Characterization of an indoor MIMO channel in Frequency Domain using the 3D-SAGE Algorithm

Michail Matthaiou and David I. Laurenson
Institute for Digital Communications
School of Engineering and Electronics
The University of Edinburgh
EH9 3JL, Edinburgh, U.K.
Email: {M.Matthaiou, Dave.Laurenson@ed.ac.uk}

Nima Razavi-Ghods and Sana Salous
Centre for Communication Systems
School of Engineering
University of Durham
DH1 3LE, Durham, U.K.
Email: {N.Razavi, Sana.Salous@dur.ac.uk}

Abstract—In this paper, the frequency domain (FD) SAGE (Space-Alternating Generalized Expectation-maximization) algorithm has been extended to the MIMO case in order to determine the angular and temporal channel characteristics at both ends of the radio link. The implementation of the SAGE algorithm relies on the serial interference cancellation (SIC) technique which outperforms the conventional parallel interference cancellation (PIC) scheme when used in the frequency domain, especially in the common case of unequal power levels. The main purpose of our investigation has been twofold. Firstly, a synthetic environment was generated to testify the efficiency of the proposed algorithm in a severe multipath scenario. Secondly, a measurement campaign was conducted and the obtained data were post-processed in order to assess the whole double-directional domain. The results revealed that the proposed algorithm demonstrates a stable performance and robustness as well as rapid convergence which are the principal criteria an estimation technique must fulfill.

I. INTRODUCTION

The recent advances in wireless communications mandate the use of a technology able to offer markedly high capacities and enhance the Quality of Service (QoS). A potential solution lies in the use of Multiple-Input Multiple-Output (MIMO) communication systems which employ antenna arrays at both ends to drastically increase the channel throughput especially in rich scattering environments [1], [2].

Taking into account the stochastic nature of radio-channels, a channel model is the most tractable tool for determining the underlying propagation mechanisms and ultimately predicting the performance of future wireless systems. In the case of MIMO systems, a double-directional channel model is essential in order to estimate the angular and temporal parameters of each wave at both ends; on this basis, the multipath components (MPCs) have to be identified and analyzed with respect to their Time of Arrival (ToA), Azimuth of Arrival (AoA), Azimuth of Departure (AoD) and complex amplitude.

Among the numerous parametric methods for signal estimation, the SAGE algorithm has been employed thanks to its accuracy and rapid convergence [3]. In contrast to the conventional estimation techniques such as MUSIC (MUltiple Signal Classification) and ESPRIT (Estimation of Signal Parameters via Rotational Invariance Technique) which perform badly in the presence of correlated signals, the SAGE algorithm is robust and yields precise estimates even in the low SNR regime [4]. In fact, no spatial smoothing is required to mitigate the effects of correlated signal sources [5]. Finally, the SAGE algorithm can be applied to any arbitrary antenna geometry provided that the array manifold is fully available.

The application of the SAGE algorithm to the problem of multipath channel parameter estimation was initially suggested by Fleury et al., in [6] and [7] where time-invariant and time-variant environments respectively were studied. The frequency domain version of SAGE was proposed by Chong et al. in [8], although the study focused merely on a Single-Input Multiple-Output (SIMO) system. In this paper, we extend this model to MIMO systems and we eventually incorporate the double directional information for joint estimation of temporal and angular variations.

The remainder of the paper is organized as follows: In Section II the channel model for a wideband MIMO system is introduced with a view to the propagation characteristics. Section III outlines the theoretical foundations of the 3D (3-Dimensional) FD-SAGE algorithm and offers an ample insight into the computational procedure. In Section IV the indoor MIMO measurement campaign which provided the necessary real-time data is discussed. In Section V the results obtained from both a simulated and a measured environment are presented in detail. Finally, Section VI concludes the paper and summarizes the key findings.

II. MIMO CHANNEL MODEL

By assuming that all signal sources as well as the scatterers are located in the corresponding far field regions, the communications between the transmitter (Tx) and the receiver (Rx) can be considered as the superposition of L propagation paths. For a time-invariant MIMO channel with N transmit and M receive elements, the double directional-delay impulse response can be succinctly modeled as [9], [10]

$$h(\tau, \theta_R, \theta_T) = \sum_{\ell=1}^{L} \beta_{\ell} \delta(\tau - \tau_{\ell})\delta(\theta_R - \theta_{R,\ell})\delta(\theta_T - \theta_{T,\ell})$$  (1)
where ($\ast$) denotes the convolution operation, $\{\tau_k\}$, $\{\theta_{R,\ell}\}$, $\{\theta_{T,\ell}\}$ and $\{\beta_{\ell}\}$ are the ToA, AoA, AoD and complex amplitude of the $\ell$th wave respectively. We note that the assumption of a time-invariant-channel implies that the Doppler effects can be ignored without loss of accuracy; this assumption is valid for the slowly varying indoor environment under consideration.

Then, the frequency-space transfer function is related to (1) via an one-dimension (1D) Fourier transform across the delay domain

$$H(f,m,n) = \sum_{\ell=1}^{L} \beta_{\ell} \delta(\theta_{R} - \theta_{R,\ell}) \delta(\theta_{T} - \theta_{T,\ell}) \cdot e^{-j2\pi f \tau_{\ell}}$$

(2)

with $f,m,n$ representing the frequency, receive and transmit spatial dimensions respectively. For clarity we concatenate all the path parameters in a vector $\varphi \triangleq [\tau_{\ell}, \theta_{R,\ell}, \theta_{T,\ell}, \beta_{\ell}]$. By considering $K$ frequency bins, the noise-corrupted transfer function at the $k$th frequency bin with $1 \leq k \leq K$, is given as

$$H(k) = \sum_{\ell=1}^{L} S(k; \varphi_{\ell}) + N(k)$$

(3)

where $S(k; \varphi_{\ell}) = \beta_{\ell} a(\theta_{R,\ell}) d^{T}(\theta_{T,\ell}) e^{-j2\pi f \tau_{\ell}}$ is the component contributed by the $\ell$th wave; the receive and transmit steering vectors are respectively expressed as $a(\theta_{R})$ and $d(\theta_{T})$ while $[1]^{T}$ is the transposition operation. Finally, the vector $N(k)$ is the zero-mean complex spatial white Gaussian noise.

III. THE 3D FD-SAGE ALGORITHM

A. Characteristics of the SAGE Algorithm

The SAGE algorithm is in principle an expansion of the typical EM algorithm which computes the ML (Maximum Likelihood) estimators of the unknown parameters in a sequential way. The derivation of the algorithm relies on the key notions of complete (unobservable) and incomplete (observable) data. In each SAGE iteration, only a subset of the parameters is updated while keeping the estimates of the remaining parameters fixed. For a detailed review of the SAGE algorithm the interested reader is referred to [3].

B. The 3D FD-SAGE Algorithm

As was first shown in [8], the SIC scheme yields more stable performance than the PIC technique in the frequency domain and hence was employed in our implementation. The main concept of the SIC technique is to order the waves with respect to their received power in a descending order. Under these conditions, we prevent the interference caused by the strong MPCs which may likely lead to inaccurate estimates of low power MPCs. Especially in indoor environments where the multipath activity is high due to the increased number of scatterers, the PIC scheme fails to resolve precisely all the MPCs. Furthermore, the PIC scheme is laborious since it iterates across the whole set of waves in order to compute the updated estimates.

By introducing the concepts of Expectation step (E-step) and Maximization step (M-step), the 3D FD-SAGE algorithm can be decomposed into two sequential procedures. In detail, the estimated contribution of the $\ell$th wave is calculated as (E-step)

$$\tilde{Y}_{\ell}(k; \hat{\phi}') = H(k) - \sum_{\ell' = 1}^{\ell-1} S(k; \hat{\phi}_{\ell'})$$

(4)

where $\tilde{Y}_{\ell}(k; \hat{\phi}')$ corresponds to the aforementioned complete data and $H(k)$ to the incomplete data. The coordinate wise updating procedure for obtaining the parameters $\hat{\phi}'$ of each wave based on all previous estimates $\hat{\phi}'$ is referred to as the M-step and expressed as

$$\hat{\tau}'_{\ell} = \arg \max_{\tau} \left\{ |z(\tau, \hat{\theta}'_{R,\ell}, \hat{\theta}'_{T,\ell}, \hat{Y}_{\ell}(k; \hat{\phi}')|^{2} \right\}$$

(5a)

$$\hat{\theta}'_{R,\ell} = \arg \max_{\theta_R} \left\{ |z(\hat{\tau}'_{\ell}, \theta_{R}, \hat{\theta}'_{T,\ell}, \hat{Y}_{\ell}(k; \hat{\phi}')|^{2} \right\}$$

(5b)

$$\hat{\theta}'_{T,\ell} = \arg \max_{\theta_T} \left\{ |z(\hat{\tau}'_{\ell}, \hat{\theta}'_{R,\ell}, \theta_{T}, \hat{Y}_{\ell}(k; \hat{\phi}')|^{2} \right\}$$

(5c)

$$\hat{\beta}'_{\ell} = \frac{1}{M N K} \cdot z(\hat{\tau}'_{\ell}, \hat{\theta}'_{R,\ell}, \hat{\theta}'_{T,\ell}, \hat{Y}_{\ell}(k; \hat{\phi}'))$$

(5d)

where $z(\tau, \theta_{R}, \theta_{T}; Y_{\ell})$ is the cost function given by

$$z \triangleq \mathbf{a}^{H}(\theta_{R}) \cdot \mathbf{U}^{*} \odot \mathbf{Y}_{\ell}(\theta_{T})$$

(6)

where ($\odot$) denotes the element wise product and $[1]^{T}$ the Hermitian transposition. The term $\mathbf{U}^{*}$ expresses the conjugate of the calibrated periodic multi-tone frequency excitation signal [9].

The execution of this update process once defines one iteration cycle of the 3D FD-SAGE algorithm. The parameter estimates are sequentially and cyclically updated until convergence is attained. The complex amplitude is then computed as the output signal normalized by the total energy. As for any other iteration method, the algorithm converges when the difference between two successive estimators becomes smaller than a predefined threshold. We have typically used a threshold of 0.5 nsec and 0.1° for the delay and angular domains respectively, as a sufficiently small change.

C. Initialization of the SAGE Algorithm

It is known that the accuracy of every iteration technique depends heavily on the initial conditions. The initialization procedure is based on a successive cancellation scheme. In particular, by starting with the pre-initial setting $\hat{\phi}' = [0, \ldots, 0]$ for $\ell = 1, \ldots L$ the initial estimates for the ToA and AoA were computed according to (7) and (8). We note that at the initialization of the $\ell$th wave the estimates related to $\ell' \geq \ell$ remain equal to 0, i.e. $\hat{\phi}' = [\hat{\phi}'_{1}, \ldots, \hat{\phi}'_{\ell-1}, 0, \ldots, 0]$. From (7) and (8) it can be derived that the ToA is calculated via incoherent summation across all antenna elements while the AoA via incoherent summation across only the transmit elements.
\[
\hat{\tau}'_\ell = \arg \max_\tau \left\{ \sum_{m=1}^M \sum_{n=1}^N \sum_{k=1}^K e^{j2\pi f_k \tau_y} Y_{\ell,m,n}(k; \hat{\phi}'') \right\}^2 
\]
\[
\hat{\theta}'_{R,\ell} = \arg \max_{\theta_R} \left\{ \sum_{n=1}^N \left| a^H(\theta_R) \sum_{k=1}^K e^{j2\pi \hat{\tau}'_\ell f_k} Y_{\ell,n}(k; \hat{\phi}'') \right| \right\}^2 
\]

IV. Measurement Campaign

An indoor measurement campaign was carried out in the Electrical Engineering Building (Vienna University of Technology) in an area with many office partitions [11], [12]. The measurements were conducted using the MEDAV RUSK ATM [9] channel sounder at a carrier frequency of 5.2 GHz and an operation bandwidth of 120 MHz. The sounder was probed at 193 equi-spaced frequency bins covering the overall bandwidth.

The receiver employed a Uniform Linear Array (ULA) of eight vertically-polarized elements with an inter-element distance of 0.4\lambda. The Rx was mounted on a wooden tripod at a height of 1.5m. Before any AoA estimation process is performed, the Rx ULA must be fully calibrated in order to remove the undesired effects of mutual coupling, non-identical element response and other array imperfections [13]. For this reason the effects of mutual coupling were mitigated using the method described in [14]. At the transmitter, an omni-directional sleeve antenna was moved on a (10\times20) rectangular grid with element spacings of 0.5\lambda, forming a virtual Tx matrix with no mutual coupling. The height of the antenna was approximately 1m. By assuming subsets of 8-elements in each row, we end up with 13\times10 = 130 spatial realizations of the 8\times8 MIMO transfer matrix.

For our study, 24 different Rx locations were investigated in several offices while the Tx was positioned at a fixed position in the hallway. In order to capture the whole azimuth domain activity, the Rx was steered to three different broadside directions (spaced by 120°), leading to the generation of 72 data sets, i.e. combinations of Rx positions and directions. The measurement layout along with the Rx locations are illustrated in Fig. 1.

V. Results

A. Simulated Environment

In order to validate the performance of the SAGE algorithm a synthetic environment has been generated. For generating a realistic indoor scenario we need a high number of MPCs and an extended delay range to account for the multiple long-delayed echoes. Intuitively, the setup is the same as the one described in Section IV. In Table I the parameter settings for the simulation environment are tabulated.

The algorithm’s performance is demonstrated in terms of the double directional power azimuth spectrum (PAS) which is depicted in Fig. 2. Simulation results revealed that the

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Simulation setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Tx antennae, N</td>
<td>8</td>
</tr>
<tr>
<td>Number of Rx antennae, M</td>
<td>8</td>
</tr>
<tr>
<td>Signal-to-Noise ratio, SNR</td>
<td>20 dB</td>
</tr>
<tr>
<td>Carrier frequency, f_c</td>
<td>5.2 GHz</td>
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<tr>
<td>Operation bandwidth, B</td>
<td>120 MHz</td>
</tr>
<tr>
<td>Total frequency bins, K</td>
<td>193</td>
</tr>
<tr>
<td>Number of MPCs, L</td>
<td>{10, 20, 30, 40, 50}</td>
</tr>
<tr>
<td>ToA, {\tau_i}</td>
<td>Uniform distribution in [0, 800] nsec</td>
</tr>
<tr>
<td>AoA, {\theta_{R,\ell}}</td>
<td>Uniform distribution in [0, 180°]</td>
</tr>
<tr>
<td>AoD, {\theta_{T,\ell}}</td>
<td>Uniform distribution in [0, 180°]</td>
</tr>
<tr>
<td>Complex amplitude, {\beta_i}</td>
<td>Complex zero-mean Gaussian RV</td>
</tr>
</tbody>
</table>
estimators are reasonably accurate even in a severe multipath scenario. However, due to the intrinsic limit in temporal and angular resolution, only paths with at least one different parameter can be accurately estimated. It is worth noting that the ToA estimates are slightly more precise than the angular estimates due to the wider lobes of the update functions of the latter, as given by (5b) and (5c). This is consistent to the results reported in [8].

We finally note that the estimator’s efficiency is strongly dependent on the number of multipaths \( L \); for higher values of \( L \) the arrays fail to resolve accurately all MPCs due to their limited size [8], [15]. A deeper insight can be obtained by calculating the root mean square error (RMSE) of all the path parameters, as a function of \( L \). The results, shown in Table II, are in satisfactory agreement with the aforementioned theoretical background. An improved performance can be achieved by using larger antenna arrays at the expense though of system complexity and cost.

### B. Measured Environment

At first, the measured SNR was enhanced by averaging the channel transfer matrix over all the 128 temporal snapshots leading to an improvement of approximately 21 dB, theoretically to values up to 50-60 dB. This increase is however inherently limited by the channel dynamic range which was about 35 dB. A crucial issue in signal-estimation algorithms is the determination of the number of waves \( L \) impinging on the receiving array. Taking into account that no \textit{a priori} knowledge of this number is available in real-time measurements, the SIC technique was employed until the signal’s power becomes smaller than -20 dB relative to the strongest peak.

For the following analysis two Rx locations have been studied, namely locations 9 (direction D1) and 12 (direction D2). In both cases, the Rx was aligned in a Non Line-of-Sight (NLoS) direction with the Tx while the surrounding environment comprised of wooden and metal furniture, monitors, book shelves and tables.

In order to validate the obtained SAGE estimates, the PAS has been also calculated using the Capon’s beamformer according to [4]

\[
P(\theta_R, \theta_T) = \frac{1}{(a(\theta_R) \otimes d(\theta_T))^H R^{-1} (a(\theta_R) \otimes d(\theta_T))}
\]

where \((\otimes)\) denotes the Kronecker product and \( R \) is the spatial covariance matrix defined as

\[
R \triangleq E \{ \text{vec}(H) \text{vec}(H)^H \}
\]

where the \text{vec}(\cdot) operator stacks the columns of a matrix into a vector and \( E \{ \cdot \} \) is the expectation operation for all the channel realizations. In general, the Capon’s beamformer achieves to suppress the undesired interference by minimizing the total output power while maintaining a constant gain in the look direction. However, its performance is strongly dependent on the input SNR and in addition it can hardly deal with coherent signals [4].

The obtained Capon spectra are depicted in Fig. 3 and 4 along with the SAGE estimates indicated by the white dots. The larger dots correspond to stronger paths. The similarity between both methods can be easily seen although the Capon beamformer fails to distinguish effectively between the closely-separated paths; this leads to the artificial generation of wide lobes which are in fact the superposition of discrete multipaths.

At location 9, two distinct clusters of AoAs are centered around \( 10 - 70^\circ \) and \( 120 - 160^\circ \) while the spread of the AoDs is about \( 90^\circ \). Both clusters can be attributed to transmission through the walls and this account for a significant portion of multipaths being blocked. At location 12, the AoDs are confined within a range of approximately \( 40 - 120^\circ \) and likewise the AoAs of \( 40 - 115^\circ \). A high concentration (dominant cluster) of impinging paths can be observed close to the end-fire direction of the receiver which correspond to second-order reflections from the wall and diffractions from objects in the corner.

It is worth noting that both locations are close to the edge of the building and thus the number of scatterers is inherently poorer. A further study revealed that locations in the middle of the building (such as 13, 25 and 26) are more susceptible to increased multipath activity.

### VI. Conclusion

A tractable approach in assessing the frequency domain performance of a MIMO channel has been proposed in this

![Fig. 2. Double directional PAS of the 3D FD-SAGE algorithm in a synthetic environment (\( L = 20 \)).](image-url)
In order to validate the statistically generated model a set of indoor measurements was conducted so as to get a deeper insight in the real-time propagation conditions. The SAGE estimates were compared with the Capon’s beamforming technique and the obtained results are in good agreement, at both locations under investigation. While Capon fails to separate the closely-spaced paths, SAGE manages to do so as long as they have at least one different parameter. It is noteworthy that the paths will be precisely estimated only when their temporal/angular characteristics differ by a fraction of the intrinsic resolution.

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REFERENCES


