Exploring the Twin Peaks Using Probabilistic Verification Techniques∗

Anitha Murugesan†
anitha@cs.umn.edu
Lu Feng∗
lufeng@cis.upenn.edu
Mats Heimdahl†
heimdahl@cs.umn.edu
Sanjai Rayadurgam†
sanjai@cs.umn.edu
Michael Whalen†
whalen@cs.umn.edu
Insup Lee∗
lee@cis.upenn.edu

†Department of Computer Science and Engineering
University of Minnesota
200 Union St. S.E., Minneapolis, MN 55455, USA
∗Department of Computer and Information Science
University of Pennsylvania
3330 Walnut St., Philadelphia, PA 19104, USA

ABSTRACT

System requirements and system architecture/design co-evolve as the understanding of both the problem at hand as well as the solution to be deployed evolve—the Twin Peaks concept. Modeling of requirements and solution is a promising approach for exploring the twin peaks. Commonly, such models are deterministic because of the choice of modeling notation and available analysis tools. Unfortunately, most systems operate in an uncertain environment and contain physical components whose behaviors are stochastic. Although much can be learned from modeling and analysis with commonly used tools, e.g., Simulink/Stateflow and the Simulink Design Verifier, the SCADE toolset, etc., the results from the exploration of the twin peaks will—by necessity—be inaccurate and can be misleading; inclusion of the probabilistic behavior of the physical world provides crucial additional insight into the system’s required behavior, its operational environment, and the solution proposed for its software. Here, we share our initial experiences with model-based deterministic and probabilistic verification approaches while exploring the Twin Peaks. The intent of this paper is to demonstrate how probabilistic reasoning helps illuminate weaknesses in system requirements, environmental assumptions, and the intended software solution, that could not be identified when using deterministic techniques. We illustrate our experience through a medical device sub-system, modeled and analyzed using the Simulink/Stateflow (deterministic) and PRISM (probabilistic) tools.

Categories and Subject Descriptors

D.2.4 [Software Engineering]: Software/Program Verification—Model checking

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General Terms

Verification

Keywords

Requirements, Architecture, Model based verification.

1. INTRODUCTION

As a development project progresses, is well known that deepening the understanding of both the problem at hand as well as the solution to be deployed leads to a co-evolution of the system requirements and system architecture/design—an exploration of the Twin Peaks [15].

Model based approaches, e.g., Simulink/Stateflow [8, 10], Rhapsody [1], or SCADE [3] are commonly used for this Twin Peaks exploration. Some approaches are supported by formal verification tools allowing an engineer to verify whether or not desirable properties (requirements) of a model are satisfied, e.g., the MathWorks Simulink Design Verifier (SLDV) [9] or the Gryphon toolsuite [13] developed at Rockwell Collins. These verification tools are generally based on some form of model-checking technology [2] and perform a deterministic determination of whether or not a property is satisfied by a model; either the property is true or it is false—there is no notion of the property being true “most of the time”. Although such analysis can be extraordinarily useful in practice [11, 13], the results from the exploration of the Twin Peaks using traditional verification tools will—by necessity—be inaccurate and can be misleading. For example, we may perform the verification of our software models under a single-failure assumption for the various sensors and actuators interacting with our system. Given this assumption, we may perform a verification of the software demonstrating that—under this assumption—the requirements are met. Since we know that this assumption is wrong—there can be multiple failures, however unlikely—we do now know for certain how our software will really behave in the face of such unlikely events; it is quite possible that the “verified” software has a low likelihood of meeting its desired requirements under realistic operating conditions. Hence, inclusion of the stochastic behavior of the physical world provides crucial additional insight into the system’s required behavior, its operational environment, and the solution proposed for its software.
In this paper, we share our initial experiences with model-based deterministic and probabilistic verification approaches while exploring the Twin Peaks. We illustrate our experiences using a Generic Patient Controlled Analgesic (GPCA) infusion pump’s empty reservoir alarm system, modeled and analyzed using the Simulink/Stateflow [8, 10] (deterministic) and PRISM [7] (probabilistic) modeling and verification approaches. The function of the sub-system considered in this paper is to raise an alarm if the remaining volume of drug in the reservoir is below a certain threshold. The possibility of sensor failures allows us to design multiple simple sensor filtering algorithms to reduce the false alarm rate (raising an alarm while there is still drug remaining in the reservoir) while maintaining the capability to detect and raise an alarm when the drug is running low. As will be shown, approaches that are seemingly sensible and can be verified to be correct using a traditional model checker may indeed have surprising behaviors when evaluated in a stochastic environment. Although we use a highly simplified version of the medical device subsystem, we show how exploration through traditional verification can give misleading results (or no useful results at all) and incorporating probabilistic reasoning can help illuminate weaknesses in system requirements, environmental assumptions, and the intended software solution.

We provide an overview of the case example in Section 2, followed by a brief description of the deterministic modeling and verification using Simulink/Stateflow tools in Section 3. In Section 4 we outline the probabilistic reasoning over the system using PRISM. We discuss our our findings and conclude in Section 5.

2. CASE EXAMPLE—THE GPCA

Infusion pumps are medical cyber-physical systems used for controlled delivery of liquid drugs into a patient’s body according to a physician’s prescription (the set of instructions that governs infusion rates for a medication). In an typical infusion system, the patient receives the drug through a intravenous needle inserted into the patient’s body and a clinician operates the device. Unfortunately, infusion pumps have been involved in numerous incidents [4] and hence there has been initiatives to improve their safety. To reduce the occurrence of critical accidents such as over- or under-infusion, modern infusion pumps are equipped with various kinds of sensors to detect and notify the clinician when hazards occur.

![Figure 1: Infusion System Overview](image)

Figure 1 shows a Generic Patient Controlled Analgesia (GPCA) infusion pump device in a typical usage environment, a hospital. In this paper we focus on the infusion pump’s reservoir sensor sub-system. The infusion pump has a drug reservoir that stores liquid drug to be delivered to the patient. The pump is equipped with a reservoir sensor that senses the drug volume in the reservoir and reports if the volume is below a certain threshold. Based on the sensor report, the system’s software commands aural and/or visual alarms to notify the clinician. The rationale for this sub-system is to avoid the hazard caused to a patient when the reservoir becomes empty while infusing a drug.

3. SIMULINK/STATEFLOW MODELING

As a part of a large ongoing initiative to analyse the complete infusion system[14], we modeled the system to better understand the requirements and verify that the requirements are met by our model. Our choice of modeling notation and tools was MathWork’s Simulink/Stateflow, that allows graphical modeling and simulation of dynamic systems. The reason for our choice of Simulink/Stateflow tool is manifold. First, its a widely used tool for modeling cyber-physical systems and since our effort was focused on providing a reference artifact that demonstrates effective modeling and verification for the infusion system as a whole, we choose a tool that is widely used. Second, the graphical modeling and expressibility provided by the notation is well suited to capture, analyse, and evaluate discrete and continuous control system behaviors. Finally, the tools provide a rich ecosystem of related tools, such as code generators, formal verification tools, test generators.

![Figure 2: System Architecture in Simulink](image)

As mentioned earlier, we in this paper include a small (and highly simplified) subset of the GPCA behaviors—the empty reservoir alarm sub-system. The simple sub-system architecture with three components: a Reservoir, a sensor and the alarm software, is shown in Figure 2. Each component is modeled as a Simulink block that has a state machine describing its functions. The high level description of the components are:

**Reservoir:** The reservoir holds the drug. The initial volume of the drug in the reservoir is set to some initial volume and the drug volume is then modeled as a non-increasing function with a floor of zero. In our simple model, there is no provision for refilling the reservoir.

**Sensor:** The sensor monitors the drug volume in the reservoir. When the volume drops below a certain threshold, the sensor outputs a boolean value: true representing that the volume is below the threshold and false otherwise. In our initial model, we assumed a perfect sensor with no failures.

1. To avoid hazard of reservoir volume becoming zero before an alarm is raised—considering the delays of the hardware and software—the threshold value is set slightly above zero.
We model the reservoir sensor system as a DTMC. PRISM is a free and open-source probabilistic model checker developed at the University of Oxford. Several types of probabilistic processes (MDPs), probabilistic automata (PAs) and probabilistic timed automata (PTAs) can be built and analyzed using PRISM. All of these models are specified using the probabilistic temporal logic of probabilistic automata (PAs) and probabilistic timed automata (PTAs). PRISM is a free and open-source probabilistic model checker, to continue our exploration and analysis of the reservoir sensor system. PRISM is a free and open-source probabilistic model checker developed at the University of Birmingham and the University of Oxford. Several types of probabilistic models including discrete-time Markov chains (DTMCs), continuous-time Markov chains (CTMCs), Markov decision processes (MDPs), probabilistic automata (PAs) and probabilistic timed automata (PTAs) can be built and analyzed using PRISM. All of these models are specified using the PRISM modeling language, a simple high-level language for model description based on a guarded command notation. We model the reservoir sensor system as a DTMC.

<table>
<thead>
<tr>
<th>Case</th>
<th>Component Behavior</th>
<th>Requirement</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sensor: Perfect. Software: Raise alarm on first “empty” reading.</td>
<td>vol &lt;= threshold ⇒ Alarm</td>
<td>Both Satisfied</td>
</tr>
<tr>
<td></td>
<td></td>
<td>vol &gt; threshold ⇒ No Alarm</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>vol &gt; threshold ⇒ No Alarm</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Sensor: Random failures. Software: Raise alarm on two consecutive “empty” reading.</td>
<td>vol &lt;= threshold ⇒ Alarm in next steps</td>
<td>Neither Satisfied. Counter example: Sensor outputs always incorrect</td>
</tr>
<tr>
<td></td>
<td></td>
<td>vol &gt; threshold ⇒ No Alarm</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Deterministic Modeling and Analysis
The move from Simulink/Stateflow to PRISM was in this initial study done manually. Given our experience with extraction of models from Simulink and Stateflow, automating this process would be relatively straightforward (but quite time consuming and labor intensive). When we transfer to a probabilistic modeling tool, we have more freedom in how we model the various components of the system. For example, where we previously modeled the reservoir as a non-increasing function, we can now model it as a non-increasing function with a probability of 0.95 that the volume decreases (there is an infusion in progress) and a probability of 0.05 that the volume stays the same (the infusion is paused for some reason)\(^2\). Similarly, we can model the failure behavior of the sensor with more fidelity. In this case, we can assign a probability of \(x\) that the sensor will signal “empty” when, in fact, the reservoir is not empty, and a probability of \(y\) that it will signal “not empty” when the drug volume is, in fact, below the threshold. The question now, of course, is what effect does this kind of probabilistic behavior have on the satisfaction of the system level requirements given a specific sensor filtering algorithm.

\[
\begin{align*}
\text{const double pr} &= 0.99; \quad \text{// prob. that we will infusion} \\
\text{const double pe} &= 0.99; \quad \text{// prob. of true positive ‘‘empty’’} \\
\text{const double pne} &= 0.95; \quad \text{// prob. of true negative ‘‘not empty’’}
\end{align*}
\]

module reservoir

\[
\begin{align*}
\text{vol: [0..inVol] init inVol;} \\
\text{//The infusion can progress (or pause) until volume is 0} \\
\text{[a] (vol>0) \rightarrow pr:(vol\rightarrow vol-drip) + (1-pr):(vol\rightarrow vol);} \\
\text{[a] (vol<0) \rightarrow (vol=0);}
\end{align*}
\]

endmodule

module sensor

\[
\begin{align*}
\text{sensor_out: bool init false;} \\
\text{[a] (vol<empty) \rightarrow pe:(sensor_out='true)} \\
\text{\quad + (1-pe):(sensor_out='false);} \\
\text{[a] (vol>empty) \rightarrow pne:(sensor_out='false)} \\
\text{\quad + (1-pne):(sensor_out='true);} \\
\end{align*}
\]

endmodule

module software

\[
\begin{align*}
\text{alarm: bool init false;} \\
\text{[a] true \rightarrow (alarm=\text{sensor_out} \mid \text{alarm});}
\end{align*}
\]

endmodule

Figure 3: PRISM model for Case 4 in Table 2

An example of our model in PRISM is shown in Figure 3. At the top we define the probabilities used in the model: the probability that we will infuse in a specific time-step (pr), the probability of a true positive reading of the sensor (pe, the probability of reservoir below the threshold and the sensor reports it), and the probability of a true negative (pne, the probability of reservoir not empty and sensor reports it). Note that the probabilities of the sensor providing false negative as opposed to a false positive are different; this allows us to provide a far more accurate model of the environment than what is possible in our Simulink/Stateflow models.

Our PRISM model has the same components as discussed previously. The reservoir contains an initial volume of drug. If the volume of remaining drug is larger than zero, the volume will be decreased by one with probability pr and remain the same (infusion is paused) with probability (1-pr). The sensor will sense the reservoir and if the remaining drug volume is less than or equal to the threshold, the sensor will accurately report that the tank is “empty” with a probability of pe and provide a false negative with probability (1-pe). Similarly, the sensor may report a false positive if the reservoir is not empty. The software will, in our simplest model, simply latch the alarm to true when an empty reservoir is detected.

Given the sensor model that has a uniform probability of failure in every step, the probability of a false alarm will be predicated on the length of the infusion interval. For example, if the infusion interval is long, the sensor will be given many opportunities to provide two consecutive false positive readings leading to a violation of our “no false positives” requirement. Of course, the likelihood of a long infusion is dependent on both the rate of infusion, the initial volume in the reservoir, and the probability that the infusion is paused, all of which will affect the probably that our requirements are actually met.

When running our analysis in PRISM, we will use a sensor model that allows arbitrary failures as in Case 3 in Section 3; the case where none of our requirements could be verified to always hold in our system. Given the new probabilistic behavior of the sensors and the reservoir, however, there is a chance that our requirements are met even with the simplistic software defined in Figure 3. Case 4 in Table 2 shows the results over our simplest model. In this analysis, we assume the initial drug volume in the reservoir to be 20 units and the infusion rate to be one unit per step (drip=1 in the model) when the pump is infusing. Further, we assume that the pump is paused 5% of the time, the probability of a false positive sensor reading is 0.05, and a false negative reading is 0.01.

As can be seen, our requirements may not hold all the time (as proved in the previous section), but we have a reasonable probability that the requirement will actually hold during operation (if our assumptions about the failure rates and infusion rates are reasonable). Given our assumptions on the environment, if the reservoir is empty, there will be an alarm with a probability of 95% and we will not raise a false alarm with a probability of 85%. This behavior, unfortunately, is deemed unacceptable and we need to improve our solution. Thus, we again explore various solutions (and relaxed requirements) as discussed in Section 3. The results of our probabilistic analysis can be seen in Cases 4-6 in Table 2 (we will discuss Case 6 shortly). All results in Table 2 use the probabilities discussed above.

To decrease the probability of false positives, we move to our solution of using two consecutive readings of “empty” to raise the alarm (Case 5). Again, we need to relax our requirements to allow for a slight delay in raising the alarm after the remaining volume becomes too low. As can be seen, we now reduce the false alarm rate to an acceptable level (4% chance of a false alarm) while boosting the probability of meeting our requirement to 98%. Of course, a direct comparison in performance with Case 4 cannot be made since we there used a different requirement (immediate action when the drug level is low). If we use our relaxed requirement and our Case 4 software, we increase the probability of correctly raising an alarm when the drug volume is low to 99%, but the false positive rate is still too high since this requirement did not change (see the two last rows for Case 4). Thus, we consult our stakeholder and the performance is deemed acceptable.

\(^2\)Note here that all numbers are entirely fictional and are included for discussion purposes only.
<table>
<thead>
<tr>
<th>Case</th>
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<th>Requirement</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td><strong>Software:</strong> Raise alarm on first “empty” reading.</td>
<td>$\text{vol} \leq \text{threshold} \Rightarrow \text{Alarm}$</td>
<td>Satisfied with $P=0.95$</td>
</tr>
<tr>
<td></td>
<td>$\text{volume} &gt; \text{empty} \Rightarrow \text{No Alarm}$</td>
<td>Satisfied with $P=0.95$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\text{vol} \leq \text{threshold} \Rightarrow \text{Alarm in next step}$</td>
<td>Satisfied with $P=0.99$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\text{volume} &gt; \text{empty} \Rightarrow \text{No Alarm}$</td>
<td>Satisfied with $P=0.85$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td><strong>Software:</strong> Raise alarm on two consecutive “empty” reading.</td>
<td>$\text{vol} \leq \text{threshold} \Rightarrow \text{Alarm in next step}$</td>
<td>Satisfied with $P=0.98$</td>
</tr>
<tr>
<td></td>
<td>$\text{vol} &gt; \text{empty} \Rightarrow \text{No Alarm}$</td>
<td>Satisfied with $P=0.96$</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td><strong>Sensors:</strong> Three redundant</td>
<td>$\text{vol} \leq \text{threshold} \Rightarrow \text{Alarm}$</td>
<td>Satisfied with $P=0.99$</td>
</tr>
<tr>
<td></td>
<td><strong>Software:</strong> Raise alarm when two out of three sensors indicate “empty”.</td>
<td>$\text{vol} &gt; \text{threshold} \Rightarrow \text{No Alarm}$</td>
<td>Satisfied with $P=0.89$</td>
</tr>
</tbody>
</table>

Table 2: Probabilistic Modeling and Analysis

At this point, the exploration of possible solutions and acceptable requirement could be over; one declares victory and moves on. Luckily, stakeholders and engineers closely inspect all assumptions made in the analysis and discover that the initial drug volume of 20 units is unrealistically low (infusion time is generally quite long) and the probability that the pump is paused is far more likely than the 5% we used in our analysis. To see if this had any impact on our analysis, we modify the initial conditions for our analysis and run our analysis with an initial drug volume of 100 units and a probability of the pump being paused at 50%. We are happy to see that under these environmental assumptions, the probability of meeting our “raise the alarm” requirement goes to 99%—we have the performance we seek. Unfortunately, given the long infusion time enabled by the larger drug volume and extensive pausing of the infusion, the probability of nuisance alarms goes from 4% per infusion session to 37%—there is simply more time for sensor failures to manifest themselves. In fact, the increase in the probability of having an alarm raised when the volume become too low is entirely caused by the increase in the false positive rate (if we erroneously raise an alarm and leave it on, it will still be on when the reservoir goes empty).

Table 3 contains the results for various initial conditions. In the case of very long infusions (1000 units initial drug volume and a 50% chance of a paused infusion), we are more or less certain of getting a false alarm during every infusion (the probably of meeting our “no false alarms” requirement goes to zero as infusion time increases). Thus, our exploration of the Twin Peaks must continue.

At this point, more radical changes to our proposed solution may be warranted. For example, one could consider an alternate system architecture with three redundant sensors and simple majority voting (two out of three raises the alarm). The performance of such an architecture and voting mechanism is included as Case 6 in Table 2. Unfortunately, the false positive rate is not acceptable and our exploration must continue. Combinations of triple redundancy and sensor filtering may yield the performance we seek with the existing sensors. Possibly, we may need to rethink our choice of sensor and aim for something far more reliable.

5. DISCUSSION AND CONCLUSION

The process of discovering and refining requirements, and exploring the architecture, design, and behavior of a system is an iterative, interactive, and highly non-trivial task. In this paper we illustrated how model based developments and verification techniques can help (and hinder) in this exploration.

As Miller et al. have pointed out, the process of discovering and refining models and requirement in tandem is a powerful process for improving both [12]. Nevertheless, the tools used in traditional verification approaches rely on “yes”—“no” answers; either a requirement is met or it is not. Since all systems really on some level of cooperation from their environments to operate as intended [5, 6], assumptions regarding how this environment operates are crucial in the verification and exploration of the system requirements and the system architecture, design, and behavior. When exploring a systems architecture containing both logical (software) and physical (electrical or mechanical) components where the system requirement—as they should [5]—are expressed in terms of the physical system, not the software, capturing assumptions about the environment and the various components involved becomes a challenge. By their nature, all physical components will experience failures in various forms and modeling and analyzing these failure modes using traditional verification techniques such as model checking and theorem is difficult; either we make unrealistic assumptions about the environment and physical components (e.g., one failure assumption) and succeed in our verification, or we make realistic assumptions (e.g., unlimited Byzantine failures) and fail in our verification.

By moving to a verification framework where we can model the stochastic nature of many system components, we believe we will be able to more effectively perform the requirement-solution exploration needed to devise an effective engineered system meeting realistic requirement. Note
here that we are not advocating abandoning traditional verification; such verification is invaluable when establishing desirable properties of many classes of models. We are advocating the inclusion of probabilistic reasoning in the process as a complement to better understand the tradeoffs involved.

The intent of this paper is to demonstrate the use of probabilistic verification techniques as a Twin Peaks exploration tool. Although the case example used was exceedingly simple and the choice of values used for analysis was random, it helps to illuminate the impact of stochastic sensor and environment behavior on the architectural choices and possibility of satisfying requirements. Our goal was to discuss the possibilities and set the stage for a broader initiative applying these stochastic techniques to explore and assess the requirements and solution spaces for Cyber Physical Systems.

6. REFERENCES