Q f(x) \, dx.$$  \hfill (10)

The total salvage value if the returned items are irrecoverable is

$$\text{SR'} = \int_0^Q S[Q - (1 - r_w - r_h)x] f(x) \, dx.$$  \hfill (20)

The total resale revenue for the recoverable items is adjusted by

$$\text{RR'} = \int_0^Q R(r_w + r_h) x f(x) \, dx + \int_Q^\infty R(r_w + r_h) Q f(x) \, dx.$$  \hfill (40)

The total return penalty considered as the revenue term becomes

$$\text{PR'} = \int_0^Q p_r (r_w + r_h) x f(x) \, dx + \int_Q^\infty p_r (r_w + r_h) Q f(x) \, dx.$$  \hfill (50)

The inventory holding costs is given by

$$\text{IC'} = \int_0^Q C(Q - x)f(x) \, dx + \int_0^Q C_j (r_w + r_h) x f(x) \, dx$$

$$+ \int_Q^\infty C_j (r_w + r_h) Q f(x) \, dx.$$  \hfill (60)

The adjusted return handling cost is given by

$$\text{HC'} = \int_0^Q C_h (r_w + r_h) x f(x) \, dx + \int_Q^\infty C_h (r_w + r_h) Q f(x) \, dx.$$  \hfill (80)

Finally, the recovery costs turn into

$$\text{RC'} = C_f + \int_0^Q C_r (r_w + r_h) x f(x) \, dx$$

$$+ \int_Q^\infty C_r (r_w + r_h) Q f(x) \, dx.$$  \hfill (90)

3.2.1. Scenario S1: returns allowed only within a time window without term penalty, and returned products are recoverable

This scenario can be applied to almost every product category for a mass merchandiser that implements a satisfaction-guaranteed policy. In this scenario, products can be returned with a full refund, with (e.g. Kmart, Costco) or without (e.g. Walmart) a receipt within a predetermined review grace period when products are in the original condition or they are unused after unwrapping. Usually, the returned product in this scenario can be re-shelved and sold as new before the season ends if it were returned intact. However, for items that have been opened and used before return, repairs and repackaging are needed to transform the item into the original condition. In order
to determine the optimal ordering quantity for this scenario, the expected profit function, \( \pi_1(Q) \), is constructed by the following revenue and cost terms:

\[
\pi_1(Q) = ER + SR + RR - PC - IC - LC - HC - RC. \tag{10}
\]

Finding and setting \( d\pi_1(Q)/dQ = 0 \) yields the necessary \( Q_1^* \), which can be obtained from (11) (see Appendix 1 for the details).

\[
F(Q_1^*) = \frac{P - C + C_l - r_w(P - R + C_j + C_h + C_r)}{P - S + C_i + C_l - r_w(P - R + C_j + C_h + C_r)}, \tag{11}
\]

3.2.2. Scenario S2: returns allowed only within a time window without penalty, but returned products are irrecoverable

In this scenario, the return of either faulty items or unsatisfactory perishable items such as cooked food, cut flowers and takeout meals for retailers is considered. For merchandisers that guarantee satisfaction, a full refund is rendered for the return. These perishable items cannot be resold and will mostly be disposed of or returned to the suppliers by the end of the selling season. To find the optimal ordering quantity for this scenario, the expected profit function, denoted by \( \pi_2(Q) \), is obtained by considering the following revenue and cost terms:

\[
\pi_2(Q) = ER + SR' - PC - IC - LC - HC. \tag{12}
\]

Finding and setting \( d\pi_2(Q)/dQ = 0 \), \( Q_2^* \) can be obtained by finding the inverse function of (13) (see Appendix 2 for the details).

\[
F(Q_2^*) = \frac{P - C + C_l - r_w(P - R + C_j + C_h)}{P - S + C_i + C_l - r_w(P - R + C_j + C_h)}, \tag{13}
\]

3.2.3. Scenario S3: return within a time window with penalty, and returned products are recoverable

Some merchandisers impose a restocking fee on several short-shelf life product categories such as electrical appliances, computers and consumer electronics if they are returned without defect. For example, Best Buy charges a 10% restocking fee for the return of Apple iPhones and 15% for computers, camcorders and digital cameras. These returned items can be cleaned and re-packaged to the original condition before they are re-shelved for sale. To determine the optimal ordering quantity for this scenario, the expected profit function denoted by \( \pi_3(Q) \) is obtained by considering the following revenue and cost terms:

\[
\pi_3(Q) = ER + SR + RR - PC - IC - LC - HC - RC. \tag{14}
\]

Finding and setting \( d\pi_3(Q)/dQ = 0 \), \( Q_3^* \) is obtained from the inverse function of (15) (see Appendix 3 for the details).

\[
F(Q_3^*) = \frac{P - C + C_l - r_w(P - R + R_r + C_j + C_h + C_r)}{P - S + C_i + C_l - r_w(P - R + R_r + C_j + C_h + C_r)}. \tag{15}
\]

3.2.4. Scenario S4: return within a time window with penalty, but returned products are irrecoverable

When the returned items appear to have obvious signs of use, a partial refund is likely to be granted. For instance, books and pre-recorded music, movie and software products such as CDs and DVDs are required to be returned intact. If the wrapping is unwrapped or if books are having obvious marks or folded pages, only a partial refund will be awarded to the customers. To determine the optimal ordering quantity for this scenario, the expected profit function denoted by \( \pi_4(Q) \) is found from the following revenue and cost terms:

\[
\pi_4(Q) = ER + SR' + PR - PC - IC - LC - HC. \tag{16}
\]

Finding and setting \( d\pi_4(Q)/dQ = 0 \), \( Q_4^* \) is obtained by finding the inverse function of (17) (see Appendix 4 for the details).

\[
F(Q_4^*) = \frac{P - C + C_l - r_w(P - R + R_r + C_j + C_h)}{P - S + C_i + C_l - r_w(P - R + R_r + C_j + C_h)} \tag{17}
\]

3.2.5. Scenario S5: returns allowed beyond a time window with no penalty, and returned products are recoverable

Some mass merchandisers give a full refund for certain product categories even after the review grace period. For instance, unworn apparel, shoes and accessories accompanied with receipts may be returned with a full refund if customers are not satisfied. To find the optimal ordering quantity for this scenario, the expected profit function denoted by \( \pi_5(Q) \) is found from the following revenue and cost terms:

\[
\pi_5(Q) = ER' + SR + RR' - PC - IC' - LC - HC' - RC'. \tag{18}
\]
Finding and setting $d\pi_5(Q)/dQ = 0$, $Q^*_5$ can be obtained by finding the inverse function of (19) (see Appendix 5 for the details).

$$F(Q^*_5) = \frac{P - C + C_l - (r_w + r_b)(P + C_j + C_h)}{P - S + C_l + C_l - (r_w + r_b)(P + C_j + C_h)}.$$  

(19)

3.2.6. Scenario S6: returns allowed beyond a time window with no penalty, but returned products are irrecoverable

Though some product categories are prone to deterioration with time, a full satisfaction-guaranteed policy offered by some mass merchandisers still accepts returns beyond the grace period. To determine the optimal ordering quantity for this scenario, the expected profit function denoted by $\pi_6(Q)$ is found from the following revenue and cost terms:

$$\pi_6(Q) = ER' + SR'' - PC - IC' - LC - HC'.$$  

(20)

Finding and setting $d\pi_6(Q)/dQ = 0$, $Q^*_6$ can be obtained by finding the inverse function of (21) (see Appendix 6 for the details).

$$F(Q^*_6) = \frac{P - C + C_l - (r_w + r_b)(P + C_j + C_h)}{P - S + C_l + C_l - (r_w + r_b)(P + C_j + C_h)}.$$  

(21)

3.2.7. Scenario S7: returns allowed beyond a time window with penalty, and returned products are recoverable

A mass merchandiser may allow customers to return unused items for certain product categories even after the review grace period. However, a restocking fee will be charged owing to the discounted value resulting from the product’s shorter product life cycle. When a product is returned beyond the time window, a greater amount of penalty is charged to penalise the customer’s late return. Examples of these include electronic gadgets, unmarked books and music CD/DVDs.

To determine the optimal ordering quantity for this scenario, the expected profit function denoted by $\pi_7(Q)$ can be obtained by considering the following revenue and cost terms:

$$\pi_7(Q) = ER' + SR + RR' + PR' - PC - IC' - LC - HC' - RC'.$$  

(22)

Finding and setting $d\pi_7(Q)/dQ = 0$, $Q^*_7$ can be obtained by finding the inverse function of (23) (see Appendix 7 for the details).

$$F(Q^*_7) = \left\{ \frac{P - C + C_l - (r_w + r_b)(P + C_j + C_h)}{P - S + C_l + C_l - (r_w + r_b)(P + C_j + C_h)} \right\} \times (P - R - p_r + C_j + C_h + C_r).$$  

(23)

3.2.8. Scenario S8: returns allowed beyond a time window with penalty, but returned products are irrecoverable

This scenario applies to certain product categories for merchandisers who carry a satisfaction-guaranteed programme. For product categories such as electronics and appliances, purchased items have been used and experienced before they are found to be unsatisfactory.

The retailers will accept the returned items while partial refund is granted to cover the transportation and processing expenses. To determine the optimal ordering quantity for this scenario, the expected profit function denoted by $\pi_8(Q)$ can be obtained by considering the following revenue and cost terms:

$$\pi_8(Q) = ER' + SR'' + PR' - PC - IC' - LC - HC'.$$  

(24)

Finding and setting $d\pi_8(Q)/dQ = 0$, $Q^*_8$ can be obtained by finding the inverse function of (25) (see Appendix 8 for the details).

$$F(Q^*_8) = \left\{ \frac{P - C + C_l - (r_w + r_b)(P + C_j + C_h)}{P - S + C_l + C_l - (r_w + r_b)(P + C_j + C_h)} \right\} \times (P - R - p_r + C_j + C_h + C_r).$$  

(25)

4. Properties of optimal order quantities for various scenarios

The closed form solutions for the eight scenarios developed above can be used to calculate the optimal order quantity when a specific scenario applies. The eight optimality conditions indicate that the optimal order quantities depend mainly on the associated cost parameters and the return rates used to compose the cdf of demand $F(Q^*_i)$. Thus, knowing $F(Q^*_i)$ can help to derive properties for the relationships among the optimal order quantities of the various scenarios. Further, for each product category, as the optimal order quantity $Q^*_i$ is found from the inverse of $F(Q^*_i)$, the scenario that has a larger $F(Q^*_i)$ will incur a larger amount of $Q^*_i$. 

(continued...
In practice, a specific product category may apply to more than one scenario. For example, electronic products such as the digital camera and camcorder can be returned intact within a grace period with a full refund (Scenario S1). The same merchandiser may also accept the returned digital camera or camcorder even after the grace period while charging a restocking fee if the item is in its original condition (Scenario S7). An apparel item can be returned within a short period of time whether it is unopened (Scenario S1) or has been tried on (Scenario S2) as long as the customer is not satisfied. When multiple scenarios are applied to a product item, different scenarios will yield different optimal ordering quantities. The largest optimal ordering quantity will be set as the actual order level of that item. The relationships among the optimal ordering quantities are studied and analysed as follows:

(1) The most obvious ordering relationships of \( F(Q^*_1) \) among eight scenarios are \( F(Q^*_1) > F(Q^*_2), F(Q^*_1) > F(Q^*_3), F(Q^*_1) > F(Q^*_4) \), and \( F(Q^*_1) > F(Q^*_5) \) due to the inclusion of the return rate beyond a time window \( r_s \) for Scenarios S5–S8 when the penalty policy and recoverability remain unchanged. For a specific product category, the relationships \( Q^*_1 > Q^*_2, Q^*_1 > Q^*_3, Q^*_1 > Q^*_7 \) and \( Q^*_1 > Q^*_8 \) hold.

(2) For scenarios that impose a penalty for the returned items, adopting a penalty \( p_r \), leads to \( F(Q^*_1) > F(Q^*_2), F(Q^*_1) > F(Q^*_3), F(Q^*_1) > F(Q^*_4) \), and \( F(Q^*_1) > F(Q^*_5) \), due to the inclusion of the return rate beyond a time window \( r_s \) for Scenarios S5–S8 when the penalty policy and recoverability remain unchanged. Thus, for a specific product category, the relationships \( Q^*_1 > Q^*_2, Q^*_1 > Q^*_3, Q^*_1 > Q^*_7 \) and \( Q^*_1 > Q^*_8 \) hold.

(3) Several model parameters appear to demonstrate specific practical conditions in the real world. For example, the resale price is usually greater than the variable recovery cost \( R > C_r \), for obvious profit maximisation reasons. Thus, the ordering relations \( F(Q^*_1) > F(Q^*_2), F(Q^*_1) > F(Q^*_3), F(Q^*_1) > F(Q^*_4) \), and \( F(Q^*_1) > F(Q^*_5) \) can be derived. Hence, for a specific product category, the relationships \( Q^*_1 > Q^*_2, Q^*_1 > Q^*_3, Q^*_1 > Q^*_4, Q^*_1 > Q^*_5, Q^*_1 > Q^*_6, \) and \( Q^*_1 > Q^*_7 \) hold.

(4) From (1), (2) and (3), the following ordering relationships can be derived: \( Q^*_1 > Q^*_2 > Q^*_3 > Q^*_4 > Q^*_5 > Q^*_6 > Q^*_7 > Q^*_8, \) \( Q^*_1 > Q^*_3 > Q^*_4 > Q^*_5 > Q^*_6 > Q^*_7 > Q^*_8, \) \( Q^*_1 > Q^*_4 > Q^*_5 > Q^*_6 > Q^*_7 > Q^*_8, \) \( Q^*_1 > Q^*_5 > Q^*_6 > Q^*_7 > Q^*_8, \) and \( Q^*_1 > Q^*_7 > Q^*_8, \) and \( Q^*_1 > Q^*_8. \)

Therefore, it can be concluded that \( Q^*_1 \) is consistently the largest while \( Q^*_7 \) is the smallest among the eight optimal order quantities. In short, merchandisers who allow for returns within a time window albeit with a penalty cost imposed (usually for handling and management costs), and the returned products are recoverable, these merchandisers should plan for larger order amounts as such products do not affect the business. The contra-positive is also true, namely, merchandisers who allow for returns beyond a grace period and without any penalty charges, but where the returned products are irrecoverable, should manage their stocks in this category more judiciously by ordering as little as possible so as to limit the amount of returns.

Therefore, based on the above discussions, a mass merchandiser should adjust the order quantity according to the return policy implemented for specific product categories. In short, the merchandiser should not have a standard run-of-the-mill return policy for all product categories. It is best to tailor the return policy according to the nature of the products and their condition upon return. For instance, from the merchandise buying perspective and from the management control perspective, a merchandiser should increase the order quantity for products that are recoverable when customers are allowed to return unwanted items within a predetermined period of time unconditionally (i.e. Scenario S3 is practised). Likewise, the mass merchandiser should limit the order quantity for items that can be returned beyond a time window but with no penalty applied if the returned products are prone to deterioration with time (i.e. Scenario S6 is practised). In general, return policies that allow products to be returned beyond a time window should have higher order quantities to cater for such eventualities. The order quantity will be even higher if the return is penalised. For a given product category that has multiple return policies, it is suggested that the merchandise buyer calculate the optimal order quantities from the associated scenarios as mentioned thus far and select the largest order quantity to process the order.

As a corollary, there are also other strategic retail management implications when we consider the eight scenarios in their totality. First, as \( Q^*_1 \) dominates the other solutions, products which can be deemed as recoverable ought to be marketed more strongly with a satisfaction-guaranteed policy as there is a resale option for such products. Better coordination in terms of the logistics is also called for to ensure an integrated product returns model (see, e.g. Yalabik, Prtruzzi, and Chhajed 2005). Second, as \( Q^*_6 \) should be as small as possible, the decision maker in the mass merchandiser should consider curtailing or doing away with the luxury of a satisfaction-guaranteed policy for this product category as it drives a strong consumer oriented return mechanism which may not profit the merchandiser in the longer term. This is especially so for deteriorating product items, a treatment of which can be found in Goh (1992).
5. Illustrative example

The proposed models for the various scenarios of a mass merchandiser’s product categories with respect to its return guidelines are illustrated by the following example, taken from a personal communication with an anonymous manager of a mass merchandiser chain in Taiwan. The mass merchandiser chain operates six stores with a centralised ordering system while selling six categories of single-period products. It is assumed that the demand for each product category at each retail store is independent and identically distributed (IID), with its random demand variable following a normal distribution. Table 2 provides the means and standard deviations of a single-period demand for the six product categories in each store.

Table 2. Means and SDs of single-period demand.

<table>
<thead>
<tr>
<th>Product category</th>
<th>Produce/flower</th>
<th>Music/video</th>
<th>Electronics</th>
<th>Appliance</th>
<th>Clothing</th>
<th>Books</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>60,000</td>
<td>18,000</td>
<td>4800</td>
<td>6000</td>
<td>15,000</td>
<td>15,000</td>
</tr>
<tr>
<td>SD</td>
<td>1715</td>
<td>612</td>
<td>171</td>
<td>490</td>
<td>490</td>
<td>735</td>
</tr>
</tbody>
</table>

Other model parameters are deliberately chosen based on the characterisation of the specified product category. The average value of the parameters for each of the selected product parameters is given in Table 3.

For each of the five product categories, only specific scenarios are applied according to the mass merchandiser’s return guidelines. Table 4 summarizes the possible application of scenarios with respect to each product category.

Applying Equations (10)–(25), the optimal order quantities and associated expected profits can be computed and are summarised in Tables 5 and 6, respectively. It is observed that the order of the optimal order quantities incurred from the different scenarios is $Q_3 > Q_7 > Q_4 > Q_2 > Q_5 > Q_6$. The order of the optimal expected profits,
however, does not demonstrate a consistent pattern since the values of the parameters vary in the different product categories.

5.1. Sensitivity analysis

A sensitivity analysis is conducted to study the effect of varying the model parameters on the optimal order quantity for the mass merchandiser. In order to examine the effects of all the eight scenarios, the product category of electronics is used in the sensitivity analysis by varying a specific parameter and keeping the rest fixed. For product categories that may not be suitable for the application of all scenarios, the results of the sensitivity analysis are not applicable. Although a total number of 13 parameters is taken into consideration in the model, only those vital to the operational performance are analysed in this article.

Varying between ±50% of the average value of the demand, the effect of the return rate within a prescribed time window \( r_w \) is depicted in Figure 1. The monotone decreasing decline of the average value with the increase in the return rate concurs with

![Figure 1. Effect of \( r_w \) on optimal ordering quantity.](image)

Table 5. Optimal ordering quantities for various product categories.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Produce/flower</th>
<th>Music/video</th>
<th>Electronics</th>
<th>Appliance</th>
<th>Clothing</th>
<th>Books</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>60,749</td>
<td>18,596</td>
<td>4927</td>
<td>6144</td>
<td>15,560</td>
<td>15,626</td>
</tr>
<tr>
<td>S2</td>
<td>60,716</td>
<td>18,491</td>
<td>4917</td>
<td>6117</td>
<td>15,433</td>
<td>15,442</td>
</tr>
<tr>
<td>S3</td>
<td>–</td>
<td>18,611</td>
<td>4928</td>
<td>6147</td>
<td>–</td>
<td>15,640</td>
</tr>
<tr>
<td>S4</td>
<td>–</td>
<td>18,512</td>
<td>4918</td>
<td>6121</td>
<td>–</td>
<td>15,465</td>
</tr>
<tr>
<td>S5</td>
<td>–</td>
<td>–</td>
<td>4923</td>
<td>6138</td>
<td>15,542</td>
<td>–</td>
</tr>
<tr>
<td>S6</td>
<td>–</td>
<td>–</td>
<td>4906</td>
<td>6101</td>
<td>15,330</td>
<td>–</td>
</tr>
<tr>
<td>S7</td>
<td>–</td>
<td>18,591</td>
<td>4925</td>
<td>6141</td>
<td>–</td>
<td>15,623</td>
</tr>
<tr>
<td>S8</td>
<td>–</td>
<td>18,440</td>
<td>4908</td>
<td>6106</td>
<td>–</td>
<td>15,396</td>
</tr>
</tbody>
</table>

Table 6. Optimal expected profits for various product categories (in dollars).

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Produce/flower</th>
<th>Music/video</th>
<th>Electronics</th>
<th>Appliance</th>
<th>Clothing</th>
<th>Books</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>53,179</td>
<td>40,622</td>
<td>1,487,461</td>
<td>734,109</td>
<td>95,696</td>
<td>129,831</td>
</tr>
<tr>
<td>S2</td>
<td>55,453</td>
<td>50,190</td>
<td>1,638,636</td>
<td>880,011</td>
<td>126,002</td>
<td>280,039</td>
</tr>
<tr>
<td>S3</td>
<td>–</td>
<td>44,156</td>
<td>1,526,622</td>
<td>773,803</td>
<td>–</td>
<td>209,653</td>
</tr>
<tr>
<td>S4</td>
<td>–</td>
<td>29,325</td>
<td>1,270,155</td>
<td>553,538</td>
<td>–</td>
<td>104,971</td>
</tr>
<tr>
<td>S5</td>
<td>–</td>
<td>–</td>
<td>1,432,467</td>
<td>696,700</td>
<td>92,308</td>
<td>–</td>
</tr>
<tr>
<td>S6</td>
<td>–</td>
<td>–</td>
<td>1,032,360</td>
<td>399,246</td>
<td>34,727</td>
<td>–</td>
</tr>
<tr>
<td>S7</td>
<td>–</td>
<td>44,665</td>
<td>1,491,859</td>
<td>747,389</td>
<td>–</td>
<td>204,743</td>
</tr>
<tr>
<td>S8</td>
<td>–</td>
<td>23,843</td>
<td>1,097,078</td>
<td>459,604</td>
<td>–</td>
<td>76,163</td>
</tr>
</tbody>
</table>
Vlachos and Dekker (2003). The rank of the optimal order quantity among the eight scenarios remains unchanged. As for the effect of variations of $r_b$ shown in Figure 2, the result is similar to the descending linear effect of variations of $r_w$, with the exception of Scenarios S1–S4, which prohibit the return of items beyond the time window.

Figure 3 shows the effect of a markup (i.e. the price deducted by the purchasing cost) on the optimal order quantity. It appears that the optimal order quantity increases with the product category’s markup. However, the rate of increase diminishes and the differences among various scenarios reduce with an increase in the markup.

Figure 4 shows the effects of resale price variation on the optimal order quantities among the various scenarios. For S1, S3, S5 and S7, the resale price has a near-linearly positive effect on the optimal order quantity while S2, S4, S6 and S8 show no effect, since the returned items cannot be resold. In Figure 5, scenarios applying the return penalty (S3, S4, S7 and S8) present a slight increase as the penalty is hiked.

Cost-related parameters also affect the optimal order quantity. In Figures 6–9, $C_i$, $C_j$, $C_h$ and $C_r$ are observed to display a slightly descending effect on the optimal ordering quantity. As for the cost of lost sales $C_p$, shown in Figure 10, a near-linearly positive effect can be found on the optimal order quantity.

The effects of the model parameters on the order quantity can also be examined by studying the expression of $F(Q^*_i)$ in Equations (11), (13), (15), (17), (19), (21), (23) and (25). The return rates, $r_w$ and $r_b$, are the negative terms in both the numerator and denominator of these equations and hence display a descending effect on optimal order quantity. The markup, $P - C$, on the numerator of the fraction for all $F(Q^*_i)$, demonstrates an ascending effect with a diminishing rate of increase since the selling price $P$ is on the denominator.
The reselling price $R$, the positive terms in both the numerator and denominator, shows a positive effect on $Q_c^*$ for Scenarios 1, 3, 5 and 7. The return penalty $p_r$ and the cost of lost sales $C_l$ are both positively related to the optimal order quantity since they are the positive terms on both sides of the fraction. Other cost terms such as $C_i$, $C_j$, $C_h$, and $C_r$, however, have a negative effect on the order quantity since they are all positive terms on both the numerator and denominator.
Figure 7. Effect of inventory cost for returned items on optimal ordering quantity.

Figure 8. Effect of handling cost on optimal ordering quantity.

Figure 9. Effect of recovery cost on optimal ordering quantity.
6. Conclusion

This article considers the complex cost structure and various return policies for a mass merchandiser making ordering decisions for a number of short-lived product categories. Models are developed and analysed in order to facilitate effective decision-making for managing the order process when certain return guidelines are applied. These models are solved by maximising the expected profit in order to obtain closed form solutions for the optimal order quantity.

The results show that various return policies will result in different optimal order quantities for the products. Hence, the buyer or manager of a mass merchandiser must carefully review the various return policies that can be applied to a specific product category before a pre-season order is made. The largest among the various optimal order quantities obtained from the proposed mathematical models will be ordered for that product category. Practically, the merchandiser needs to enhance the order quantity for products that are recoverable when the unsatisfied products are allowed to return within a period of time unconditionally. The order quantity should be limited when the return of irrecoverable items beyond a time window with no penalty is practised.

Determining a suitable scenario to apply on a product category relies on the product category’s characteristics, inventory constraints, as well as the applicable return guidelines for it. For perishable products such as produce, delicatessen and fresh flowers, Scenarios S1 or S2, which allow return within a very short period of time despite the product condition may be applied. Books, music CDs, DVDs, computer software and video games and other software-oriented items are likely to suit Scenarios S3 or S7 when handling return orders. Fashion apparel and accessories are frequently guaranteed returns with a full refund and hence can be exercised by Scenarios S1, S2, S5 and S6.

Future research may incorporate multiple period return policies and a dynamic cost structure for a particular product category or class (Smith, Limón, and Cárdenas-Barrón 2009). Further, the integration of ordering decisions with upstream suppliers for multiple product categories may be studied so that the expected profit along the supply chain can be optimised due to better joint order optimisation and returns management (see, e.g. Smith, Martínez-Flores, and Cárdenas-Barrón 2007).

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References


Appendix 1

Proposition 1: The expected profit for Scenario 1 can be maximised by Equation (10) since its second derivative with respect to \( Q \) is negative.

Proof: The first derivative of \( \pi_1(Q) \) with respect to \( Q \) is given by:

\[
\frac{d\pi_1(Q)}{dQ} = P - C + C_l - r_s(P - R + C_j + C_h + C_l)
\]

\[-[P - S + C_l + C_l - r_s(P - R + C_j + C_h + C_l)]f(Q)\]

The second derivative of \( \pi_1(Q) \) with respect to \( Q \) can be computed by:

\[
\frac{d^2\pi_1(Q)}{dQ^2} = -[P - S + C_l + C_l - r_s(P - R + C_j + C_h + C_l)]f(Q)
\]

\[-r_s(P - R) + (C_l + C_l)\]
In practice, $S < R$. Therefore, $P - S > P - R > r_u(P - R)$. It is also practical to conclude that the cost of lost sales $C_l$ is significantly higher than $C_j$, $C_h$, and $C_r$. Hence, the following relation holds given that $r_u$ is less than 1:

$$C_l > C_j + C_h = C_r > r_u(C_j + C_h + C_r).$$

Therefore, $d^2\pi_1(Q)/dQ^2 < 0$ and $\pi_1(Q)$ is maximised at the optimal order quantity $Q^*_1$.

### Appendix 2

**Proposition 2:** The expected profit for Scenario 2 can be maximised by Equation (12) since its second derivative with respect to $Q$ is negative.

**Proof:** The first derivative of $\pi_2(Q)$ with respect to $Q$ is

$$\frac{d\pi_2(Q)}{dQ} = P - C - C_l - r_u(P + C_j + C_h)$$

$$- (P - S + C_i + C_l - r_u(P + C_i + C_j))f(Q).$$

The second derivative of $\pi_2(Q)$ with respect to $Q$ can be computed by

$$\frac{d^2\pi_2(Q)}{dQ^2} = -(P - S + C_i + C_l - r_u(P + C_i + C_j))f(Q).$$

In practice, $S$ can be neglected since the irrecoverable and perishable food and farm products have little or no salvage value. Therefore, $(1 - r_u)P > S$. It is also practical to conclude that the cost of lost sales $C_l$ is significantly higher than $C_j$ and $C_h$. Hence the following relation holds as $r_u$ is less than 1:

$$C_l > C_j + C_h = r_u(C_j + C_h).$$

Therefore, $d^2\pi_2(Q)/dQ^2 < 0$ and $\pi_2(Q)$ is maximised at the optimal order quantity $Q^*_2$.

### Appendix 3

**Proposition 3:** The expected profit for Scenario 3 can be maximised by Equation (14) since its second derivative with respect to $Q$ is negative.

**Proof:** The first derivative of $\pi_3(Q)$ with respect to $Q$ is

$$\frac{d\pi_3(Q)}{dQ} = P - C - C_l - r_u(P - R - P_r + C_j + C_h + C_r)$$

$$- (P - S + C_i + C_l - r_u(P - R - P_r + C_i + C_j + C_h))f(Q).$$

The second derivative of $\pi_3(Q)$ with respect to $Q$ can be computed by

$$\frac{d^2\pi_3(Q)}{dQ^2} = -(P - S + C_i + C_l - r_u(P - R - P_r + C_i + C_j + C_h))f(Q).$$

In practice, $S < R$. Therefore, $P - S > P - R > r_u(P - R)$. It is also practical to conclude that the cost of lost sales $C_l$ is significantly higher than $C_j$, $C_h$ and $C_r$. Hence, the following relation holds given that $r_u$ is less than 1:

$$C_l > C_j + C_h + C_r > r_u(C_j + C_h + C_r).$$

Therefore, $d^2\pi_3(Q)/dQ^2 < 0$ and $\pi_3(Q)$ is maximised at the optimal order quantity $Q^*_3$.

### Appendix 4

**Proposition 4:** The expected profit for Scenario 4 can be maximised by Equation (16) since its second derivative with respect to $Q$ is negative.

**Proof:** The first derivative of $\pi_4(Q)$ with respect to $Q$ is

$$\frac{d\pi_4(Q)}{dQ} = P - C - C_l - r_u(P - P_r + C_j + C_h)$$

$$- (P - S + C_i + C_l - r_u(P - P_r + C_i + C_j + C_h))f(Q).$$

The second derivative of $\pi_4(Q)$ with respect to $Q$ can be computed by

$$\frac{d^2\pi_4(Q)}{dQ^2} = -(P - S + C_i + C_l - r_u(P - P_r + C_i + C_j + C_h))f(Q).$$

In practice, $S$ has little or no value since the returned products can only be sold as used or second-hand goods. Thus, $(1 - r_u)P > S$. Also, the cost of lost sales $C_l$ is significantly higher than $C_j$ and $C_h$. Hence, the following relation holds given that $r_u$ is less than 1:

$$C_l > C_j + C_h > r_u(C_j + C_h).$$

Therefore, $d^2\pi_4(Q)/dQ^2 < 0$ and $\pi_4(Q)$ is maximised at the optimal order quantity $Q^*_4$.

### Appendix 5

**Proposition 5:** The expected profit for Scenario 5 can be maximised by Equation (18) since its second derivative with respect to $Q$ is negative.

**Proof:** The first derivative of $\pi_5(Q)$ with respect to $Q$ is

$$\frac{d\pi_5(Q)}{dQ} = P - C - C_l - (r_u + r_h)(P - R - C_j + C_h + C_r)$$

$$- (P - S + C_i + C_l - (r_u + r_h)(P - R - C_i + C_j + C_h + C_r))f(Q).$$

The second derivative of $\pi_5(Q)$ with respect to $Q$ can be computed by

$$\frac{d^2\pi_5(Q)}{dQ^2} = -(P - S + C_i + C_l - (r_u + r_h)(P - R - C_i + C_j + C_h + C_r))f(Q).$$

In practice, $S < R$. Therefore, $P - S > P - R > (r_u + r_h)(P - R)$. It is also practical to conclude that the
The first derivative of \( \pi_7(Q) \) with respect to \( Q \) can be computed by
\[
\frac{d^2\pi_7(Q)}{dQ^2} = -\frac{P - S + C_i + C_l - (r_w + r_b)}{(P - R - p_r + C_j + C_h + C_r)}f(Q)
\]

Therefore, \( \frac{d^2\pi_7(Q)}{dQ^2} < 0 \) and \( \pi_7(Q) \) is maximised at the optimal order quantity \( Q^*_7 \).

Appendix 7

**Proposition 7:** The expected profit for Scenario 7 can be maximised by Equation (22) since its second derivative with respect to \( Q \) is negative.

**Proof:** The first derivative of \( \pi_7(Q) \) with respect to \( Q \) is
\[
\frac{d\pi_7(Q)}{dQ} = P - C + C_l - (r_w + r_b)(P + C_j + C_h)
\]

Therefore, \( \frac{d\pi_7(Q)}{dQ} < 0 \) and \( \pi_7(Q) \) is maximised at the optimal order quantity \( Q^*_7 \).

Appendix 8

**Proposition 8:** The expected profit for Scenario 8, \( \pi_8(Q) \), can be maximised by Equation (24) since its second derivative with respect to \( Q \) is negative.

**Proof:** The first derivative of \( \pi_8(Q) \) with respect to \( Q \) is
\[
\frac{d\pi_8(Q)}{dQ} = P - C + C_l - (r_w + r_b)(P - p_r + C_j + C_h)
\]

Therefore, \( \frac{d\pi_8(Q)}{dQ} < 0 \) and \( \pi_8(Q) \) is maximised at the optimal order quantity \( Q^*_8 \).