Abstract

In this paper we propose a method for image analysis, processing and coding, based on physical computation of signal distortion. A binary tree data structure of coupled system of data sets was initially proposed in [9, 10], derived from the statistical physics model of free energy. We assess the scale invariance, in the method, by hierarchically clustering data. Theoretical model of error propagation is given in such a computational scheme. This decomposition of image information is analyzed by multifractal model formalism. We study how it correlates with the convective structure in clouds, that is associated with rain. The results are shown for MeteoSat IR images, provided by Thalweg ARC project.

The regularity constraints of data are used in the hierarchical scale decomposition of images. Accordingly, the reconstruction formula is derived based on the Laplacian system of diffusion of the residual information from the most singular sets. This gives us an effective way of compressing and progressive coding of information in image sequences. The proposed algorithm, also, is suitable for the implementation in parallel computer architectures.

1. Introduction

Information theoretical approach to statistical mechanics was introduced in the work of Jaynes [8], and the principle of maximum entropy was proposed as an inference procedure. Clustering techniques are applied in many problems (like pattern recognition, learning, source coding, image and signal processing [7, 9, 10, 11, 12, 14]) where a priori knowledge about the distribution of the data is not available. Clustering of data is widely used tool for analyzing multidimensional data in diverse disciplines such as engineering, biology, social science, and astronomy.

Various approaches to the probabilistic and fuzzy inference in clustering are presented in literature [7, 14]. The computation of the clustering parameters becomes more effective on the adaptively selected, local windows of computation in our clustering algorithm, as compared to the other techniques. We treat the propagation of signal distortion in such an approach globally, in space and in scale, that results in clustering spatial and temporal features in image data as distinct singular sets. The hierarchical decomposition of image data enables progressive coding of image sequences, as well as, the implementation of the algorithm in parallel computer architectures.

A multifractal model formalism is derived during my posdoc stage at “Thalweg ARC.” project, to explain the decomposition of image sequences into the singular data sets. The partition function describes the probabilistic model of data clusters and is analyzed as a multifractal measure in the method. Singularity analysis of computational maps of clustering vectors is derived to describe the computational means of decomposing the image information into different singular sets. We show also that the propagation of information in image sequences is governed by the scale-space wave equation, therefore enabling us to treat singular frequencies of data clusters in an unified way, both in space and in time.

Contextual information of the spatial coherency of data is used in the segmentation process in the hierarchical scale computation of feature vectors. The spatial segmentation of images is performed while using the Green’s function, parameterized with the scale parameter, as the integration function in the segmentation process. The scale information is evaluated by conjoining the two parameters: the scale parameter $\beta$ of the signal distortion, and the spatial scale parameter $r$. A larger extent of spatial integration of the motion information is used on a larger scale, while it becomes effectively more local in space as we decrease the scale of segmentation.

Distinct singular features are segmented on a certain scale and the least singular feature become segmented in two spatial windows with the Laplacian system regularity constraints, in the hierarchical scale computation. Accordingly, the reconstruction formula is derived based on the
Laplacian system of the diffusion of the residual information from the most singular sets. This gives us an effective way of compressing and progressive coding of the information in image sequences. The binary tree data structure of the clustering parameters is suitable in the coding schemes that use the hierarchical structure of the binary images of the spatial distribution of cluster windows, along with the feature vectors and residual image information that make up for the point feature vector estimation.

Image motion has been an important problem in computer vision and image analysis since the results can be applied in the analysis of scene segmentation, coding, shape recovery, target tracking, or as a module in parallel algorithms for the recovery of the information about scene, integrating several visual cues [6, 9, 13, 15]. Accurate computation of displacement parameters from one image frame to the next is, however, a very difficult task. We use the cluster windows, in the algorithm, to address the question of the appropriate integration of the information.

The multiscale approach in the estimation of the motion vectors in image sequences enables us, also, to solve the so-called aperture problem. Sufficient texture content is required in order to robustly compute motion vector for a window of pixels. Two questions have to be answered before we select these cluster windows: what is the criteria of having sufficient texture for a window of pixels, and what size of the window we choose. To answer these questions we start off by intuitive reasoning of what constitute good image invariants to be traced from one frame to the next with a reasonable accuracy. Due to the aperture problem we know that we can not trace the window of pixels with the uniform intensity, while if we have a straight edge, we can only determine the motion component orthogonal to that edge. On the other hand if a window contains strong surface markings, like a corner for example, we can uniquely find the displacement parameters for that window and therefore solve the aperture problem. However, we still have to address the question of the first-order deformation of intensity due to the noise in the system, resulting in shortening/dilation in the image points sampling introduced by the rotation of camera. An appropriate window size is required to suffice for the estimation of introduced parameters.

Tomasi and Kanade [15], in their algorithm, used a fixed size windows to estimate motion vectors. We base our algorithm on adjustable size windows to suffice for the estimation of the cluster vectors. In our approach a limit constraint on the robustness of the estimate of cluster vectors is used across the selected cluster windows, that cover the whole image space.

The method of singular features decomposition of image sequences by hierarchical clustering is derived in Section 2. The results of the application of the algorithm are shown in Section 3. The scale invariance of the feature vectors mapping is evaluated for the still images in space, and for a longer image sequences, both in space and in time. The results are discussed in Section 4. Concluding remarks are given in Section 5.

2. Method

2.1. Maximum entropy inference

The principle of maximum entropy inference states the following: among all the probability density functions (pdf), that satisfy a given set of constraints, choose that that maximizes the entropy. The chosen pdf is agreeable with all the knowledge available (a priori knowledge, or that obtained by the estimation), and at the same time keeps the maximal uncertainty towards anything else (a posteriori knowledge, or the future results of the estimation). This, also, means that such a chosen pdf is maximally unbiased toward any future solution that includes the future knowledge obtained about the problem. Any other pdf is biased toward some of the possible solutions.

We define a cluster here with its computed cluster vector representative \( \mathbf{y} \), and the selected cluster window of computation, \( W \). Let \( d(\mathbf{x}, \mathbf{y}) \) denotes a distortion measure introduced to a data point \( \mathbf{x} \) by the representation \( \mathbf{y} \). The distortion energy, or variance \( V \) of a cluster is defined by:

\[
V = \int_W d(\mathbf{x}, \mathbf{y})P(\mathbf{x})
\]

It can be shown [8] that the probability density function that maximizes the entropy: \( \max \{ -\int_W P(\mathbf{x}) \log P(\mathbf{x}) \} \) subject to: \( V = \int_W d(\mathbf{x}, \mathbf{y})P(\mathbf{x}) \), and \( \int_W P(\mathbf{x}) = 1 \), is the Gibbs distribution:

\[
P(\mathbf{x}) = \frac{e^{-\beta d(\mathbf{x, y})}}{Z} = \frac{e^{-\beta d(\mathbf{x,y})}}{\int_W e^{-\beta d(\mathbf{x,y})}}
\]

where \( Z \) is the partition function, and \( \beta \) is a Lagrange multiplier.

2.2. Clustering motion information in image sequences

If we limit our attention to an image sequence of small inter-frame displacements we derive first the necessary condition for computing the common motion vector for a window of pixels. Then, we shall describe the derivation of the algorithm for space-motion quantization of image sequences.

Let \( I(\mathbf{x}, t) \) denote the image brightness of some scene point \( \mathbf{x} = (x, y) \) at the time \( t \). Also, let assume that the scene point \( (x, y) \) projects onto a new point \( (x + dx, y + dy) \) at the time \( (t + dt) \). The brightness change model for computing the small image point displacements, based on the gradient method [5], can be written as:

\[
I(\mathbf{x}, t + \delta t) = I(\mathbf{x}, t) - \mathbf{gd},
\]
and for a feature point corresponding to a window of pixels \( W \), we can write,
\[
\varepsilon = \int_W (I(\vec{x}, t) - g d - I(\vec{x}, t+dt))^2 \omega \, d\vec{x},
\]
or in a more compact form,
\[
\varepsilon = \int_W (h - g \vec{d})^2 \omega \, d\vec{x}, \tag{1}
\]
where \( h = I(\vec{x}, t) - I(\vec{x}, t+dt), \) \( g = [\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}] \), \( \vec{d} = [dx, dy]^T \), and \( \omega \) is a weighting factor. For a unique time frame period of image sampling we can conveniently substitute displacement vector with an image motion vector \( \vec{v} = [u, v]^T \), where \( u \) and \( v \) correspond to the \( x \) and \( y \) components of the motion vector, respectively.

The residue \( \varepsilon \) in the equation 1 can be locally minimized by setting the result of differentiation equal to zero:
\[
0 = \int_W (h - g \vec{v}) \omega \, d\vec{x}.
\]
By assuming \( \vec{v} \) to be constant within the selected window \( W \), we have,
\[
(\int_W g g^T \omega \, dA) \vec{v} = \int_W h g \omega \, dA.
\]
Or, equivalently, it can be written as a \( 2 \times 2 \) matrix equation:
\[
G \vec{v} = \vec{f}, \tag{2}
\]
where \( G = \int_W g g^T \omega \, dA, \) and \( \vec{f} = \int_W h g \omega \, dA. \)

As we can see, in order to solve the equation 2, the matrix \( G \) must be nonsingular. Also, we want to select a robust feature in scale, particularly if it is to be tracked through the sequence of images. We propose an algorithm for the selection of space-motion information features across the hierarchy of scales, as described below.

### 2.3. Hierarchical scale decomposition

The nonlinear dynamics of clustering, in this work is derived from the model of “free energy”, originally used in statistical physics to model different complex systems. In this section we shall describe the mathematical model of clustering the motion information, as well as the points of discontinuities, when new clusters emerge from the existing clusters, described by the cooling and melting procedures.

The free energy describes the state of a cluster for a given parameter \( \beta \),
\[
F(\beta) = -\frac{1}{\beta} \log \rho Z, \tag{3}
\]
The parameter \( \beta \) is inversely proportional to temperature \( (\beta = 1/T) \), in physical analogy. At the equilibrium, the cluster settles in the state that minimizes its free energy.

The distortion measure, applied in the algorithm, is chosen to be the constraint equation on the motion vector \( \vec{v} \), also known as the extended optical flow constraint equation:
\[
d = \varepsilon^2 = (I_t + \nabla I \vec{v} + I d v(\vec{v}))^2, \tag{4}
\]
which provides the mass conservation principle [1]. In this work the coherency of data is estimated with its Green’s function, to control the smoothness of the optical flow in the computational scheme, adaptively in scale.

The largest scale computation is performed on the whole window \( W \), starting from initial point \( (\vec{v}, \beta) = (0^+, 0^+) \), and the initial equation of motion:
\[
\dot{\vec{v}} = -\frac{\partial F}{\partial \vec{v}} = \int_W 2\varepsilon \frac{\partial^2 F}{\partial \vec{v}^2} \vec{p}, \tag{5}
\]
\[
\beta = +\frac{\partial F}{\partial \beta} > 0. \tag{6}
\]

The nonlinear map in 3 exhibits no chaotic behavior [17]. This gives a fixed-point iteration:
\[
\vec{v} = \vec{v} + \frac{\int_W \varepsilon \frac{\partial^2 F}{\partial \vec{v}^2} \vec{p}}{\int_W \frac{\partial^2 F}{\partial \vec{v}^2}} = g(\vec{v}, \beta). \tag{7}
\]

For a given parameter \( \beta \), this map is stable if the Hessian of the free energy, \( \frac{\partial^2 F}{\partial \vec{v}^2} \), is positive definite. The system of equation 3 can be adaptively “cooled” (increasing \( \beta \)) up to the point when it becomes unstable.

The point in scale \( \beta_o \), when some nonconvex component becomes dominant in the estimation process indicates the point of instability of the map. This is the point of discontinuity in the algorithm, which is followed with the procedure of phase transition - splitting of the cluster.

The cooling procedure is defined by the equation:
\[
\beta' = \frac{\partial F}{\partial \beta} > 0. \tag{8}
\]
At the equilibrium point,
\[
\frac{\partial F}{\partial \vec{v}} = 0,
\]
if the Hessian of the free energy is positive definite, we compute:
\[
\delta \vec{v} = -\frac{\partial V}{\partial \vec{v}}, \tag{9}
\]
and update,
\[
\Delta \beta = \sqrt{\frac{1}{2} \delta \vec{v} \frac{\partial^2 F}{\partial \vec{v}^2} \delta \vec{v}} > 0. \tag{10}
\]
Note that this way we keep the integral:
\[
\int_S dU = \int_S \frac{\partial F}{\partial \beta} d\vec{v} + \frac{\partial V}{\partial \vec{v}} d\beta = 0, \tag{11}
\]
and,
\[ \Delta F|_{\phi=const.} = \Delta F|_{\beta=const.} \]
if we neglect the higher order terms. The same potential levels difference the equilibrium point moves away by the change of the parameter $\beta$ (6), as with the change of the direction of computation (5), what is expressed by the equation of continuation (7). These equations enables the equilibrium point to escape potential barrier in the free energy landscape, such that the local minimum is avoided, as it is shown on figure 1.

If the Hessian of the free energy is negative definite for some of the clusters, at the critical value of the scale parameter $\beta_c$, the condition of phase transition is reached and that cluster is splitted along the principal component vector corresponding to the maximal singular value of the scatter matrix:

\[ 1 - \frac{\partial^2 F}{\partial \bar{\xi}^2}, \]

according to:

\[ R_1(\bar{x}) = \int G(\bar{x} - \bar{y})[e_x \ e_y][E_x \ E_y]^T \]

\[ = \begin{cases} > 0 & \bar{x} \in W_1, \\ < 0 & \bar{x} \in W_2. \end{cases} \]  

$R_1(\bar{x})$ is the point residual motion information of the projections of the distortion vectors along the principal component vector, $[E_x \ E_y]$, corresponding to the maximal singular value of the scatter matrix. From equation 4 we write the point distortion vector as:

\[ [e_x \ e_y] = z [\frac{I_x}{(I_x^2)_{avg.}} \quad \frac{I_y}{(I_y^2)_{avg.}}]. \]

The integration is obtained by summing up the projections of the distortion vectors multiplied by the Green’s function $G(\bar{x} - \bar{y})$ in the equation 8. The Gaussian function is used in this work $G(\bar{x} - \bar{y}) = e^{-\beta|\bar{x} - \bar{y}|^2}$. The parameter $\beta$ here plays the role of the spatial extent of integration. On a lower value of the scale parameter $\beta$, the cluster windows are formed by using a larger extent of the spatial integration of the projections of distortion vectors. As we gradually increase $\beta$, the integration becomes effectively more local in space.

Let’s analyze now the entries of the scatter matrix of the map, as written in equation 4. After few lines of derivation we have,

\[ \frac{\partial g_u}{\partial u} = 2\beta \int_W z^2 I_x^2 d^2 x \quad \frac{\partial g_v}{\partial v} = 2\beta \int_W z^2 I_y^2 d^2 x \quad \frac{\partial g_w}{\partial w} = 2\beta \int_W z^2 I_z^2 d^2 x. \]

And by the symmetry of the partial derivatives,

\[ \frac{\partial g_u}{\partial u} = \frac{\partial g_u}{\partial v} = \frac{\partial g_v}{\partial v} = \frac{\partial g_u}{\partial w}, \]

We can further simplify the notation by deriving the normalized sensitivity coefficients. We have,

\[ S_u = \frac{\partial z}{\partial u} = I_x, \quad S_v = \frac{\partial z}{\partial v} = I_y, \]

and we write,

\[ S_{u,u} = \int_W z^2 (S_u^2) d^2 x, \quad S_{u,v} = \int_W z^2 (S_u S_v) d^2 x, \quad S_{v,v} = \int_W z^2 (S_v^2) d^2 x. \]

And by the symmetry,

\[ S_{u,v} = S_{v,u}. \]

We can now write the scatter matrix in the form of the normalized sensitivity coefficients as:

\[ \mathbf{S} = 2\beta \begin{bmatrix} S_{u,u} & S_{u,v} \\ S_{v,u} & S_{v,v} \end{bmatrix}. \]

### 2.4. The multifractal model formalism

Singularity of data clusters is evaluated by the means of the scalability of the maps of feature vector representatives in scale-space. For a given maximal value of the distortion energy the minimal number of singularity manifolds is obtained in the hierarchical scale computation.

For a given cluster the partition function $Z(\beta, \tau)$ describes the distribution of the data points with respect to the cluster vector representative. The scale information is evaluated by conjoining the two parameters: the signal energy distortion scale parameter $\beta$, and the spatial scale parameter $\tau$, which equals to the number of data points inside the spatial window of computation $W$. For a given point distortion measure of the signal, $z^2 = d(x, y)$, the partition function is written by:

\[ Z(\beta, \tau) = \int_W \tau^{-\beta z^2}. \]
and a data point belongs to the cluster in probability, with the probability density function:

\[ P = \frac{r^{-\beta_2}}{Z}. \]

The partition function can be conveniently written as:

\[ Z = r^{-2H} = r^{1-\beta V}. \]

This function is a multifractal measure, giving a way of decomposing the signal into feature vector clusters ordered by the singularity exponents, \( H = 2\beta V < 1 \), and singular frequency factors, \( F/V < 1 \), as written in:

\[ Z = r^{-\frac{1}{2}}H \left( \frac{2}{\beta} \right). \]

The nonlinear dynamics of clustering is governed by the two energy functions. For a given cluster its free energy, and the distortion energy is defined by:

\[ F(\beta, \gamma) = -\frac{1}{\beta} \log r Z, \quad V = \int_W d(x, y)P. \]

We relate the mechanism of the multifractal decomposition of the signal to the stability analysis of the map, as written in the equation 3. The stability condition of this map is given by the relation: \( 2\beta V < 1 \). We limit the singularity exponent of the clusters with the maximal value \( H \), by splitting that cluster in two for which the condition is reached: \( 2\beta V = H \), at the critical value of the scale parameter \( \beta_c \).

The cooling and melting procedures describe an adaptive multiscale method for processing of multidimensional data. A binary tree of splitting clusters gives a representation of data in the hierarchy of scales. On figure 2 is shown a tree structure with 5 clusters, corresponding to the leaves of the tree. The cluster with the group window equal to the whole image frame, \( W_0 \), corresponds to the root of the tree. At the critical value of the scale parameter \( \beta_c^0 \), the computed cluster vector representative is \( y^0 \). The root cluster is splitted in two at the critical value of the scale parameter \( \beta_c^0 \). The structure of the tree is formed at the increasing values of the scale parameter \( \beta_1^0 < \beta_2^0 < \beta_3^0 < \beta_4^0 \), splitting one of the clusters, that reaches the critical value of computation, in two. The depth of the tree and the number of the clusters, corresponding to the leaves of the tree, are determined by the error value of the coding scheme.

The estimation of the code vectors for every node of the tree is obtained with separately defined maps and on the selected group windows of computation, what makes this algorithm suitable for the implementation in parallel computer architectures. This data structure enables, also, the coding of the spatial distribution of cluster windows to be carried out by the hierarchy of binary images.

Figure 2: The binary tree structure of the distribution of clusters.

### 2.5. Temporal aspects

For a longer sequence of images we propose an alternative scheme, by considering the splitted clusters with the constrained equations of motion:

\[ \dot{\beta}_1 = -\frac{\partial F_1}{\partial \beta_1} \pm \beta \frac{\partial F_1}{\partial \beta_2}, \]
\[ \dot{\beta}_2 = -\frac{\partial F_2}{\partial \beta_1} \pm \beta \frac{\partial F_2}{\partial \beta_2}, \]
\[ \beta' = \pm \frac{\partial F}{\partial \theta}. \]

where the plus sign corresponds to the cooling part and minus to the melting part of the algorithm. This system of equations can be analysed by the series expansion of the system’ free energies:

\[ \Delta F = \left[ \Delta_1 \quad \Delta_2 \right] \left[ F_1 \mp \beta F_2 \right] \]
\[ = \left( \frac{\partial^2 F_1}{\partial \beta_1^2} \delta \beta_1^2 + \frac{\partial^2 F_2}{\partial \beta_2^2} \delta \beta_2^2 \right) \]
\[ + \frac{1}{2} \left( \delta \beta_1 \delta \beta_2 \right)^2 \left[ \frac{\partial^2 F_1}{\partial \beta_1^2} \mp \beta \frac{\partial^2 F_2}{\partial \beta_1^2} \right]. \]

This gives an update formula for the parameter \( \beta \):

\[ \Delta \beta = + \sqrt{\frac{\partial F_1}{\partial \beta_1} \delta \beta_1 T - \beta \frac{\partial F_1}{\partial \beta_2} \delta \beta_2 T} \]
\[ = + \sqrt{\frac{\partial F_1}{\partial \beta_1} \delta \beta_1 T - \beta \frac{\partial F_1}{\partial \beta_2} \delta \beta_2 T} \]
\[ = - \sqrt{\frac{\partial F_2}{\partial \beta_1} \delta \beta_1 T + \beta \frac{\partial F_2}{\partial \beta_2} \delta \beta_2 T} \]
\[ = - \sqrt{\frac{\partial F_2}{\partial \beta_1} \delta \beta_1 T + \beta \frac{\partial F_2}{\partial \beta_2} \delta \beta_2 T} \]
The above system of equations results in the determinant of the map:

\[ D = \mathbf{A}^2 + \left( \frac{\partial^2 F_1}{\partial \theta_1^2} + \frac{\partial^2 F_2}{\partial \theta_2^2} \right) \mathbf{A} + \left( 1 - \beta^2 \right) \frac{\partial^2 F_1}{\partial \theta_1^2} \frac{\partial^2 F_2}{\partial \theta_2^2}. \]

For \( 0 \leq \beta < 1 \) the eigenvalues of the determinant of the map have negative values if the Hessians of the free energies, \( F_1 \) and \( F_2 \), are positive definite, giving the conditions of the numerical stability of the coupled system of equations.

The use of the constrained equations of motion 2.5 can be seen as to enhance the cluster independence in the cooling part, while it, also, enhances melting of the similar motion components of the splitted clusters, in the melting part of the algorithm.

Interesting points of observation become the “signal energy levels”:

\[ F - V = const., \quad (A.1) \]

and its propagation in time. The time derivative of this equation becomes:

\[ \left[ (\frac{E}{V} - 1) \frac{\partial}{\partial \theta} \log V + \frac{\partial}{\partial \theta} \left( \frac{E}{V} \right) \right] \dot{\beta} = - \left[ (\frac{E}{V} - 1) \frac{\partial}{\partial \theta} \log V + \frac{\partial}{\partial \theta} \left( \frac{E}{V} \right) \right] \dot{\theta} \quad (A.2) \]

The Green’s formula:

\[ \int_S \frac{\partial^2 f}{\partial \theta^2} d\theta + \frac{\partial V}{\partial \theta} d\beta = 0, \]

as well as, the accompanying wave equation:

\[ \frac{\partial^2 F}{\partial \beta^2} = \nabla^2 V, \]

are used in the computational scheme for the estimation of the clustering parameters, and are still part of my ongoing research.

At the “signal scale equilibrium” \( \frac{\partial E}{\partial \theta} = 0 \), an isolated cluster can be modeled by the equations:

\[ \frac{\partial V}{\partial \beta} = \beta \nabla^2 V, \]

and,

\[ \int_S \frac{\partial V}{\partial \theta} d\beta = \int_S \frac{||\nabla V||^2}{\beta \nabla^2 V} d\theta = 0, \]

The Green’s function gives a model of spatial coherency of information for a data cluster:

\[ G(\beta, \theta) = \frac{||\nabla V||^2}{\beta \nabla^2 V}, \]

and is applied locally in the segmentation process.
3. Results

Gray scale sequences of images of small inter frame displacements are used to show the singularity decomposition of image sequences, for the cooling part of the algorithm, only. We shall explain how the selection of spatial windows along with the motion information in the space-motion quantization algorithm depend on the spatial content of image gradients \((I_x, I_y)\), in an image, and on the robustness of the estimate of the quantum image motion signal \((u, v)\).

Test image sequence is designed with the test image pattern \(I_1(x, y) = \sin(\pi x) \cdot \sin(\pi y), (0 \leq x, y \leq 1)\). This gives a 2D half periode originating in the upper left corner and having the maximum brightness in the middle of the image, as shown on figure 3 (a). This test pattern is expanded radially, and deformed toward lower right corner with radially graded motion originating in the upper left corner. The disparity vectors \(d_1 = E[2x - 1 \cdot 2y - 1] \) and \(d_2 = D[x \cdot y]\), expand and deform the test pattern in the first image into the image pattern \(I_2(x, y) = \sin(\pi(x - d_{x1} - d_{x2})) \cdot \sin(\pi(y - d_{y1} - d_{y2})\).

The computation of image motion vectors starts with the value of the scale parameter \(\beta = 0\), computing the average displacement parameter \((u^0, v^0)\) of the whole image frame \((W^0)\) - the root window, in figure 2. The value of the scale parameter is monotonically increased up to the point when the process reaches the critical value of the scale parameter \(\beta^c\), and the point of the formation of new group windows of computation, by using the discriminant function, given in equation 8. We continue next with the convex minimization of the free energy and the estimation of the group vectors \(y^1\) and \(y^2\) on separately defined maps, as in equation 3, on the selected group windows of computation, \(W^1\) and \(W^2\).

The hierarchical clustering algorithm is applied here for the formation of the tree structure of the distribution of clusters. The gray level values used for labeling of the group windows, indicate the singularity level of the corresponding cluster of data points. The gray scale code is used uniformly to label more singular clusters with the brighter value. The residual motion information along the major and complement principal components, \(R_1(\mathbf{x})\) and \(R_2(\mathbf{x})\), are shown with the bluish shade corresponding to the positive, and the reddish to the negative values of the functions. The results in figure 3 are shown for up to 8 clusters computed to describe the dependencies of the feature vectors on the complex image motion pattern in the test image and the image gradients distribution in the image.

The “vortex” sequence of 6 images is used to evaluate the spatial distribution of the singular sets in scale, both in spectral and time domain. For a still image decomposition the distortion measure \(d = |\mathbf{I} - \mathbf{e}|^2\) is applied as in [12]. The 5th image in sequence is shown in figure 4 (a), for which the spectral singular sets are shown in 4 (b). The distribution of the singular sets for the motion information is shown in 4 (c), evaluating the scale behavior of this image sequence in the time domain. A distortion measure \(d = |\mathbf{I} - \mathbf{c}|^2 + \lambda(\mathbf{I}_1 - \mathbf{I}_2 - \mathbf{I}_3 \cdot u - \mathbf{I}_4 \cdot v)^2\) is used in mapping the joint motion and spectral information feature vectors. The joint motion and spectral information singular sets of the image sequence are shown in figure 4 (d).

The Meteosat infrared image displays the IR data acquisition where the gray level intensities correspond to cloud temperature. We decompose the images into the scale singular feature sets and study its relation to the pluimetry. The image in figure 5 (a) is decomposed into the singular sets with up to the 4 data clusters computed. The distribution of the temperature fronts is gray scale coded in accordance with spectral scalability data clusters. The resulting images is shown in figure 5 (b-f).

4. Discussion

In the method of hierarchical decomposition of image data, developed in this work, convex optimization of the free energy of data clusters is achieved by the adaptation in the cooling schedule and with the adaptive selection of spatial windows of computation. The cooling process is implemented to evaluate the scale invariance of the group vector representative for a cluster of data points. In the process of phase transition, the least singular cluster of data points, for which the condition of phase transition is reached, is divided in two newly created windows, according to the rule given in equation 8. This way we achieve a minimax optimization of the error function. For every value of the scale parameter, a minimal number of the feature vector data clusters is obtained for a given limit constraint of the maximal scaling exponent \(H\), in the data clusters decomposition.

The symmetry of the space-motion features can be observed for the first 4 clusters computed, as shown on figure 3. The cluster windows are splitted along the diagonal line in figures 3 (b and e), since the largest variance in the estimate the motion vector goes along the direction of distortion of the test pattern. The major principal component vector of the data scatter matrix is orthogonal to the diagonal line of segmentation. The expansion motion of the test pattern determines dominantly the shape of the residual information along the complement principal component vector, for the first 4 clusters computed, as shown on figures 3 (d and g).

A disk form structure is formed around the vanishing point of image gradients in the middle of figure 3 (b). Also, due to the image gradients distribution, a circular structure in the cluster windows distribution is shown in figure 3 (e). The motion field probability distribution is restricted in the newly formed cluster windows, and the average motion vectors define the circular structure in the equation \(z = 0\).
Figure 4: The “vortex” sequence. For the still image in (a) the distribution of the singular sets is shown in (b). In the sequence of 6 images the distribution of the singular sets for the motion information is shown in (c), and for the joint motion and spectral information in (d).

Figure 5: The MeteoSat infrared image of the convective clouds (a), is decomposed in temperature fronts (b-f). The temperature fronts in figures (b-f) are gray scale coded according to their singularity factors. The rain process is shown associated with the perturbation propagating through the fusion of convective clouds.
The image in figure 3 (h) shows the refinement of the space-motion features, dominated by the distortion motion in the test image. The newly formed spatial features can be observed in the lower right quadrant of the image, where the distortion motion is the strongest.

The results of processing the MeteoSat infrared images are shown in figures 4 and 5. Decomposition of the still image in figure 4 (a) shows that the most singular manifolds distribute along the strongest temperature fronts of the cloud, as it is also observed in the work [16]. For the IR image of the clouds in figure 5 (a) we find however an alternation of the singular sets along the edges of the clouds, as shown in figures 5 (b-f). This can be explained by the multipolar character of those edges, that is associated, in our view, to the turbulent exchange of energy along the vertical layers inside the clouds.

The space-motion clusters decomposition of the image sequence, is shown in figure 4 (c). In the observation of the motion in the image sequence we find that the elongation of the most singular sets goes along the direction orthogonal to the major component of motion vector as well as to the stronger image gradient content. This effect can be also observed in figure 3 (h). The top of the cloud forming the vortex movement shows greater stability in scale as compared to the bottom of the cloud. This can be also observed in figure 4 (d), where joint motion and spectral information is evaluated in scale for the sequence of 6 images. The most singular manifold for the joint motion and spectral information features decomposition is formed more in the cloud like shape on top of the cloud, as compared to the bottom part where it is formed dominantly by the scale invariant motion information features.

The estimation of the feature vector data clusters, by our algorithm, can be effectively used in compressing the image motion information in video sequences. The spatial features of the group windows distribution are arranged in the hierarchical structure of binary images. Based on the Laplacian diffusion system of equations:

\[ \nabla^2 R = 0 \quad \vec{x} \in \Omega \]
\[ R = h(\vec{x}) \quad \vec{x} \in \Gamma, \quad (9) \]

the reconstruction formula: \( R(\vec{x}) = -\int_\Gamma \frac{\partial g(\vec{\xi})}{\partial n} h(\vec{\xi}) d\Sigma_\xi \) gives us a way of diffusing the residual information from the most singular sets, denoted by \( \Gamma \), into the smoother regions, denoted by \( \Omega \).

The image motion vectors and the residual motion information are estimated, in this work, from the linear constraint equation on the motion vector components. The smoothing in the optical flow field estimation is controlled adaptively by the hierarchical scale computation. A larger order constraint equation, that uses the information of an image stimulus, results in spatial clustering of features along the object stimulus line as shown in [9].

5. Concluding remarks

In this paper, we have described a new method for analysis, processing and coding of image information in image sequences. The hierarchical scale decomposition of image information into different singular sets is obtained by evaluating scale-space scalability of singular features for the clusters of data points. The information content is evaluated by the scale-space frequencies of respective data cluster.

For every level of signal distortion, the minimal data clusters decomposition is obtained in hierarchy of scales. A limit constraint of the scaling exponents is used in the decomposition scheme. The minimax optimization is achieved in the hierarchy of scales of computation, progressively always decreasing the uncertainty in the estimation.

The Green’s function expresses spatial coherency of data clusters and is used in the segmentation process as a smoothing function. In the method proposed, harmonic decomposition of images is achieved in hierarchy of scales. A better spatio-temporal resolution and small code size of data features is obtained, that can be used effectively in compressing image information by our method. The image reconstruction formula is based on the Laplacian system of the diffusion of the residual information from the most singular sets. We intend to investigate the reconstruction part of our method in our future work. We also intend to investigate a parallel computer implementation of the algorithm along with the refinement of our computational scheme, in our future work.

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References


