Time-dependent analysis and simulation-based reliability assessment of suspended cables with rheological properties

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Abstract

The intention of the paper is to illustrate the ability of the probabilistic time-dependent reliability assessment procedure applied to non-linear suspended cable structure with rheological properties, when a rope made from the high strength synthetic fibres is used in order to demonstrate the new qualitatively different concept. Attention is turned to the individual main steps in the assessment procedure, i.e. to the selection of an appropriate method of structural analysis and to derivation of an appropriate closed-form and discrete analytical models, analysis of random variables representing individual actions (four basic random variables, such as structural geometry, cable’s modulus of elasticity and creep strain increments and loading, are considered), evaluation of the structural response with respect to the interaction of the random variables considering a history of the time-dependent action effects and to the definition of the limiting values considering serviceability of cable structure. The potential of the method using direct Monte Carlo technique as one of the possible alternatives for simulation-based time-dependent reliability assessment as a powerful tool is emphasized. The influence of an excessive deflection of suspended cable (caused by creep of cable and rheologic changes) on its serviceability in required time is investigated and illustrative examples are performed.

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1. Introduction

Ropes made from high strength synthetic fibres may soon be preferred for use in cable suspension bridges and roofs. They have many advantages over traditional materials and could be used to replace high tensile steel cables in many application areas of tension structures, particularly where low weight and corrosion resistance are of important concern [1]. It is clear that in contrast to the classical tension steel rods and bars, which operate in the linear elastic range, steel cables and mainly fibre ropes have time-dependent non-linear viscoelastic properties. To predict the structural response and assess the structural reliability and serviceability of tension structures with suspended fibre cables during their entire service life, adequate closed-form and numerical analytical models for time-dependent analysis and adequate methods for estimating the probability of failure must be available.

Various papers and books have been published concerning closed-form [2–5] and numerical [6–14] methods for analysis of suspended cable structures considering geometrical and material non-linearities. Most of the recent methods of non-linear analysis of cable structures are based on the discretisation of the equilibrium equations using FEM and solving the resulting non-linear algebraic equations by numerical methods. The non-linear material model of cable structural element was proposed by Jonatowski and Bîrnstiel [6]. The complementary energy principle for a cable modelled as one-dimensional continuum has been presented for large deflection analysis by Cannarozzo [15]. Kanno et al. [16] derived a special method for friction or

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The common approach to these investigations is to study the cable structure as a geometrically non-linear system. However, only a little attention is paid to the time-dependent analyses of suspended cable with rheological properties. Therefore the purpose of this paper is to derive and present non-linear time-dependent closed-form and discrete static solutions of a suspended cable with viscoelastic properties considering the creep effects of the synthetic fibre ropes. For the time-dependent analysis of a suspended cable, the time domain is divided into a discrete number of time steps. The creep theory is adopted for rheological analysis. In the case of closed-form analysis the Irvine’s convenient form of the cable equation [2] is modified because the effects of a creep strain need to be incorporated to the cable and deflection equation. For the time-dependent discrete non-linear analysis of the suspended cable, a finite element method based on the displacement formulation is used. A time-dependent tangential stiffness matrix of the cable element is defined. Incremental and iterative solution strategies have been implemented to solve the geometrically and physically non-linear behaviour problem.

Structural reliability theory has received considerable attention in the literature [20–23]. The general principles for a probabilistic design of bearing structures were published by the Joint Committee on Structural Safety and can be found in [24].

The available methods for estimating the probability of failure $P_f(t)$ can be roughly classified into two groups, which can be marked as gradient-based (FORM and SORM approaches) and simulation-based methods (Monte Carlo method). The progress in the use for bearing structures and elements reached the Monte Carlo simulation method and its modifications [25–27]. Simulation-based methods hinge upon the creation of a set of $N(t)$ response samples on which the probability of failure can be estimated at time $t$ as $P_f(t) = N_f(t)/N(t)$, where $N_f(t)$ is the number of samples lying in the failure domain at time $t$. The last from the described methods will be applied in the paper.

The intention of the paper is to illustrate the ability of the probabilistic time-dependent reliability assessment procedure applied to non-linear suspended cable structure with rheological properties, when a rope made from the high strength synthetic fibres is used in order to demonstrate the new qualitatively different concept. Attention is turned to the individual main steps in the assessment procedure, i.e. to the selection of an appropriate method of the time-dependent structural analysis and to derivation of an appropriate closed-form and discrete analytical models, analysis of random variables representing individual actions, evaluation of the structural response with respect to the interaction of the random variables considering a history of the time-dependent action effects and to the definition of the limiting values considering serviceability of cable structure.

Finally, it is, perhaps, necessary mentioning that although the engineering theory of a suspended cable was well developed, despite numerous examples in the literature, no other closed-form model has as yet been proposed which can predict the creep and associated deflections in suspended cables composed of synthetic fibres. An area, included improvement of theoretical approaches (which unlike the previous solutions, include creep) for predicting the time-dependent behaviour of suspended cable composed of synthetic fibres, can be considered as distinct in this work.

2. Time-dependent closed-form analysis

All the geometrical and force quantities and equations of the suspended cable, geometrical–deformational equations considering large deflections, physical and constitutive equations as well as all deflections and state cable equations are expressed as the time and stress functions respecting non-linear creep and rheological parameters. They can be expressed by the only one unknown value at the investigated time $t$, i.e. by the horizontal component of the cable force $H(t) = H(t_0) \pm \Delta H(t)$, where $H(t_0)$ is the horizontal component of tension force of the cable loaded only by its self-weight $g_0$ at the starting initial time $t_0$ and $\Delta H(t)$ is the increment or decrement in the horizontal component of cable tension at the investigated time $t$.

2.1. Basic geometric and force quantities

Consider that the profile of a uniform cable hanging under its own weight between two supports is flat (cables of a small sag measured from the connecting line of suspension points are considered), so that the ratio of sag to span is 1:8 or less (Fig. 1). The initial starting reference equation

![Fig. 1. Initial geometry of suspended cable under self-weight.](image-url)
of the cable sag curve at the time \( t_0 \) and temperature \( T(t_0) \), which all the state changes (changes caused by additional load, creep and temperature) are to be confronted with, at the corresponding time increments, is in the following form \[2\]:

\[
z(x, t_0) = \frac{g_0}{2H(t_0) \cos \beta} x(l - x) + x \tan \beta,
\]

where \( l \) is the span of cable and \( \beta \) is the inclination of the connecting line of the suspension points of the cable with the horizontal axis \( x \). The horizontal component of cable tension force of the cable loaded only by its self-weight \( g_0 \) per unit length at the starting initial time \( t_0 \), \( H(t_0) \) is generally defined as ratio of bending moment \( M(x) \) at investigated cross-section \( x \) of the simple horizontal beam with the same span and load as the investigated cable to sag \( d_s \) measured from connecting line of suspension points. The cable is perfectly flexible, i.e. bending moments in all cable cross-sections are equal to zero. It works only in tension, having zero stiffness in compression and bending. \( H(t_0) \) can be written in the form \( H(t_0) = M(x)/d_s \). In the case of a uniformly distributed weight of intensity \( g_0 \) per unit span of the cable with the sag \( d \) at the mid-span valid: \( H(t_0) = gq^2/(8d) \). Increments or decrements of the horizontal component of the cable force \( \pm \Delta H(t) \) at the time of investigation \( t \) corresponding to the state changes are determined from the cable equations.

The cable axial tensile force in an arbitrary cross-section \( x \) and time \( t \) can be determined from the relationship

\[
N(x, t) = H(t) \sqrt{1 + \left( \frac{Q(x)}{H(t) \cos \beta} + \tan \beta \right)^2}
\]

where \( Q(x) \) is shear force in cross-section \( x \) of horizontal simple supported beam with the same span and load as the investigated cable.

2.2. Cable deflection as the time function

Suppose that under applied vertical loading, the shear force at some cross-section \( x \) along the span is \( Q \). General vertical equilibrium at a cross-section of the deformed cable at time \( t \) is in the form

\[
(H(t_0) + \Delta H(t))(\frac{dz(x, t_0)}{dx} + \frac{dw(t)}{dx}) = Q + (H(t_0) + \Delta H(t)) \tan \beta + \frac{g_0 l}{2 \cos \beta} \left(1 - \frac{2x}{l}\right)
\]

where \( w(t) \) is the additional vertical cable deflection and \( \Delta H(t) \) is the increment in the horizontal component of cable tension \( N(t) \) at the investigated time \( t \) (note that the vertical component of cable tension on the left-hand side of Eq. (3) is determined as \( V = Hdz/dx \), where \( dz/dx \) is the tan of the angle subtended to the horizontal by the tangent to the profile of the cable). The right-hand side of the equilibrium equation (3) is analogous to the shear force in the cross-section \( x \) in a simple supported horizontal beam of uniform weight under the action of the same loading and with the same span as the investigated suspended cable. Eq. (3) may be integrated directly, and after the boundary conditions have been satisfied the deflection equation for the various types of characteristic vertical loads is in the form as follows:

\[
w(x, t) = \frac{1}{H(t_0) + \Delta H(t)} \left\{ Q' - \frac{\Delta H(t)g_0 l}{2H(t_0) \cos \beta} \left(1 - \frac{x}{l}\right) \right\}
\]

(4)

The relationships for values of the \( Q' \) of some significant loads occurring in construction practice are in Table 1.

Under creep and temperature rise the deflection equation is given by

\[
w(x, t) = \frac{1}{H(t_0) - \Delta H(t)} \left\{ \frac{\Delta H(t)g_0 l}{2H(t_0) \cos \beta} \left(1 - \frac{x}{l}\right) \right\}
\]

(5)

and for temperature fall the expression is

\[
w(x, t) = \frac{1}{H(t_0) + \Delta H(t)} \left\{ \frac{\Delta H(t)g_0 l}{2H(t_0) \cos \beta} \left(1 - \frac{x}{l}\right) \right\}
\]

(6)

To complete the solution, \( \Delta H(t) \) must be determined \[2\]. Use is made of a cable equation that incorporates elastic (application of the Hooke’s law), creep and temperature strain to provide a closure condition at investigated time \( t \) relating the changes in cable tension force to the changes in cable geometry when the cable is displaced from its initial original equilibrium profile.

2.3. Cable equation as the time function

If \( \Delta s(t_0) \) is the original length of the cable element and \( \Delta s(t) \) is its new length at the time \( t \), then \( (\Delta s(t_0))^2 = dx^2 + dz^2 \) and \( (\Delta s(t))^2 = (dx + dw(t))^2 + (dz + dw(t))^2 \), where \( u(t) \) and \( w(t) \) are the longitudinal and vertical components of the displacements of the cable element, respectively. If the profile of the cable is flat, so that the ratio of its sag to span is 1:8 or less, the fractional change in its length, correct to the second order of small quantities, is

\[
\frac{\Delta s(t) - \Delta s(t_0)}{\Delta s(t_0)} = \frac{\Delta u(t)}{\Delta s(t_0)} \frac{dx}{\Delta s(t_0)} + \frac{\Delta w(t)}{\Delta s(t_0)} \frac{dz}{\Delta s(t_0)} + \frac{1}{2} \left(\frac{\Delta w(t)}{\Delta s(t_0)}\right)^2
\]

(7)

Because, in the cable element of the vertically loaded suspended cable of appreciable curvature, the longitudinal component of displacements is much smaller than the transversal one, the axial strain defined as \( \varepsilon(t) = \Delta s(t) - \Delta s(t_0)/\Delta s(t_0) \) related to the current displacement of the cable element is evaluated through the following simplified expressions \( \varepsilon_{z}(t) = \partial \varepsilon(t)/\partial z + (1/2)\partial \varepsilon(t)/\partial z) \) and \( \varepsilon_{z}(t) = \partial \varepsilon(t)/\partial z \). Thus the corresponding axial strain \( \varepsilon(t) \) is obtained from the solution of the quadratic equation in the form as \( \varepsilon^2(t)(\Delta s(t_0))^2 + 2\varepsilon(t)(\Delta s(t_0))^2 - 2(\varepsilon_{z}(t)\Delta s^2 + \varepsilon_{z}(t)\Delta s^2) = 0 \).
Table 1
Relationships for the values \( Q' \) of some significant loading types

<table>
<thead>
<tr>
<th>Loading diagram</th>
<th>( Q' )</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Loading Diagram" /></td>
<td>( Q' = \frac{x}{2} \left( 1 - \frac{x}{q} \right) ) for ( x \in (0,a) )</td>
</tr>
<tr>
<td><img src="image" alt="Loading Diagram" /></td>
<td>( Q' = \frac{qx}{2l} \left( \frac{a^2 - b^2}{2} - q(x - a) \right) ) for ( x \in (a,b) )</td>
</tr>
<tr>
<td><img src="image" alt="Loading Diagram" /></td>
<td>( Q' = \frac{qx}{2l} \left( \frac{a^2 - b^2}{2} - \frac{q}{2} x^2 + a^2 - 2xb \right) ) for ( x \in (b,l) )</td>
</tr>
<tr>
<td><img src="image" alt="Loading Diagram" /></td>
<td>( Q' = \frac{qx}{2l} \left( \frac{a^2 - b^2}{2} - \frac{q}{2} (a^2 - b^2) \right) )</td>
</tr>
<tr>
<td><img src="image" alt="Loading Diagram" /></td>
<td>( Q' = \frac{lx}{6} \left( 2q_1 + q_2 \right) - \frac{q_1 x^2}{6l} (q_2 - q_1) ) for ( q_1 &lt; q_2 )</td>
</tr>
<tr>
<td><img src="image" alt="Loading Diagram" /></td>
<td>( Q' = \frac{q_s x}{6l} (l^2 - x^2) )</td>
</tr>
<tr>
<td><img src="image" alt="Loading Diagram" /></td>
<td>( Q' = P_s \left( 1 - \frac{a}{l} \right) ) for ( x \in (0,a) )</td>
</tr>
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</tr>
</tbody>
</table>

Hooke’s law, applied to the element, requires that

\[
\frac{ds(t) - ds(t_0)}{ds(t_0)} = \frac{\Delta H(t)}{E(t)A} \frac{ds(t_0)}{dx}
\]

(8)

where \( E(t) \) is the modulus of elasticity of the cable at the time \( t \) and \( A \) is the cross-sectional area of the cable.

If the effect of creep strain \( \varepsilon_c(t) \) of the cable at the time \( t \) and the effect of a uniform temperature difference of \( \Delta T(t) = T(t) - T(t_0) \) (where \( T(t_0) \) and \( T(t) \) are the initial and design temperatures, respectively) need to be incorporated, we add the terms \( \varepsilon_c(t) \) and \( \Delta T = \frac{x}{2} \Delta T(t) \), where \( \alpha \) is the coefficient of expansion, to the elemental equations. Consequently, considering Eqs. (7) and (8) the cable equation for the element is in the form as follows:

\[
\frac{\Delta H(t) \left( \frac{ds(t_0)}{dx} \right)^3}{E(t)A} + \varepsilon_c(t) \left( \frac{ds(t_0)}{dx} \right)^2 + \alpha \Delta T \left( \frac{ds(t_0)}{dx} \right)^2 = \frac{dw(t)}{dx} + \frac{dw(t)}{dx} \frac{dz}{dx} + \frac{1}{2} \left( \frac{dw(t)}{dx} \right)^2
\]

(9)

After multiplication of Eq. (9) by \( (ds(t_0)/dx)^2 \) one can obtain

\[
\frac{\Delta H(t) \left( \frac{ds(t_0)}{dx} \right)^3}{E(t)A} + \varepsilon_c(t) \left( \frac{ds(t_0)}{dx} \right)^2 + \alpha \Delta T \left( \frac{ds(t_0)}{dx} \right)^2 = \frac{dw(t)}{dx} + \frac{dw(t)}{dx} \frac{dz}{dx} + \frac{1}{2} \left( \frac{dw(t)}{dx} \right)^2
\]

(10)

If the effects of elastic cable deformation, assuming Hooke’s law, creep strain, temperature change and the fractional change in length of the cable, correct to the second order of small quantities are considered, the cable equation follow the general integrated form:

\[
\int_{t_0}^{t} \frac{\Delta H(t)L_c}{E(t)A} dt + \int_{t_0}^{t} \varepsilon_c(t)L_c dt + \int_{t_0}^{t} \alpha \Delta T(t)L_T dt = \int_{t_0}^{t} \int_{x_0}^{x} \frac{dw(t)}{dx} dx dt + \int_{t_0}^{t} \int_{x_0}^{x} \left( \frac{dw(t)}{dx} \frac{dz(x,t_0)}{dx} \right) dx dt + \frac{1}{2} \int_{t_0}^{t} \int_{x_0}^{x} \left( \frac{dw(t)}{dx} \right)^2 dx dt
\]

(11)

where \( L_c, L_c \) and \( L_T \) are members connected with length of the cable under its own weight \( g_0 \) at the time \( t_0 \) in the forms as follows:
\[ L_e = \int_0^l \left( \frac{ds(t_0)}{dx} \right)^3 \, dx = l \left( 1 + \frac{3}{2} t_i^2 \beta + \frac{g_0^2 l^2}{8H^2(t_0) \cos^2 \beta} \right) \quad (12) \]
\[ L_e = L_T = \int_0^l \left( \frac{ds(t_0)}{dx} \right)^2 \, dx = \frac{1}{\cos \beta} \left( 1 + \frac{g_0^2 l^2}{12H^2(t_0)} \right) \quad (13) \]

The increment or decrement in the horizontal component of cable tension force \( \Delta H(t) \) at the investigated time \( t \) can be found by a solution of a cable equation (11). After the cable equation (11) is adjusted for continuous and non-continuous load and after Eqs. (1) and (4) are substituted into Eq. (11) and the integration is performed, the following general cubic equation for \( \Delta H(t) \) as a time function is derived

\[ \Delta H^3(t) + B(t) \Delta H^2(t) + C(t) \Delta H(t) + D(t) + \overline{Q}(t) = 0 \quad (14) \]

in which for the individual coefficients \( B(t) \), \( C(t) \) and \( D(t) \) is valid as follows:

\[
B(t) = \frac{E(t)A}{L_e} \left\{ \frac{g_0^3 l^3}{24H^2(t_0) \cos^2 \beta} + \frac{2L_e}{E(t)A} H(t_0) + \varepsilon_3(t)L_e + \varepsilon_T(t)L_T \right\} 
\]

\[
C(t) = \frac{E(t)A}{L_e} \left\{ \frac{g_0^3 l^3}{12H(t_0) \cos^2 \beta} + \frac{L_e}{E(t)A} H^2(t_0) \right\} + 2\varepsilon_3(t)L_e + 2\varepsilon_T(t)L_T H(t_0) \]

\[
D(t) = \frac{E(t)A}{L_e} \left\{ \varepsilon_3(t)L_e H^2(t_0) + \varepsilon_T(t)L_T H^2(t_0) \right\} 
\]

The relationships for values of the \( \overline{Q}(t) \) of some significant loads are in Table 2.

If horizontal supports flexibilities of \( f_{ux}(t) \) and \( f_{ux}(t) \) occur at each end, respectively, as shown in Fig. 2 one may replace the axial tension stiffness \( E(t)A \) of the cable by the modified stiffness at time \( t \)

\[
\overline{E(t)A} = \frac{E(t)A}{1 + E(t)A f_{ux}(t) f_{ux}(t)} 
\]
where \( f_s(t) = f_{ax}(t) + f_{bx}(t) \) and then proceed as if the supports were unyielding.

3. Time-dependent numerical FEM solution

3.1. Description of computational model

A computational model of the geometrically and parametrically non-linear cable structures with the rheologic properties is based on the deformation variant of the FEM [6–8]. Time-dependent behaviour of structure during the construction process as well as during the service life of structure can be analyzed through the derived model. For the time-dependent analysis of the cable structure, the time domain is divided into a discrete number of time steps. At each time increment, the structure is analyzed under the external applied loads and imposed current geometry, strain and stress originated from the previous time interval. The model enables the continuous transfer of the current structural behaviour data during the required time period.

A Lagrangian formulation is adopted for the stress and force analyses. Elements of the cable structure are straight between the nodes, which have 3 degrees of freedom. Inelastic and slackening effects of the cables are considered. Various boundary conditions (homogeneous or non-homogeneous) and effects of the temperature change are accounted for. In the present model, an explicit creep constitutive equation for synthetic fibre ropes, recently developed by Guimarães and Burgoyne [28], is used and incorporated into the structural analysis program.

An Updated Lagrangian formulation is used to solve geometrically non-linear effects. In this approach the initial geometry of a structure is taken as the reference configuration. The incremental and Newton–Raphson iterative solution strategies or their combination have been implemented to solve the non-linear problems. Within an increment of load, the geometry and stress update are performed once the iterative process has converged. The convergence is verified with the deflections and forces criteria formulated independently.

The created transformation model has been implemented into the software product LANSTAT by means of the programming language Microsoft Fortran Power Station for the computational part and the language Pascal Borland Delphi for the graphical environment of the preprocessor and postprocessor, while the object oriented programming with the expert system element introduction was used. The algorithm in the software LANSTAT employed the Skyline method and Collin’s procedure [29], which minimizes the width of the half-band of the global stiffness matrix by a renumbering the node numbers of the individual elements of the structure.

3.2. Governing equations – finite element formulation

Consideration is given to the single cable element and the analysis is provided at a studied time \( t \). The displacements \( \mathbf{u}(t) \) within an element are given as a function of the nodal displacements \( \Delta(t) \) as

\[
\mathbf{u}(t) = \mathbf{M}(t) \cdot \Delta(t)
\]

where \( \Delta(t) \) for a cable element is in the form

\[
\Delta(t) = \{u_i(t), v_i(t), w_i(t), u_{i+1}(t), v_{i+1}(t), w_{i+1}(t)\}^T
\]

and \( \mathbf{M}(t) \) is a shape matrix with the coefficients which are functions of \( x \).

The Green’s strain related to the current total displacements is defined in the form

\[
\varepsilon(t) = \frac{\partial \mathbf{u}(t)}{\partial x} + \frac{1}{2} \left[ \left( \frac{\partial \mathbf{v}(t)}{\partial x} \right)^2 + \left( \frac{\partial \mathbf{w}(t)}{\partial x} \right)^2 \right]
\]

For the strain energy of a cable element with the cross-section \( A \) is

\[
U_{\text{INT}}(t) = \int_{t_0}^{t} \left( A \int_{0}^{l} E(\sigma, \varepsilon, t) \cdot \varepsilon(t) \cdot d\mathbf{v} \right) dt
\]

For a current modulus of elasticity \( E(\sigma, \varepsilon, t) \) as the function of stress, strain and time, at the investigated time \( t_k \) is

\[
E(\sigma, \varepsilon, t) = \frac{\partial \sigma}{\partial \varepsilon} = \frac{d(f(\varepsilon(t_k)) + \Delta \varepsilon(t_k))}{d\varepsilon(t_k)}
\]

Full derivation and details of numerical evaluation of the expression (23) can be found in [10]. After substituting
the expression of Green’s strain defined by Eq. (21) into the equation for strain energy (22) and taking into account Eq. (20) one can obtain the cable strain energy as the function of displacements parameters $\Delta(t)$. According to the Castigliano’s theorem, the governing fundamental system of non-linear equilibrium equations for the cable finite element can be expressed as

$$\frac{\partial U_{\text{INT}}(t)}{\partial \Delta(t)} = K_T(t, \Delta) \cdot \Delta(t) = P(t)$$

(24)

where $K_T(t, \Delta)$ is a tangential stiffness matrix of an element in the current time $t$ and $P(t)$ is an external nodal load vector. The tangential stiffness matrix is defined as the sum of three sub-matrices: the elastic stiffness matrix $K_e(t, \Delta)$, the geometric stiffness matrix $K_g(t, \Delta)$ and the initial stiffness matrix $K_0(t_0)$. The change of an initial length of the cable element $L(t_0)$ at the time $t_0$ into a current length $L(t)$ at the time $t$ is expressed in the form

$$L(t) = L(t_0)(1 + e_0(t_0) + e_e(t) + e_c(t) + e_T(t))$$

(25)

where $e_0(t_0)$ is a permanent strain caused by initial stretching of cable including its anchor, $e_e(t)$ is a creep strain, $e_c(t)$ is an elastic strain corresponding to the load changes in the investigated time and $e_T(t)$ is caused by the temperature changes $\pm \alpha \Delta T(t)$. The described current length $L(t)$ defined by Eq. (25) is updated in the mentioned stiffness matrices of cable element.

### 3.3. Verification of computational model and analytical procedure

In order to verify the accuracy of the computational techniques and the computer program $\text{LANSTAT}$ developed in this study, solutions have been generated for the suspended cable structure previously examined by Rakowski [4]. The closed-form and numerical models were developed and used by Rakowski. The structure analyzed was a single parabolic cable with a sag to span ratio of 0.05. The initial geometry, member properties and discrete loading conditions used in this study and shown in Fig. 3 were the same as used previously in [4].

According to the quantity of the mid-span discrete loading force of 100 kN relatively strong geometrically non-linear response in suspended cable can occur. This was the reason that the mentioned cable was chosen for the verification of computational model and for a comparison of the results obtained by the various independent approaches. The geometrically non-linear behaviour of cable structure was generated under several load increments and Newton–Raphson iterations. The vertical deflections of the characteristic nodes of the suspended cable thus obtained were compared to and found to be in close agreement with, that reported by Rakowski [4], as shown in Fig. 4. Deflection at the mid-span node of the suspended cable in the form of a graphical output of a postprocessor is shown in Fig. 5.

The possibilities of use of the computational model and software $\text{LANSTAT}$ for 2D and 3D cable structures are shown in Fig. 6.

The second example is that of an initially prestressed cable subjected to uniformly distributed load as shown in Fig. 7. Consider the prestressed cable with a span $l = 10,000$ in and cross-sectional area $A = 0.065$ in.$^2$. Modulus of elasticity of cable is $E = 20 \times 10^6$ lb/in.$^2$ and its initial prestress is $\sigma = 20 \times 10^3$ lb/in.$^2$. Initial length of the cable is $L_0 = 9990.0099$ in and its prestressing force is $H(t_0) = N(t_0) = \sigma(t_0)A = 20 \times 10^3 \times 0.065 = 1300$ lb. There is a vertical uniformly distributed load $q = 0.02$ lb/in. To check how well do results from the closed-form and discrete solution presented compare with results in literature a problem depicted in Fig. 7 considered by Ozdemir [7], Jayaraman and Knudson [8] and Cannarozzi [15] was solved. An exact solution performed by Sinclair and Hoder is available [3]. On purpose the original units were used in this example. The length $L_n$ of the cable in this problem is less than the distance $l$ between the supports, hence the cable is initially prestressed. When a cable is initially straight, one may set $z(x, t_0)$ and $g_0$ to zero and, in the closed-form time-independent solution the simplified deflection and cable equations (4) and (14), respectively,
can be used. The horizontal line joining the supports has been used as the starting geometry for all the idealisations considered here. In the case of FEM the prestressed cable was divided into 500 finite elements.

In Table 3, vertical deflection $w$ at mid-span of the initially prestressed cable obtained by closed-form solution and FEM presented is compared with the values given in literature [3,7,8,15]. Cross-check with literature results confirmed a good agreement.

Unfortunately, there is no of cross-check with literature results from point of view of the time-dependent analysis, in the paper. Account for this is that studies on behaviour of suspended cable are restricted to the cases where only cables with time-independent properties, i.e. without con-
consideration of rheological properties and/or of creep effects are applied.

4. Numerical applications and discussions of results

The following examples are given to briefly illustrate the application of the closed-form and numerical theories to practical problems. Synthetic fibre ropes are an attractive practical alternative to steel wire ropes and they are used in prestressed suspension roofs and bridges. It is clear that for these applications a good understanding of the creep behaviour of material and rheological behaviour of structure is highly desirable, because load durations of many decades are required. The present examples are concerned with the rheological time-dependent behaviour of suspended synthetic fibre cable, when the creep of the rope is considered.

The cable used in these examples is parallel lay aramid rope constructed from the basic 30,000 N Type G Parafil rope with cross-sectional area of the yarn in the core of 15.28 mm². The creep tests of these ropes were carried out by Guimaraes and Burgoyne at Imperial College in London [28] and an empirical expressions for prediction of long-term creep at ambient temperature were obtained. The pre-tensioning load was 60% of the nominal breaking load of ropes.

The span of suspended cable, with the suspension points at the same level (that is \( b = 0 \) and \( \cos \beta = 1 \)), in its free-hanging position is \( l = 60 \) m and the sag to span ratio is 1:15, so that the mid-span sag is \( d = 4 \) m (Fig. 8). The compliance of the support points is ignored, for this case. The vertical uniformly distributed load \( q \) is applied over the entire span. Properties of the cable constructed from the basic 30,000 N Type G Parafil parallel lay aramid ropes are specified as follows: its self-weight \( g_0 = 0.004 \) N mm⁻¹, its cross-sectional area is \( A = 264.62 \) mm² and its modulus of elasticity is \( E = 130,000 \) MPa.

The problem is to find the additional horizontal component of cable tension \( D_H(t) \) and the associated mid-span deflection \( w(t) \) at the investigated times \( t = t_0 = 0 \) days (initial time), \( t = 10^4 \) days; \( 10^5 \) days; \( 10^6 \) days when the vertical uniformly distributed loads \( q = 1.75 \) N mm⁻¹; \( 2.185 \) N mm⁻¹; \( 2.865 \) N mm⁻¹; \( 3.155 \) N mm⁻¹ and \( 3.595 \) N mm⁻¹ applied over the entire span and the corresponding creep strain increments were assumed. These load levels correspond to 35%; 43.7%; 57.3%; 63.1% and 71.9% of the actual ultimate load carrying capacity of cable \( \sigma_u = 2200 \) MPa. For these stress levels, based on the experimental results [28], we can determine the creep strains from the following logarithmic creep constitutive equations:

\[
v_c(t) = \beta(\sigma) \cdot 10^{-6} \log_{10}(t)
\]  

(26)

where the creep coefficient \( \beta(\sigma) \) (creep rate parameter) with the dimension of \([\text{decade}^{-1}]\) depends on the stress level. In the present study the time-dependent analytical and numerical solutions for the suspended cable were gradually generated for the creep coefficient \( \beta(\sigma) = 74; 104; 91; 106 \) and 117 according to the required stress and/or load levels \( q = 1.75 \) N mm⁻¹; \( 2.185 \) N mm⁻¹; \( 2.865 \) N mm⁻¹; \( 3.155 \) N mm⁻¹ and \( 3.595 \) N mm⁻¹, respectively. The units of time into Eq. (26) is necessarily in seconds, thus the resultant \( v_c(t) \) is dimensionless (e.g. \( 10^{-6} \) days = 8.64 s).
The time-dependent responses to the loading at studied levels, i.e. the mid-span deflections of suspended cable at the investigated times are obtained using two developed transformation models. First, the response is calculated in a closed-form model, when Eq. (4) (with a corresponding $Q^*$ from Table 1) for the vertical deflections under the relevant stress levels and times are used. Secondly FEM is applied and in this case the suspended cable was divided into 50 finite elements and 50 increments of load with Newton–Raphson iterations on every load step was considered for solving the non-linear problem at every investigated time $t$. Resulting responses, i.e. deflections determined according to mentioned two models are, compared and presented in Fig. 9. The results are in very good agreement with each other.

The obtained results confirm the correctness of the derived equations and techniques as well as their physical importance. The cable deflections increase under the influence of the creep strain increments as time increases. The values correspond quantitatively to the relevant stress levels.

5. Simulation-based time-dependent reliability assessment of suspended cable with rheological properties

The fully probabilistic simulation-based time-dependent reliability assessment is applied on the suspended cable shown in Fig. 8.

At this point, it is, worthwhile explaining the difference between a classical time-dependent reliability analysis and a time-dependent reliability analysis in the case presented in the paper. In classical time-dependent reliability problems, interest often lies in estimating the probability of failure $P_f(t)$ over an investigated time interval, say from $t_0$ to $t_k$. This could be obtained by integrating $P_f(t)$ over the interval $(t_0, t_k)$, considering the correlation characteristics in time of the basic variables process $X(t)$, or, sometimes more conveniently, the resistance process $R(t)$, the load effect process $S(t)$, as well as any cross-correlation between $R(t)$ and $S(t)$ [24]. The calculation model for each limit state considered contains a specified set of basic variables, i.e. physical quantities which characterize material properties and geometrical quantities as well as actions and environmental influences. There are also parameters, which describe the requirements (e.g. serviceability constrains in the form of limiting deflections) as well as parameters, which characterize the calculation model itself and which may be treated as basic variables. It is generally the case that the basic variables are functions of time. Changes in both mean values and standard deviations could occur for the basic variables during a time period. They may be random variables (including the special case deterministic variables) or stochastic processes or random fields. Note that the load effect process $S(t)$ in classical time-dependent reliability analysis is composed of additive components, $S_1(t), S_2(t), \ldots, S_n(t)$, for each of which the time fluctuations may have different features (e.g. continuous variation, pulse-type variation, spikes). A complete description of loading generally requires knowledge of its history, illustrating the characteristics of the action during the entire assumed service-life-span of the structure. Basic properties of loading are occurrence, recurrence, intensity, duration,
direction and position of action, variability and variability rate. In special cases such as fire or explosion an adequate reliability assessment format must be selected and applied. It is evident from above remarks that the appropriate approach for solving a time-dependent reliability is a difficult problem. It would depend on a number of considerations, including the time frame of interest, the nature of the load and resistance processes involved, their correlation properties in time, time-dependent reference values as well as target probabilities in both groups of limit states criteria, and the confidence required in the probability estimates.

Another class of problems calling for a time-dependent reliability analysis are those related to damage accumulation, such as rheological effects (e.g., creep), fatigue, corrosion, material degradation and fracture [24]. This is the case presented in the paper. In comparison with a mentioned classical general time-dependent reliability analysis, it is not so complex and consequently not so complicated. Serviceability of the suspended cable is assessed through a fixed threshold (critical creep size resulting to the critical total strain and/or to the corresponding critical, i.e., limiting deflection, which characterizes the tolerable limit of

Fig. 10. Density functions of (a) the geometrical variable \(d(t)\) at the mid-span of the suspended cable, (b) the modulus of elasticity \(E(t)\) (Weibull distribution), (c) the creep strain \(\varepsilon_c(t)\) (Weibull distribution) and (d) a loading \(q\) of the suspended cable and the corresponding distribution functions.
an excessive deflection accumulation) and a monotonically increasing time-dependent creep function (actual creep size under a corresponding stress and/or load at any given time).

5.1. Density and distribution functions of the random variables

For the purpose of quantifying uncertainties at the design stage of suspended cable structure for probability reliability assessment, it is necessary to define a set of basic variables and to decide which quantities should be modelled by random variables or by deterministic parameters. The set of the basic random variables which affect the structural behaviour may be defined as follows: geometrical properties of the suspended cable characterized by the mid-span sag $d(t)$ at the investigated time $t$; mechanical properties of the cable characterized by the modulus of elasticity $E(t)$ at the investigated time $t$; creep strain $\varepsilon_c(t)$ at the investigated time $t$ and external loading $q$. The basic set of each of the described random variable is characterized by the probability density function in the form of continuous normal Gaussian and Weibull distribution. The random variables are taken into account as the independent quantities. A statistical evaluation of the random variable material characteristics of synthetic fibre cable (parallel lay aramid rope) is based on the experimental research [28].

Although the following types of loads and actions are important in the design of cable structure: dead loads, live loads (short- and long-lasting), wind and snow loads, thermal actions, etc., in this example a load of the permanent type with the mean value of 1.75 N mm$^{-1}$ (the expected value) is applied.

The density functions of the described random variables, such as geometrical quantity, modulus of elasticity, creep strain and loading as well as their corresponding distribution functions at the investigated time $t$ are shown in Fig. 10. In the bottom part of the figures is the graphic description, i.e. time record of selection of random variable value from the corresponding set. All data are outputs from the developed software ProbabStat.

Two cases of the time-dependent changes in the distribution functions of the random variables during investigated time period as shown in Fig. 11 will be considered. This means that two independent examples on simulation-based time-dependent reliability assessment of suspended cable will be performed and compared.

5.2. Simulation-based time-dependent reliability assessment of suspended cable

The fully probabilistic simulation-based method using direct Monte Carlo technique is applied on the time-dependent reliability assessment of the suspended cable shown in Fig. 8. Experience with a number of cable structures with synthetic fibre ropes has shown that their failure is characterized by the time-dependent rheologic changes in the tension stiffness of the cables. This is the reason why the serviceability limit states are assessed in the present study. Probability time-dependent reliability assessment of the analysed suspended cable is based on its serviceability and not on ultimate limit states.

When the creep data of a cable component are available, we can find the reliability $P_s(t)$ or failure probability $P_f(t)$ of the suspended cable at an arbitrary time $t$ from the required time interval during its service life. The time interval is divided into the required time domains of the lengths $\Delta t_1, \Delta t_2, \ldots, \Delta t_n$. Then the Monte Carlo method is used to estimate the reliability or failure probability of the suspended cable, with the corresponding random variables, at the considered time $t$.

The two suspended cables with the different properties used as the examples were selected for the investigation of their time-dependent reliability and for the illustration of the mentioned procedure and capability of the closed-form analytical model. The first suspended cable analysed was the structure with the time-dependent changes characterized by the distribution functions of the random variables during investigated time period according to Fig. 11, example (a). As the second case, the suspended cable with time-dependent properties according to Fig. 11, example (b) was analysed. Reliability of the suspended cables with the fixed, i.e. unyielding, supports and defined properties according to Fig. 11 at the investigated times $t = t_0 = 0$ days (initial time), $t = 10^{-4}; 10^{-3}; 10^{-2}; 10^{-1}; 10^0; 10^1; 10^2; 10^3$ days when the vertical uniformly distributed load $q = 1.75$ N mm$^{-1}$ applied over the entire span were assessed.

The program simulates the uncertainty of geometry of cable vertical position by varying its mid-span sag as well as length of the cable in the analytical model and the corresponding total geometry in the case of application of discrete model and FEM.
Serviceability assessment refers to the vertical deflection limit value \( w_{\text{lim}}(t) = 0.860 \text{ m} \) at the mid-span of the suspended cable. This limit deflection was calculated for the design failure probability \( P_{\text{fd}} = 14 \times 10^{-2} \) (design index of reliability \( \beta_d = 1.05 \)), with the corresponding design reliability \( P_{\text{ad}} = 1 - P_{\text{fd}} = 1 - 14 \times 10^{-2} = 0.86 \), for the normal reliability level from the point of view serviceability limit states and for the design technical service life of the structure \( t_d = 80 \) years.

In this method the pseudo-random numbers generator is used to select a simulation value of each of the random parameters of the structure for the corresponding distribution function (see Section 5.1). The number of selections is equal to the number of one million simulations. The simulation system’s response, i.e., the resulting vertical deflections, at the investigated times \( t = t_0 = 0 \) days (initial time), \( t = 10^{-4}; 10^{-3}; 10^{-2}; 10^{-1}; 10^{0}; 10^{1}; 10^{2}; 10^{3} \) and \( 10^{4} \) days, are compared with the specified limit deflection. After generating the predetermined number of simulations, the overall reliability (or failure probability) of the suspended cable, at the investigated time \( t \), is computed. ProbabStat program output, i.e., serviceability reliability function of the suspended cable is shown for \( t = 10^{4} \) days, as an illustration, in Fig. 12. The boundary between successful and unsuccessful simulations is marked by the line.

The minimum \( w_{\text{min}}(t) \) and maximum \( w_{\text{max}}(t) \) deflections (obtained from one set of simulations at time \( t \)) at the mid-span of the suspended cable at the investigated times \( t = 0 \) (initial time); \( t = 10^{-4}; 10^{-3}; 10^{-2}; 10^{-1}; 10^{0}; 10^{1}; 10^{2}; 10^{3} \) and \( 10^{4} \) days, obtained for example (a) and (b) are shown in Fig. 13. The corresponding time-dependent reliabilities \( P_{\text{s}}(t) \) are shown in Fig. 14. The deflections of the suspended

![Fig. 12. ProbabStat program output, i.e. the serviceability function of the suspended cable.](image1)

![Fig. 13. Minimum \( w_{\text{min}}(t) \) and maximum \( w_{\text{max}}(t) \) deflections at the mid-span of the suspended cable at the investigated times \( t \) under 1,000,000 simulations.](image2)
cable increased with the time increment due to the creep influences and reliability of the cable structure is decreased, when serviceability limit states assessment is conducted. Influence of the different time-dependent changes of the random variables during investigated time period (example (a) and (b)) on the final values of deflection and reliability is clear in Figs. 13 and 14. After the time \( t = 10^2 \) days in the example (b) the distance between minimum \( w_{\text{min}}(t) \) and maximum \( w_{\text{max}}(t) \) deflections of structure are increased and reliabilities \( P_s(t) \) are rapidly decreased.

Using the time-dependent simulation-based reliability assessment concept, based on the Monte Carlo method, the closed-form analytical model seems to be in this particular case more convenient and practical compared to FEM model. It does not need much CPU time.

6. Conclusions

In this paper the non-linear time-dependent closed-form static solution of suspended cable with unmovable and elastic yielding supports subjected to various types of loads has been presented. Irvine’s forms of the deflection and the cable equations were modified because of the effects of non-linear creep.

For the non-linear time-dependent discrete analysis of the suspended cable, a finite element method based on the displacement formulation was used. Modelling of the rheological cable properties is based on the non-linear creep theory. The derived time-dependent tangential stiffness matrices of the cable element are functions of stress, strain and time. Verification of the results was performed. First, the response of suspended cable, considering the creep of synthetic fibre rope, was calculated using an obtained closed-form model, and secondly the response was calculated using FEM. The results are in very good agreement with each other.

The substance of a probabilistic structural simulation-based time-dependent reliability assessment method based on the Monte Carlo technique was explained and the strategy of its application was outlined using serviceability assessment of the suspended cable with rheological properties as an example.

It is believed that the presented time-dependent solutions will lead to an improved closed-form and discrete analysis of the suspended cable with rheological properties in the non-linear range and to an improvement of its reliability assessment.

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