Probabilistic Detection of Morphologic Indicators for Beach Segmentation With Multitemporal LiDAR Measurements

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Abstract—Airborne light detection and ranging (LiDAR) surveys provide a rich data source of topographic information. However, beach monitoring with LiDAR data has been mostly limited to visualization and first-order measures derived from digital elevation models (DEMs). To exploit more information from multitemporal LiDAR data acquired over a beach, we extract surface features to detect morphologies that are indicative of shoreline change patterns. First, through cross-shore profile sampling of LiDAR-derived DEMs, the continuous 3-D beach surface is parameterized into several 1-D morphologic features progressing alongshore. Profiles are subsequently partitioned into binary erosion or accretion classes dependent on measured shoreline change between surveys. Then, a feature’s class separability is quantified using information divergence measures nonparametrically constructed via Parzen windowing. The more interclass separation provided by a feature, the greater its discriminative potential, and the higher its ranking as a morphologic indicator. Rankings are computed across the survey epochs to evaluate performance stability of the features. Finally, the top-ranked features are im-plemented within a naive Bayes classifier to assess their ability to segment terrain more likely to erode. Results demonstrate the utility of the developed framework to systematically extract and incorporate useful morphologic information from LiDAR data for discerning patterns in beach change.

Index Terms—Divergence, light detection and ranging (LiDAR), pattern classification, shoreline erosion.

I. INTRODUCTION

B EACHES define a geomorphic barrier between land and sea, protecting vital infrastructure and natural habitat. However, sandy beaches are dynamic landforms, constantly evolving with time and subject to the continual threat of erosion and coastal flooding [1]. This evolution constitutes a highly variable 3-D process, consisting of rapid variations due to storm events, seasonal variations due to changes in wave and wind climate, and long-term variations due to sea-level rise, longshore sediment transport, anthropogenic influences, and extreme storm events [2]. Monitoring and analysis of beach evolution is critical to the future sustainability of these environments and the economies that depend on them [3].

Airborne scanning light detection and ranging (LiDAR) has revolutionized coastal monitoring, making it possible to measure 3-D changes in topography at spatial resolutions needed to advance science and monitor erosion along coastlines efficiently and accurately [4]. This revolution has been propelled by topographic LiDAR systems that operate in the near-IR portion of the electromagnetic spectrum and by bathymetric systems that operate in the blue-green range of the spectrum [5], [6]. The focus herein is on the application of data from small-footprint discrete-return topographic LiDAR systems [7], which typically record two or more returns per emitted pulse (including first and last). Small-footprint systems enable beach mapping with average spatial resolutions greater than 1 point/m² and achievable positional accuracy values of 15–30 cm horizontal (x, y) and 5–10 cm vertical (z) [8]. However, point density will vary locally depending on flight parameters, scan angle, beam divergence, surface properties, and pulse repetition rate, among other factors [9], [10].

Numerous studies have demonstrated the application of repeat-coverage LiDAR surveys for quantifying changes in beach terrain (e.g., [4] and [11]–[15]). Generally, this is accomplished by differencing LiDAR-derived digital elevation models (DEMs) or contour vectors to estimate change in sediment volume or shoreline position between surveys. In addition to 67 elevation change, many different features can be derived, where 68 a feature in this context refers to a property of the sampled surface, such as slope or curvature [16]. For coastal analysis, 70 feature extraction has been used to track changes in nearshore 71 geomorphology, such as berm formation or dune migration 72 and coastal flooding [1]. This evolution constitutes a highly variable 3-D process, consisting of rapid variations due to storm events, seasonal variations due to changes in wave and wind climate, and long-term variations due to sea-level rise, longshore sediment transport, anthropogenic influences, and extreme storm events [2]. Monitoring and analysis of beach evolution is critical to the future sustainability of these environments and the economies that depend on them [3].

Relatively unexplored are analysis methods that venture beyond change detection [19]. With the accumulation in coastal LiDAR time series data, the exploration of enhanced analysis methods to exploit more information from the data is warranted [15], [19]. In this regard, data mining and pattern classification techniques offer great potential to progress beach monitoring with LiDAR data beyond visualization and simple (first-order) measurements made from DEMs.
In the following discussion, we develop a systematic framework to mine multitemporal LiDAR data acquired over a beach and detect morphologies (surface features) indicative of observed patterns in shoreline change behavior. The problem is approached as one of optimal feature selection for binary classification. Several different morphologic features are extracted and then ranked based on their ability to separate shoreline erosion and accretion zones (profiles) along the beach. A feature’s probabilistic relation to class occurrence is estimated from the data and used to evaluate their discriminative potential. The top-ranked features are then implemented within a naïve Bayes classifier to assess their effectiveness in discerning shoreline change patterns alongshore.

This paper is structured as follows. Section II outlines the feature extraction and evaluation methodology. Section III describes the LiDAR data set and morphologic features examined with the developed framework. Section IV presents feature analysis and performance results for the study area. Section V provides concluding remarks.

II. METHODOLOGY

Given a time series of airborne LiDAR surveys acquired over a beach, our approach seeks connections between variation in morphology and shoreline change patterns observed along the beach. Those morphologies most indicative of shoreline change behavior are systematically revealed. Fig. 1 outlines our approach, and the steps are detailed below.

A. Morphologic Data Mining

DEM Generation: The raw LiDAR data consist of point clouds of irregularly spaced \( x, y, z \) values providing a 3-D representation of the ground and land cover. For multireturn data, the last (or single) return data are utilized to minimize the probability of land cover biasing the surface elevations. Filtering is applied to remove nonground points [20], and the points are then interpolated into bare-earth elevation grids (e.g., [15] and [21]) at a desired spatial resolution (e.g., 1 m). The result is a sequence of LiDAR-derived DEMs representing the beach surface and surrounding terrain at the time of each survey.

Profile Sampling: The coordinate system traditionally used for studying shoreline change consists of a local 2-D Cartesian system oriented with alongshore (shore parallel) and cross-shore (shore perpendicular) axes [2]. A MATLAB program was developed to sample the LiDAR-derived DEMs by extracting elevation values along cross-shore profile lines oriented orthogonal to a shore-parallel landward baseline [19]. This provides the \( x, y \) coordinates and elevation values along each profile at a user-defined spacing (see Fig. 2). The same baseline is used to extract profiles across the surveys, and the algorithm can determine the intersection of each profile with a shoreline contour or other vector (such as dune line). The cross-shore profile method provides a standardized reference frame from which to extract morphology and relate to shoreline change.

Effective beach management relies upon detecting and monitoring zones of beach more prone to erosion, which motivated the use of LiDAR-derived data to detect shoreline changes.
While it is possible that feature transformation methods, such as principal components [24], or methods that project the original features to a higher dimensional space, such as support vector machines [25], could lead to features that produce even greater class separation, such methods were not used here. Our intent is to assess the value of individual features relative to each other and retain clear physical meaning for greater benefit to coastal researchers.

**Density Estimation:** Airborne LiDAR data in feature space can exhibit multimodal and distinctly non-Gaussian properties [16], [19]. To avoid distributional assumptions, the nonparametric Parzen window method [23] is employed to estimate the class-conditional pdfs $p(f_j|C_i)$ from the data. The Parzen density estimate $\hat{p}(\vec{x})$ can be written as

$$\hat{p}(\vec{x}) = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{h^d} K_h (\frac{\vec{x} - \vec{x}_i}{h})$$

where $K_h$ is the kernel function, $h$ controls the length of the $d$-dimensional kernel window commonly referred to as the kernel bandwidth (here $d = 1$), $N$ is the number of points in the data set, $\vec{x}_i$ is the vector of current feature values $(f_j)$, and $\vec{x}$ are the points at which the pdf is to be estimated along each feature dimension.

Bias in the Parzen estimate can be asymptotically reduced to zero by selecting a symmetric unimodal kernel function [26]; therefore, $K_h$ is selected to be a 1-D Gaussian kernel. The Gaussian kernel has been shown to perform well despite the presence of correlated samples, which is evident in our features [27], [28]. Furthermore, it provides infinite support and useful analytic properties the importance of which will become clear below. Setting $K_h$ to a 1-D Gaussian kernel, the Parzen density estimate at a point $x$ is

$$\hat{p}(x) = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{\sqrt{2\pi}h} e^{-u^2/2}$$

where $u = x - x_i/h$ and $h$ acts similar to the familiar standard deviation of the normal distribution.

Crucial to Parzen estimation is the selection of the bandwidth $h$ [23]. Bandwidths that are too small over-fit the samples yielding pdf estimates that are spiky in appearance, and 222
bandwidths that are too large yield pdf estimates that are overly smoothed. Its optimal value (the value that minimizes the measure of dissimilarity between the estimated pdf and the true pdf) strongly depends on the type of data, the number, and the amount they are corrupted by noise [29]. Automatic bandwidth selection methods can be applied to estimate \( h \) directly from the sample data with various tradeoffs in performance [30], [31].

For this work, a maximum likelihood (ML) approach is implemented [32]. Given the Parzen density estimator, i.e., \( \hat{p}(x) \), the goal is to maximize the expected value of the log-likelihood, i.e., \( E_x [\log \hat{p}(x)] \), with respect to \( h \). The expectation is approximated by the sample mean, resulting in

\[
J_{ML}(h) = \frac{1}{N} \sum_{i=1}^{N} \log (\hat{p}(x_i)).
\]

Substituting in the Parzen density estimate of (2), this becomes

\[
J_{ML}(h) = \frac{1}{N} \sum_{i=1}^{N} \log \left( \frac{1}{N} \sum_{j=1}^{N} \frac{1}{\sqrt{2\pi h}} e^{-\frac{(x_i-x_j)^2}{2h^2}} \right). \tag{3}
\]

However, this criterion exhibits an undesirable global maximum at the null kernel size. To avoid this situation, (3) is modified in accordance with the leave-one-out cross-validation estimate, \( \hat{p}_{-i}(x) \) [26]. This yields

\[
J_{ML}(h) = \frac{1}{N} \sum_{i=1}^{N} \log \left( \frac{1}{N-1} \sum_{j=1, j \neq i}^{N} \frac{1}{\sqrt{2\pi h}} e^{-\frac{(x_i-x_j)^2}{2h^2}} \right). \tag{4}
\]

The optimal bandwidth based on the ML criteria is then

\[
h_{ML} = \arg \max_h J_{ML}(h).
\]

Equation (4) can be directly maximized with respect to \( h \) through the derivative and numerically solved, which is an attractive property for coupling the Gaussian kernel with an ML approach. An iterative Newton–Raphson scheme is implemented to solve for \( h_{ML} \). To avoid ill effects on bandwidth convergence due to data with ties (equivalent values stemming from the precision level used for feature discretization), the feature data are corrupted with uniformly distributed random noise at a level that preserves original sample integrity following an approach outlined in [33]. The optimal ML bandwidth is then selected as the average bandwidth based on several simulations.

The ML bandwidth is equivalent to choosing the bandwidth that minimizes the Kullback–Leibler divergence between the unknown true density \( p(x) \) and the Parzen estimate \( \hat{p}(x) \). A potential tradeoff with using the ML criterion is that it can be affected by tail behavior of the true density \( p(x) \) and result in an overly smoothed estimate [34]. Although a concern, estimated bandwidths generally fell within about 1/40th the range of feature values or less for those investigated in this analysis, which is not overly smoothed.

Once the class-conditional pdfs \( p(f_j|C_i) \) are estimated, each feature’s class separability is quantified using divergence measures described in Section IV. The more interclass separation provided by a feature, the greater its ranking and discriminative potential as a morphologic indicator of erosion or accretion occurrence.

### III. DESCRIPTION OF DATA SET AND FEATURES

The developed framework is applied to a time series of 271 seven airborne LiDAR surveys acquired along the St. Augustine 272 Beach region of northeast Florida, USA, between August 2003 and February 2007. This resulted in six sequential data epochs of varying temporal lengths (see Table I), ranging from a 275 maximum of almost two years to less than two months. A 276 10-km long stretch of the coastline comprises our study area 277 (see Fig. 5). This region was selected because it contains both a historical accretion zone and a highly erosive pier zone, 279 which is armored with revetments and requires periodic beach

<table>
<thead>
<tr>
<th>Time period</th>
<th>Dates</th>
<th>Length (years)</th>
</tr>
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<tbody>
<tr>
<td>Epoch 1</td>
<td>August 16, 2003 to March 10, 2005</td>
<td>~1.6</td>
</tr>
<tr>
<td>Epoch 2</td>
<td>March 10, 2005 to May 07, 2005</td>
<td>~0.2</td>
</tr>
<tr>
<td>Epoch 3</td>
<td>May 07, 2005 to October 11, 2005</td>
<td>~0.5</td>
</tr>
<tr>
<td>Epoch 4</td>
<td>October 11, 2005 to January 27, 2006</td>
<td>~0.3</td>
</tr>
<tr>
<td>Epoch 5</td>
<td>January 27, 2006 to June 14, 2006</td>
<td>~0.4</td>
</tr>
<tr>
<td>Epoch 6</td>
<td>June 14, 2006 to February 19, 2007</td>
<td>~0.8</td>
</tr>
</tbody>
</table>

Fig. 5. LiDAR-derived bare-earth DEM of the August 2003 survey showing the entire 10-km study area and a zoomed-in view of the pier region showing dunes, beach, and filtered buildings. Elevation is relative to NAVD88. The \( x \) and \( y \) axes are measured in UTM Zone 17 m.
nourishment to replace lost beach area. Effects from two nour-ishments conducted in 2003 and 2005 are observed in the data. Both nourishment focused on the approximately 2-km revetment region of the pier.

The surveys were conducted by the University of Florida using an Optech International ALTM 1233 airborne LiDAR system operating at a laser pulse rate of 33 kHz and a near-infrared wavelength of 1064 nm. The system can record up to two returns per an emitted pulse. Surveys were conducted during low tide to capture more exposed land surface. Nominal values for the surveys included a flying height of 600 m above ground level, flight speed of 60 m/s, and a mirror scan rate of 28 Hz resulting in a mean ground point density of approximately 1.3 points/m². Only the last-return data points were utilized for this analysis.

296 A. Data Processing

Vertical bias in the LiDAR observations stemming mostly from GPS-induced trajectory errors are sometimes present [8]. The offsets between flights were removed by vertically shifting the LiDAR points to match higher accuracy kinematic GPS calibration lines acquired along static road surfaces within the survey area. For the LiDAR system, calibration, and flight parameters used in this work, this correction generally yields vertical accuracy values of 5–10 cm relative to ground-survey measurements over slowly varying surfaces [8]. The initial LiDAR elevations were provided as ellipsoid heights and were therefore transformed relative to the North American Vertical Datum of 1988 (NAVD88) using the GEOID03 model [35]. The data were then filtered using an adaptive algorithm in [36] to remove nonground points without altering the natural beach morphology. The filtered points were interpolated into 1 m × 1 m bare-earth DEMs (see Fig. 5) using ordinary kriging [14], [37]. The data processing procedure and parameters were kept constant for all data sets.

Shoreline Delineation: To minimize impact from wave run-up and tidal fluctuations, the mean higher high water (MHHW) tidal datum was selected as the proxy for shoreline [38], [39]. MHHW is the average of the higher high water height of each tidal day observed over a 19-year tidal epoch. The nearest NOAA tidal gauge to the study area was used to determine the MHHW elevation of ∼0.6 m relative to NAVD88 [14]. The 0.6-m contour was delineated to create a contiguous shoreline vector in each DEM.

To gauge the fluctuation in MHHW–NAVD88 datum separation over the entire 10-km study area, the VDatum software tool was used (NOAA, http://vdatum.noaa.gov, 2011). The appropriate VDatum grid covering the northeast Florida coast was used as input. MHHW elevations relative to NAVD88 were computed at the northern and southern edge of the study area. The difference in elevation was approximately 0.05 m, which falls within the range of VDatum’s vertical uncertainty for the region. This result validates the use of a constant 0.6-m NAVD88 reference elevation for MHHW shoreline in the study area.

Additional uncertainty in shoreline position is introduced by LiDAR positional errors, the DEM gridding process, and correcting for GPS-induced trajectory errors are sometimes present [8]. Nonetheless, the expected uncertainty in LiDAR-derived DEM shoreline positions, even for gently sloping beaches, is more than acceptable for erosion monitoring where the concern is with significant changes in shoreline, typically several meters. For a more detailed treatment of positional uncertainty in LiDAR-derived shorelines, refer to [40].

B. Morphologic Feature Extraction

Cross-Shore Profile Sampling: Following the steps in our framework (see Fig. 1), cross-shore profiles were extracted every 5 m in the alongshore direction and 1-m point spacing in the cross-shore direction extending to the MHHW shoreline (see Fig. 2). The 5-m alongshore spacing was selected to provide dense coverage without over redundancy of information due to the high spatial correlation of beach topography and shoreline change. A 1-D moving average filter of length 5 m was applied to each profile to reduce any residual high-frequency artifacts that may sometimes be present due to scan line artifacts or isolated nonground points that the filter failed to remove.

Profile Segmentation: Profiles were then segmented into “erosion” or “accretion” tending classes (C₁ or C₂), depending on the shoreline change experienced between each survey epoch (see Table I). For certain data epochs, the majority of the 359 shoreline either eroded or accreted. In such cases, the median value was used to segment the classes. C₁ still corresponds to profiles with the most negative tending shoreline change (high erosion or low accretion), and C₂ still corresponds to the most positive tending shoreline change (low erosion or high accretion). Fig. 6 shows the variability in shoreline change and

Fig. 6. Shoreline change computed alongshore for each epoch. Solid line is the class segmentation line (above = “accretion,” below = “erosion”). The erosion hot spot in the pier vicinity is located in the range of ∼3000 to 5000 m alongshore from north to south. Note that Epoch 6 only covers the northern section of beach because of limited coverage in the February 2007 survey.
Examined Morphologies: A total of nine features (see Table II) were extracted to evaluate their discriminative potential as morphologic indicators for the region. The nine features were selected based on their historical usage or suspected potential as a measure to describe the alongshore character of the subaerial beach surface (see Fig. 3).

- **Beach slope** was estimated using a linear fit through the profile elevation values from shoreline to dune toe line and represents the slope of the subaerial beach profile. Beach width is the width of the subaerial beach. Near-shoreline slope is a linear fit through the profile elevation values from shoreline to a few meters landward. It approximates the slope in the more steeply sloping foreshore region of the subaerial profile. Volume-per-width is the volume per unit width (area) under the subaerial profile above 0 m NAVD88 computed using trapezoidal integration. Mean profile curvature is the mean value of surface curvature estimated every 3 m along the subaerial profile using a central-difference approximation. Orientation is another measure to describe the alongshore character of the subaerial beach surface (see Fig. 3).

- **Deviation-from-trend** is the deviation of elevation values along the subaerial profile and provides a measure of profile surface roughness. Orientation is the angle in degrees the local shoreline makes relative to azimuth (clockwise from north) estimated by piecewise linear fitting along the shoreline. Deviation-from-trend (m) is the relative alongshore distance (m) from north to south. The reduction in beach width and reaction of curvature near 4000 m alongshore coincides with the beginning of the revetment wall in the pier region.

### Table II

<table>
<thead>
<tr>
<th>Feature ($f_j$)</th>
<th>Units</th>
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<tbody>
<tr>
<td>Beach slope</td>
<td>(m/m)</td>
</tr>
<tr>
<td>Beach width</td>
<td>(m)</td>
</tr>
<tr>
<td>Near-shoreline slope</td>
<td>(m/m)</td>
</tr>
<tr>
<td>Volume-per-width</td>
<td>(m$^3$/m)</td>
</tr>
<tr>
<td>Mean gradient</td>
<td>(m/m)</td>
</tr>
<tr>
<td>Mean curvature</td>
<td>(m/m$^2$)</td>
</tr>
<tr>
<td>Orientation</td>
<td>(degrees)</td>
</tr>
<tr>
<td>Standard deviation of</td>
<td>(m)</td>
</tr>
<tr>
<td>height</td>
<td></td>
</tr>
<tr>
<td>Deviation-from-trend</td>
<td>(m)</td>
</tr>
</tbody>
</table>

Fig. 7. Three features extracted from the August 2003 profiles; y axes are the relative alongshore distance (m) from north to south. The reduction in beach width and reaction of curvature near 4000 m alongshore coincides with the beginning of the revetment wall in the pier region.
variable $x$ can be approximated by a discrete sum using the 445 sampled distributions as follows:

$$D_{kl}(P, Q) = \sum_x P(x) \Delta x \log \frac{P(x)}{Q(x)}$$  \hspace{1cm} (5)

where $\Delta x$ is the interval between samples of the quantized 447 random variable [16]. In this work, $\Delta x$ was scaled by the range 449 of values for each feature to reduce effects stemming from 450 inconsistent binning on the resultant divergence calculations. 451 $D_{kl}$ is measured in bits of information, nonnegative, and equal 452 to zero if and only if $P = Q$ [22], [23].

JSD: JSD [42] can be expressed as

$$JSD(P, Q) = \pi_1 D_{kl}(P, M) + \pi_2 D_{kl}(Q, M)$$  \hspace{1cm} (6)

where $\pi_1 + \pi_2 = 1$, and $M = \pi_1 P + \pi_2 Q$. JSD is a nonnega-
455 tive symmetric bounded form of $D_{kl}$ between two pdfs and their 456 weighted average, $M$. It is equal to zero if and only if $P = Q$

The form of JSD in (6) naturally fits to our two-class 457 decision problem by letting $P = p(f_j | C_1)$ and $Q = p(f_j | C_2)$, 458 and $\pi_1$ and $\pi_2$ are the binary class priors, respectively.

By applying JSD within a decision-theoretic framework as we have done here, it is equivalent to the well-known mutual 461 information measure $I(f_j; C)$ between the random feature 462 variable $f_j$ and the random class variable $C$ (see Appendix). 463 We are explicitly quantifying the amount of information gained 464 from feature $f_j$ on knowing the class label $C$ through the JSD 465 measure. In this case, the question being asked is whether the 466 profile being queried is of class “erosion” tending or “accre-
467 tion” tending. For this binary question, the maximum amount 468 of information possible is 1 bit. $JSD(P, Q) \in [0, 1]$, which 469 means we expect good features (more interclass separability) to 470 yield $0 \ll JSD < 1$.

Normalized JSD: Separability based on $D_{kl}$ and, subse-
471 quently, JSD does not directly consider the inherent complexity 472 of each feature [44]. To account for feature complexity in our 474 rankings, we construct a normalized information divergence 475 [45]. We select a feature’s probabilistic entropy, i.e., $H(f_j)$, as 476 the complexity measure

$$C_{P,Q} = H(f_j) = H(\pi_1 p(f_j | C_1) + \pi_2 p(f_j | C_2))$$

$$= H(\pi_1 P + \pi_2 Q)$$

where $H$ is the information-theoretic entropy function [22], 478 and JSD of (6) as the information term, $I_{P,Q}$, to construct an 479 information divergence as follows:

$$NJSD(P, Q) = \frac{I_{P,Q}}{C_{P,Q}} = \frac{JSD(P, Q)}{H(f_j)} = \frac{JSD(P, Q)}{H(\pi_1 P + \pi_2 Q)}$$  \hspace{1cm} (7)

The measure is referred to as Normalized JSD (NJSD). From 481 (7), we see that NJSD is bounded by 1 and equal to 0 if and only 482 if $P = Q$, which implies values close to 0 have poor class sep-
483 aration. More tightly bounded, $NJSD(P, Q) \in [0, 1/H(f_j)]$. 484 Therefore, we expect good features to yield larger NJSD values, 485 $0 \ll NJSD < 1/H(f_j)$.

To better understand the behavior of (7), take, for example, 487 the following two cases given two features, i.e., $f_1$ and $f_2$: 488

Case 1: $f_1$ and $f_2$ have equal entropy, i.e., $H(f_1) = H(f_2)$, 489 but $JSD(f_1) < JSD(f_2)$, $f_2$ is selected as the more opti-
490 mal feature because $f_2$ provides more separation between 491 classes based on JSD.

Case 2: $f_1$ and $f_2$ have equal divergence, i.e., $JSD(f_1) = 493 JSD(f_2)$, but $H(f_1) < H(f_2)$, $f_1$ is selected as the more 494 optimal feature because it requires less complexity to de-
495 scribe its behavior.

NJSD selects the feature that provides a compromise be-
497 tween least uncertainty and most information gained on know-

ing the class label. This behavior is desirable in feature selection 499 problems. Features with lower relative entropy (less uncer-

ainty) are more stable and, in return, can potentially enable 501 more reliable classification outside of the training data.

Median Metric: For comparison to divergence-based rank-

ings, a simple median-based measure originally constructed for 504 LiDAR-derived (non-Gaussian) samples was selected to assess 505
Fig. 10. Ranking of each feature’s interclass separation between erosion and accretion tending profiles by epoch and for each of the four measures. For NJSD, JSD, and median, 1 indicates the feature that is most separable and 9 least separable. For $R^2$, 1 indicates the feature most correlated to shoreline change and 9 least correlated.

1-D interclass separation. The median-based measure is unitless and has the following form [16]:

$$d_j = \left| \frac{\text{median} \left( f_j^{\text{erosion}} \right) - \text{median} \left( f_j^{\text{accretion}} \right)}{\sqrt{\left( \text{mad} \left( f_j^{\text{erosion}} \right) \right)^2 + \left( \text{mad} \left( f_j^{\text{accretion}} \right) \right)^2}} \right|$$

where $f_j^{\text{erosion}}$ represents feature $j$ for all points in class erosion. Index $j$ runs from 1 to 9 for all nine features, and mad is the median absolute deviation about the cluster median value. Equation (8) is a measure of the separation of the cluster centers in feature space relative to their spread that is robust to outliers and computationally fast. There is no theoretical maximum value for (8) in the limit as cluster spreads vanish. In general, the larger the value in (8), the more separable the classes for a given feature [16].

B. Feature Performance

Each feature’s separability between “erosion” and “accretion” tending profiles ($C_1$ or $C_2$) was ranked for the six data epochs using the two divergence measures, namely, JSD and NJSD, and the median metric. Those features that rank highest yield the most interclass separation according to that measure. Divergences, i.e., Pearson’s squared correlation coefficient ($R^2$) was computed to evaluate the strength of a linear relationship between the features and shoreline change [23]. In general, a feature that is well correlated with measured shoreline change (i.e., change viewed as a continuous random variable rather than a binary class label) would be expected to also exhibit a significant divergence in the class-conditional pdfs. Instances in which the feature’s ranking from the divergences is different from those suggested by correlation indicate potential cases where using the full pdf instead of merely second-order descriptions is important.

Individual Epochs: Fig. 10 shows the rankings of each feature’s interclass separation based on JSD, NJSD, and median metric, and based on correlation, by epoch. From the rankings, we can assess the relative importance of the morphologic features for a given epoch as well as assess temporal stability of the features and ranking stability of the measures. Instances in which a feature was ranked lower by NJSD compared with JSD indicate cases where a feature’s performance was penalized for its inherent entropy.

Take, for example, Epoch 1, the longest epoch of $\sim1.6$ years. As shown in Fig. 10, deviation-from-trend (DT) and width (W) ranked highest for all measures indicating good agreement. In contrast, for Epoch 2, the shortest epoch, only JSD and NJSD ranked DT high, whereas all four measures ranked W high. This indicates that DT is performing quite different as a feature. There is also a pattern of decreased performance (lower rankings) of W for Epochs 3, 4, and 6 due to exclusion of profiles in all or part of the nourishment region of the beach. Spreading of postnourishment sediment away from the region results in a strong retreat of the shoreline saturating the W relationship. However, DT maintained a strong performance across the epochs, indicating it is useful as a feature both within the nourishment zone and outside the region.

Overall Rankings: Table III shows the overall rankings for each measure computed by averaging the rankings across all 58 successive epochs (see table for feature abbreviations). Divergences, i.e., JSD and NJSD, closely agreed and ranked DT highest followed by S. On average, only the rankings for G and V differed between the two measures. This suggests that G provided a better balance between feature entropy and class divergence across the six epochs. Median metric ranked S highest followed by G. Correlation ranked DT highest followed by V. These results suggest that inclusion of the entire pdf shape is important in determining the relative importance of S and G.
Deviation-from-trend performed best overall as a feature and was ranked high by all measures over the longest survey epochs. This is noteworthy in that the pier region’s deviation from the natural trend of the beach is believed by coastal researchers to be a strong contributing factor to it being a long-term erosion zone [46]. Other strong performers included beach width and volume, whose values are more directly linked to shoreline position. Beach slope also performed well, except as measured by correlation, and outperformed near-shoreline slope. Orientation performed worst overall. A potential explanation is that orientation effects on shoreline response patterns are scale dependent, manifesting at much larger distances alongshore compared with the finer scales considered in this work.

Those features that rank highest do not imply a causal relationship to shoreline erosion. Their utility as a morphologic indicator is analogous to how a physical symptom manifests as a result of a particular disease, providing the doctor an indication of its origin. In this case, the examined morphologies are governed by physical processes in the region, some of which also drive shoreline dynamics. This connection can result in morphologic indicators (symptoms) of shoreline change patterns in the region, which is what our method seeks to detect. Such features can provide important metrics for characterizing a beach and be incorporated for segmentation of erosion-prone zones, as shown next.

C. Classification

To assess the potential utility of the morphologic features for segmenting zones of beach more likely to erode, the top two ranked features selected by each measure (labeled \( f_1 \) and \( f_2 \)) are used to implement a two-class naive Bayes classifier [23], [47], i.e.,

\[
P(C_i|f_1, f_2) = \frac{p(f_1|C_i)p(f_2|C_i)P(C_i)}{\sum_{i=1}^{2} p(f_1|C_i)p(f_2|C_i)P(C_i)} \quad (9)
\]

for \( i = 1 \) to \( 2 \). \( C_i \) represents an “erosion” or “accretion” tending profile as previously defined. In spite of its naive\( i \)ve design, such a classifier often works quite well in practice and suits our objective. We are concerned only with each feature’s individual influence (not joint effect) on class occurrence learned from the data.

The classifier is trained by estimating class-conditional pdfs \( p(f_j|C_i) \) using the nonparametric Parzen approach developed in Section II. The prior probabilities \( P(C_i) \) are simply the ratio of the number of class erosion or class accretion profiles to the total number of training profiles. In this example, we test the classifier on each epoch having trained it using only the 611 profiles from the other survey epochs. For instance, if we are testing on Epoch 1 (August 2003 to March 2005), we train the classifier using profiles from all other epochs, then classify the August 2003 profiles using the extracted features. The class 615 with the maximum \( a \) posteriori (MAP) probability is then selected for each profile, and results are compared with the actual 617 class occurrence for each profile in Epoch 1. Classification is, therefore, based on the beach’s current morphologic state given prior training data regardless of their occurrence in time.

This differs from classic time series prediction, which is not conducingiven the limited temporal coverage and deviates from our objective.

Due to differences in temporal length and physical conditions experienced between epochs, the training data were normalized by linear scaling to ensure correspondence in the range of 626 feature values while preserving the distribution. There are 2010 627 profiles for each period to test except for Epoch 6, which had 628 852 profiles, and Epochs 3 and 4, which had ~1600 and 1850 629 profiles because of the nourishment exclusion. 630

### Classification Results

The top two ranked features selected 631 by divergence (DT, S) had an average success rate (number 632 of correctly classified profiles divided by the total number 633 of profiles for a given epoch) of 74%. Success rates of 80% 634 were achieved for Epochs 4 and 6 and a maximum of 84% for 635 Epoch 1. In comparison, classification results based on the top 636 two features selected by the median metric (S, G) and corre- 637 lation (DT, V) had an average success rate of 61% and 69%, 638 respectively, supporting the utility of the divergence method.

To better assess classifier performance, Fig. 11 shows the variation in classification results alongshore for each epoch 641 using DT, S selected by divergence. The plots are grayscale 642 color-coded by the four possible binary classification outcomes 643 as found in a confusion matrix: true positive (correctly classified “erosion” tending), true negative (correctly classified as “accretion” tending), false positive (Type I error), and false negative (Type II error). The classifier does well within the highly erosive pier zone (star), correctly segmenting the majority of 648 the region as erosion tending for each epoch. Outside the pier 649

### TABLE III

<table>
<thead>
<tr>
<th>Feature ((f_j))</th>
<th>JSD</th>
<th>NJSD</th>
<th>Median</th>
<th>(R^2)</th>
<th>(\hat{\rho})</th>
<th>JSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beach slope (S)</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>6</td>
<td>0.34</td>
<td>0.40</td>
</tr>
<tr>
<td>Beach width (W)</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>-0.45</td>
<td>0.38</td>
</tr>
<tr>
<td>Near-shoreline slope (NS)</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>7</td>
<td>-0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>Volume-per-width (V)</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>-0.48</td>
<td>0.37</td>
</tr>
<tr>
<td>Mean gradient (G)</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>0.38</td>
<td>0.36</td>
</tr>
<tr>
<td>Mean curvature (C)</td>
<td>8</td>
<td>8</td>
<td>9</td>
<td>4</td>
<td>0.43</td>
<td>0.24</td>
</tr>
<tr>
<td>Orientation (O)</td>
<td>9</td>
<td>9</td>
<td>8</td>
<td>9</td>
<td>-0.13</td>
<td>0.14</td>
</tr>
<tr>
<td>Std. deviation of height (SD)</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>8</td>
<td>0.23</td>
<td>0.30</td>
</tr>
<tr>
<td>Deviation-from-trend (DT)</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>-0.55</td>
<td>0.51</td>
</tr>
</tbody>
</table>

For \(i = 1 \) to 2, \( \hat{\rho} \) indicates average degree of linear dependence to shoreline change, and \(\text{JSD} \) indicates average bits of information on determining class \(C\).
Fig. 11. Classification results alongshore for each epoch using DT and S. True Positive = erosion tending, True Negative = accretion tending. Strong performance is evident in the erosive pier zone (star). The rectangles outline the region of beach excluded in the analysis.

For most of the epochs, the majority of the beach experienced relatively subtle changes in shoreline position. Only the pier region exhibited consistent loss of shoreline for the majority of the epochs, and DT enabled the classifier to correctly segment the region. Additional information provided by S helped the classifier, in some instances, discern the more subtle zones of erosion occurrence outside the region.

Example of Classifier Utility: Fig. 12 shows the posteriori probability of erosion occurrence alongshore for the August 2003 and March 2005 beach using DT and S. Notice the difference in probability of erosion within the pier region between the two dates. The August 2003 survey followed previous nourishment; hence, the beach was very wide with high deviation in the pier zone. This resulted in high probabilities of erosion occurrence in the region. In contrast, the March 2005 beach was much narrower and had a reduced deviation in the pier region. This terrain configuration was unique in the data and resulted in a much lower certainty of erosion occurrence.

Although presented as examples, such classifiers offer many possibilities as coastal LiDAR data coverage continues to grow. For example, trained classifiers can be used to simulate the probabilistic response of shoreline for different beach modifications or magnitudes of erosion occurrence or to generate probability plots to detect potential coastal hazards and direct mitigation efforts directly after a prestorm LiDAR acquisition.

Fig. 12. Probability of erosion occurrence alongshore for August 2003 and March 2005 beach surface using DT and S. The zoomed-in view of the pier region shows the difference in erosion probability between the beaches due to differing terrain states.
V. CONCLUSION

We have developed a framework to exploit information from multitemporal LiDAR data acquired over a beach with the objective of systematically extracting and detecting morphologies that are indicative of observed patterns in shoreline change. By extracting features along cross-shore profile lines, the beach surface can be parameterized into several meaningful 1-D morphometrics and partitioned into binary erosion or accretion zones (classes) dependent on shoreline change between LiDAR acquisitions. This provides a mechanism for examining the relationship between morphology and shoreline change patterns alongshore. By incorporating full pdf information, the developed method proved to be more powerful in revealing indicative morphologies compared with first- and second-order relational measures alone.

The entire time period spanned by the LiDAR observations considered in this paper is only about 3.5 years. This is, arguably, too short a period from which to derive strong conclusions regarding the long-term underlying coastal processes in the study region. Therefore, we focused our analysis on evaluating the ability of the extracted features to discern short-term patterns in shoreline change based on the time periods spanned by the successive LiDAR acquisitions. The portability of those features found most effective in our study region to another beach will depend on the similarity in physical characteristics and coastal processes. The nine features examined represent a subset of the many that could potentially be extracted and coastal processes. The nine features examined represent the single most meaningful measure to assess the relative importance of the features because it uses the entire pdf and is the single most meaningful measure to assess the relative importance of the features because it uses the entire pdf and normalizes on the basis of entropy. JSD can be also equally considered in cases where the entropy of the features is similar across the feature space. Correlation and the median-based measure can serve as a reasonable tool for ranking features when computational limitations preclude the estimation of the pdfs and the underlying pdfs are expected to be highly non-Gaussian. High divergence values do not guarantee that a simple binary decision boundary will yield accurate classification. High rankings from the correlation measure can, however, suggest features that are amenable to simple binary decision rules, which is useful if such rules are to be used in a subsequent classification.

The classification results that were presented evaluate the ability to predict solely based on subtle variations in beach morphology without explicitly accounting for spatial correlation in the binary class variables. Overall, the results are promising; however, they must be understood in the context for which they were intended. The predictions are based on probabilities of erosion occurrence for a given morphologic feature learned from the LiDAR data with no direct inclusion of time, bathymetry, or the governing physics of sediment transport. Therefore, the classification results are data dependent and not intended as a stand-alone tool for predicting shoreline change. Rather, the success of the results demonstrates that certain morphologies can be systematically extracted from LiDAR data sets and incorporated to discern patterns in beach change, supporting the notion of morphologic change indicators and their potential utility for beach characterization.

Finally, the focus here was on assessing individual morphologies rather than their joint effect. Methods such as mutual information or other approaches could be applied to examine the utility of feature pairs for discerning erosion and accretion zones. Additionally, the effect of scale dependence on a feature’s performance is an additional area of further investigation.

APPENDIX A

Let $P_1 = P(x|C_1)$ and $P_2 = P(x|C_2)$ be the likelihood for a discretized feature $x$, and $\pi_1 + \pi_2 = 1$ be the prior probabilities for class 1 and 2, respectively. Generalized JSD can then be written as

$$JSD = \pi_1 D_{kl}(P_1, \pi_1 P_1 + \pi_2 P_2) + \pi_2 D_{kl}(P_2, \pi_1 P_1 + \pi_2 P_2)$$

$$= \sum_x \pi_1 P_1 \log \left( \frac{P_1}{\pi_1 P_1 + \pi_2 P_2} \right) + \sum_x \pi_2 P_2 \log \left( \frac{P_2}{\pi_1 P_1 + \pi_2 P_2} \right)$$

$$= \sum_x \pi_1 P_1 \log(P_1) - \sum_x \pi_1 P_1 \log(\pi_1 P_1 + \pi_2 P_2) + \sum_x \pi_2 P_2 \log(P_2) - \sum_x \pi_2 P_2 \log(\pi_1 P_1 + \pi_2 P_2)$$

$$= H(\pi_1 P_1 + \pi_2 P_2) - \pi_1 H(P_1) - \pi_2 H(P_2)$$

where $H$ is the information-theoretic entropy function. From the law of total probability, the first term in (10) is equal to

$$H(\pi_1 P_1 + \pi_2 P_2) = H(\pi_1 P(x|C_1) + \pi_2 P(x|C_2)) = H(x).$$

Furthermore, we get the following result for the second terms in (10):

$$-\pi_1 H(P_1) - \pi_2 H(P_2) = \sum_{c=1}^2 \pi_c \sum_x P(x|C) \log(P(x|C))$$

$$= -H(x|C).$$

Replacing the terms in (10) with (11), and (12), we get the mutual information equivalence, i.e.,

$$JSD = H(x) - H(x|C) = I(x; C).$$

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Dr. Slatton was named the 2006 winner of the Presidential Early Career Award for Scientists and Engineers for work on predicting signal propagation in highly cluttered environments using remotely sensed geometry from ALSM. On March 30, 2010, he passed away from cancer, and the world lost a brilliant researcher, talented teacher, and loving father.
AUTHOR QUERIES

AUTHOR PLEASE ANSWER ALL QUERIES

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END OF ALL QUERIES
Probabilistic Detection of Morphologic Indicators for Beach Segmentation With Multitemporal LiDAR Measurements

Michael J. Starek, Member, IEEE, Raghav Vemula, and K. Clint Slatton, Senior Member, IEEE

Abstract—Airborne light detection and ranging (LiDAR) surveys provide a rich data source of topographic information. However, beach monitoring with LiDAR data has been mostly limited to visualization and first-order measures derived from digital elevation models (DEMs). To exploit more information from multitemporal LiDAR data acquired over a beach, we extract surface features to detect morphologies that are indicative of shoreline change patterns. First, through cross-shore profile sampling of LiDAR-derived DEMs, the continuous 3-D beach surface is parameterized into several 1-D morphologic features progressing alongshore. Profiles are subsequently partitioned into binary erosion or accretion classes dependent on measured shoreline change between surveys. Then, a feature’s class separability is quantified using information divergence measures nonparametrically constructed via Parzen windowing. The more interclass separation provided by a feature, the greater its discriminative potential, and the higher its ranking as a morphologic indicator. Rankings are computed across the survey epochs to evaluate performance stability of the features. Finally, the top-ranked features are implemented within a naïve Bayes classifier to assess their ability to segment terrain more likely to erode. Results demonstrate the utility of the developed framework to systematically extract and incorporate useful morphologic information from LiDAR data for discerning patterns in beach change.

Index Terms—Divergence, light detection and ranging (LiDAR), pattern classification, shoreline erosion.

I. INTRODUCTION

EACHES define a geomorphic barrier between land and sea, protecting vital infrastructure and natural habitat. However, sandy beaches are dynamic landforms, constantly evolving with time and subject to the continual threat of erosion and coastal flooding [1]. This evolution constitutes a highly variable 3-D process, consisting of rapid variations due to storm events, seasonal variations due to changes in wave and wind climate, and long-term variations due to sea-level rise, longshore sediment transport, anthropogenic influences, and extreme storm events [2]. Monitoring and analysis of beach evolution is critical to the future sustainability of these environments and the economies that depend on them [3].

Airborne scanning light detection and ranging (LiDAR) has revolutionized coastal monitoring, making it possible to measure 3-D changes in topography at spatial resolutions needed to advance science and monitor erosion along coastlines efficiently and accurately [4]. This revolution has been propelled by topographic LiDAR systems that operate in the near-IR portion of the electromagnetic spectrum and by bathymetric systems that operate in the blue-green range of the spectrum [5], [6]. The focus herein is on the application of data from small-footprint discrete-return topographic LiDAR systems [7], which typically record two or more returns per emitted pulse (including first and last). Small-footprint systems enable beach mapping with average spatial resolutions greater than 1 point/m² and achievable positional accuracy values of 15–30 cm horizontal (x, y) and 5–10 cm vertical (z) [8]. However, point density will vary locally depending on flight parameters, scan angle, beam divergence, surface properties, and pulse repetition rate, among other factors [9], [10].

Numerous studies have demonstrated the application of 62 repeat-coverage LiDAR surveys for quantifying changes in beach terrain (e.g., [4] and [11]–[15]). Generally, this is accomplished by differencing LiDAR-derived digital elevation models (DEMs) or contour vectors to estimate change in sediment volume or shoreline position between surveys. In addition to elevation change, many different features can be derived, where a feature in this context refers to a property of the sampled surface, such as slope or curvature [16]. For coastal analysis, feature extraction has been used to track changes in nearshore geomorphology, such as berm formation or dune migration [17], [18].

Relatively unexplored are analysis methods that venture beyond change detection [19]. With the accumulation in coastal LiDAR time series data, the exploration of enhanced analysis methods to exploit more information from the data is warranted [15], [19]. In this regard, data mining and pattern classification techniques offer great potential to progress beach monitoring with LiDAR data beyond visualization and simple (first-order) measurements made from DEMs.
In the following discussion, we develop a systematic framework to mine multitemporal LiDAR data acquired over a beach and detect morphologies (surface features) indicative of observed patterns in shoreline change behavior. The problem is approached as one of optimal feature selection for binary classification. Several different morphologic features are extracted and then ranked based on their ability to separate shoreline erosion and accretion zones (profiles) along the beach. A feature’s probabilistic relation to class occurrence is estimated from the data and used to evaluate their discriminative potential. The top-ranked features are then implemented within a naïve Bayes classifier to assess their effectiveness in discerning shoreline change patterns alongshore.

This paper is structured as follows. Section II outlines the feature extraction and evaluation methodology. Section III describes the LiDAR data set and morphologic features examined with the developed framework. Section IV presents feature analysis and performance results for the study area. Section V provides concluding remarks.

II. METHODOLOGY

Given a time series of airborne LiDAR surveys acquired over a beach, our approach seeks connections between variation in morphology and shoreline change patterns observed along the beach. Those morphologies most indicative of shoreline change behavior are systematically revealed. Fig. 1 outlines our approach, and the steps are detailed below.

A. Morphologic Data Mining

DEM Generation: The raw LiDAR data consist of point clouds of irregularly spaced $x, y, z$ values providing a 3-D representation of the ground and land cover. For multireturn data, the last (or single) return data are utilized to minimize the probability of land cover biasing the surface elevations. Filtering is applied to remove nonground points [20], and the points are then interpolated into bare-earth elevation grids (e.g., [15] and [21]) at a desired spatial resolution (e.g., 1 m). The result is a sequence of LiDAR-derived DEMs representing the beach surface and surrounding terrain at the time of each survey.

Profile Sampling: The coordinate system traditionally used for studying shoreline change consists of a local 2-D Cartesian system oriented with alongshore (shore parallel) and cross-shore (shore perpendicular) axes [2]. A MATLAB program was developed to sample the LiDAR-derived DEMs by extracting elevation values along cross-shore profile lines oriented orthogonal to a shore-parallel landward baseline [19]. This provides the $x, y$ coordinates and elevation values along each profile at a user-defined spacing (see Fig. 2). The same baseline is used to extract profiles across the surveys, and the algorithm can determine the intersection of each profile with a shoreline contour or other vector (such as dune line). The cross-shore profile method provides a standardized reference frame from which to extract morphology and relate to shoreline change patterns alongshore.

Profile Segmentation: The temporal change in shoreline between successive surveys (data epochs) is quantified using pairs of profiles by computing the distance between the shoreline position $x, y$ coordinates (see Fig. 2). Profiles are then segmented into binary classes, $C_1$ for “erosion” tending or $C_2$ for “accretion” tending, depending on whether the shoreline had moved landward (− change) or seaward (+ change) during that period. For instance, if a hypothetical profile 1 in survey 1 experienced negative shoreline change for the survey 1 to survey 2 temporal epoch, it is classified as an erosion profile.

Effective beach management relies upon detecting and monitoring zones of beach more prone to erosion, which motivated...
While it is possible that feature transformation methods, such as Gaussian pdfs with similar support (see Fig. 4) [16], [23], where geometric measures of separation can fail, such as non-geo-
metric measures of separation can fail to capture important differences.

**Density Estimation:** Airborne LiDAR data in feature space can exhibit multimodal and distinctly non-Gaussian properties [16], [19]. To avoid distributional assumptions, the nonpara-
metric Parzen window method [23] is employed to estimate the class-conditional pdfs \( p(f_j|C_i) \) from the data. The Parzen density estimate \( \hat{p}(\vec{x}) \) can be written as

\[
\hat{p}(\vec{x}) = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{h^d} K_h \left( \frac{\vec{x} - \vec{x}_i}{h} \right)
\]

where \( K_h \) is the kernel function, \( h \) controls the length of the d-
dimensional kernel window commonly referred to as the kernel bandwidth (here \( d = 1 \)), \( N \) is the number of points in the data set, \( \vec{x}_i \) is the vector of current feature values \( f_j \), and \( \vec{x} \) are the points at which the pdf is to be estimated along each feature dimension.

Bias in the Parzen estimate can be asymptotically reduced to zero by selecting a symmetric unimodal kernel function [26]; therefore, \( K_h \) is selected to be a 1-D Gaussian kernel. The Gaussian kernel has been shown to perform well despite the presence of correlated samples, which is evident in our features [27], [28]. Furthermore, it provides infinite support and useful analytic properties the importance of which will become clear below. Setting \( K_h \) to a 1-D Gaussian kernel, the Parzen density estimate at a point \( x \) is

\[
\hat{p}(x) = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{\sqrt{2\pi}h} e^{-u^2/2}
\]

where \( u = x - x_i/h \) and \( h \) acts similar to the familiar standard deviation of the normal distribution.

Crucial to Parzen estimation is the selection of the band-
width \( h \) [23]. Bandwidths that are too small over-fit the sam-
}

---

Fig. 3. Example of a cross-shore beach profile showing the subaerial section of the profile where features are extracted. This example is more typical of a summer profile because of the berm formation.

Fig. 4. Example of two likelihood for a single feature \( f_j \) given data from two classes, i.e., \( C_1 \) and \( C_2 \). Divergence measures quantify the degree to which the two pdfs differ and, thus, serve as indicators of the \( j \)th feature’s potential value for classification. When the pdf mean values are similar, as depicted here, geometric measures of pdf separability can fail to capture important differences.
bandwidths that are too large yield pdf estimates that are overly smoothed. Its optimal value (the value that minimizes the measure of dissimilarity between the estimated pdf and the true pdf) strongly depends on the type of data, the number, and the amount they are corrupted by noise [29]. Automatic bandwidth selection methods can be applied to estimate \( h \) directly from the sample data with various tradeoffs in performance [30], [31].

For this work, a maximum likelihood (ML) approach is implemented [32]. Given the Parzen density estimator, i.e., \( \hat{p}(x) \), the goal is to maximize the expected value of the log-likelihood, i.e., \( E_p[\log \hat{p}(x)] \), with respect to \( h \). The expectation is approximated by the sample mean, resulting in

\[
J_{\text{ML}}(h) = \frac{1}{N} \sum_{i=1}^{N} \log (\hat{p}(x_i)).
\]

Substituting in the Parzen density estimate of (2), this becomes

\[
J_{\text{ML}}(h) = \frac{1}{N} \sum_{i=1}^{N} \log \left( \frac{1}{N} \sum_{j=1}^{N} \frac{1}{\sqrt{2\pi h}} e^{-\frac{(x_i - x_j)^2}{2h^2}} \right).
\]

However, this criterion exhibits an undesirable global maximum at the null kernel size. To avoid this situation, (3) is modified in accordance with the leave-one-out cross-validation estimate, \( \hat{p}_{-i}(x) \) [26]. This yields

\[
J_{\text{ML}}(h) = \frac{1}{N} \sum_{i=1}^{N} \log \left( \frac{1}{N-1} \sum_{j=1, j \neq i}^{N} \frac{1}{\sqrt{2\pi h}} e^{-\frac{(x_i - x_j)^2}{2h^2}} \right).
\]

The optimal bandwidth based on the ML criteria is then

\[
h_{\text{ML}} = \arg \max_h J_{\text{ML}}(h).
\]

Equation (4) can be directly maximized with respect to \( h \) through the derivative and numerically solved, which is an attractive property for coupling the Gaussian kernel with an ML approach. An iterative Newton–Raphson scheme is implemented [32] to solve for \( h_{\text{ML}} \). To avoid ill effects on bandwidth convergence due to data with ties (equivalent values stemming from the precision level used for feature discretization), the feature data are corrupted with uniformly distributed random noise at a level that preserves original sample integrity following an approach outlined in [33]. The optimal ML bandwidth is then selected as the average bandwidth based on several simulations.

The ML bandwidth is equivalent to choosing the bandwidth that minimizes the Kullback–Leibler divergence between the unknown true density \( p(x) \) and the Parzen estimate \( \hat{p}(x) \). A potential tradeoff with using the ML criterion is that it can be affected by tail behavior of the true density \( p(x) \) and result in an overly smoothed estimate [34]. Although a concern, estimated bandwidths generally fell within about 1/40th the range of feature values or less for those investigated in this analysis, which is not overly smoothed.

Once the class-conditional pdfs \( p(f_j|C_i) \) are estimated, each feature’s class separability is quantified using divergence measures described in Section IV. The more interclass separation provided by a feature, the greater its ranking and discriminative potential as a morphologic indicator of erosion or accretion occurrence.

### III. DESCRIPTION OF DATA SET AND FEATURES

The developed framework is applied to a time series of 271 seven airborne LiDAR surveys acquired along the St. Augustine 272 Beach region of northeast Florida, USA, between August 2003 273 and February 2007. This resulted in six sequential data epochs 274 of varying temporal lengths (see Table I), ranging from a 275 maximum of almost two years to less than two months. A 276 10-km long stretch of the coastline comprises our study area 277 (see Fig. 5). This region was selected because it contains both 278 a historical accretion zone and a highly erosive pier zone, 279 which is armored with revetments and requires periodic beach

![Fig. 5. LiDAR-derived bare-earth DEM of the August 2003 survey showing the entire 10-km study area and a zoomed-in view of the pier region showing dunes, beach, and filtered buildings. Elevation is relative to NAVD88. The x and y axes are measured in UTM Zone 17 m.](image)
nourishment to replace lost beach area. Effects from two nourishment projects conducted in 2003 and 2005 are observed in the data. Both nourishment focused on the approximately 2-km revetment region of the pier. The surveys were conducted by the University of Florida using an Optech International ALTM 1233 airborne LiDAR system operating at a laser pulse rate of 33 kHz and a near-IR wavelength of 1064 nm. The system can record up to two returns per an emitted pulse. Surveys were conducted during low tide to capture more exposed land surface. Nominal values for the surveys included a flying height of 600 m above ground level, flight speed of 60 m/s, and a mirror scan rate of 28 Hz resulting in a mean ground point density of approximately 1.3 points/m². Only the last-return data points were utilized for this analysis.

### A. Data Processing

Vertical bias in the LiDAR observations stemming mostly from GPS-induced trajectory errors are sometimes present [8]. The offsets between flights were removed by vertically shifting the LiDAR points to match higher accuracy kinematic GPS calibration lines acquired along static road surfaces within the survey area. For the LiDAR system, calibration, and flight parameters used in this work, this correction generally yields vertical accuracy values of 5–10 cm relative to ground-survey measurements over slowly varying surfaces [8].

The initial LiDAR elevations were provided as ellipsoid heights and were therefore transformed relative to the North American Vertical Datum of 1988 (NAVD88) using the GEOID03 model [35]. The data were then filtered using an adaptive algorithm in [36] to remove nonground points without altering the natural beach morphology. The filtered points were interpolated into 1 m × 1 m bare-earth DEMs (see Fig. 5) using ordinary kriging [14], [37]. The data processing procedure and parameters were kept constant for all data sets.

**Shoreline Delineation:** To minimize impact from wave runup and tidal fluctuations, the mean higher high water (MHHW) tidal datum was selected as the proxy for shoreline [38], [39]. MHHW is the average of the higher high water height of each tidal day observed over a 19-year tidal epoch. The nearest NOAA tidal gauge to the study area was used to determine an MHHW elevation of ~0.6 m relative to NAVD88 [14]. The 0.6-m contour was delineated to create a contiguous shoreline vector in each DEM.

To gauge the fluctuation in MHHW–NAVD88 datum separation over the entire 10-km study area, the VDatum software tool was used (NOAA, http://vdatum.noaa.gov, 2011). The appropriate VDatum grid covering the northeast Florida coast was used as input. MHHW elevations relative to NAVD88 were computed at the northern and southern edge of the study area. The difference in elevation was approximately 0.05 m, which falls within the range of VDatum’s vertical uncertainty for the region. This result validates the use of a constant 0.6-m NAVD88 reference elevation for MHHW shoreline in the study area.

Additional uncertainty in shoreline position is introduced by LiDAR positional errors, the DEM gridding process, and concomitant error propagation in shoreline positioning. Gently sloping beaches will further amplify the total propagated uncertainty. Nonetheless, the expected uncertainty in LiDAR-derived DEM shoreline positions, even for gently sloping beaches, is more than acceptable for erosion monitoring where the concern is with significant changes in shoreline, typically several meters. For a more detailed treatment of positional uncertainty in LiDAR-derived shorelines, refer to [40].

### B. Morphologic Feature Extraction

**Cross-Shore Profile Sampling:** Following the steps in our framework (see Fig. 1), cross-shore profiles were extracted every 5 m in the alongshore direction and 1-m point spacing in the cross-shore direction extending to the MHHW shoreline (see Fig. 2). The 5-m alongshore spacing was selected to provide dense coverage without over redundancy of information due to the high spatial correlation of beach topography and shoreline change. A 1-D moving average filter of length 5 m was applied to each profile to reduce any residual high-frequency artifacts that may sometimes be present due to scan line artifacts or isolated nonground points that the filter failed to remove.

**Profile Segmentation:** Profiles were then segmented into “erosion” or “accretion” tending classes (C₁ or C₂), depending on the shoreline change experienced between each survey epoch (see Table I). For certain data epochs, the majority of the shoreline either eroded or accreted. In such cases, the median value was used to segment the classes. C₁ still corresponds to profiles with the most negative tending shoreline change (high erosion or low accretion), and C₂ still corresponds to the most positive tending shoreline change (low erosion or high accretion). Fig. 6 shows the variability in shoreline change and 365...
class membership alongshore for each survey epoch. Extensive 

The 2005 nourishment to replace the lost beach can be observed 
in the large accretion values for Epoch 3 within the pier region 
(~3000 to 5000 m alongshore) and a portion of Epoch 4 
(~5500 m to 5700 m alongshore). Profiles located in the zones 
of nourishment for Epochs 3 and 4 were omitted from the 
analysis.

Examined Morphologies: A total of nine features (see Ta-
ble II) were extracted to evaluate their discriminative potential 
as morphologic indicators for the region. The nine features were 
selected based on their historical usage or suspected potential as 
a measure to describe the alongshore character of the subaerial 
beach surface (see Fig. 3).

Beach slope was estimated using a linear fit through the 
profile elevation values from shoreline to dune toe line and rep-
resents the slope of the subaerial beach profile. Beach width is 
the width of the subaerial beach. Near-shoreline slope is a linear 
fit through the profile elevation values from shoreline to a few 
meters landward. It approximates the slope in the more steeply 
sloping foreshore region of the subaerial profile. Volume-per-
width is the volume per unit width (area) under the subaerial 
profile above 0 m NAVD88 computed using trapezoidal inte-
gration. Mean profile gradient is the mean value of surface 
slope estimated every 3 m along the subaerial profile using a 
central-difference approximation. Mean profile curvature is the 
mean value of surface curvature estimated every 3 m along the 
subaerial profile using a central-difference approximation to the 
second derivative. Standard deviation of height is the standard 
device of elevation values along the subaerial profile and 
provides a measure of profile surface roughness. Orientation 
is the angle in degrees the local shoreline makes relative to 
azimuth (clockwise from north) estimated by piecewise linear 
fitting along the shoreline. Deviation-from-trend (m) is the 
400 orthogonal distance the shoreline deviates from the natural 
trend of the beach over the distance of a few kilometers. It is 
a measure created to capture the deviation of the highly erosive 
pier region (see Fig. 5). The beach trend was estimated by a 
linear fit through the shoreline contour points, excluding the 
405 roughly 2-km revetment region.

Mean gradient and mean curvature are novel measures con-
structed to better characterize the variability in the subaerial 
profile observed along the beach. Large values for mean gra-
dient indicate narrow sections of the beach with a large scarp 
formation, whereas small values can indicate sections of beach 
with little to no berm formation. A positive value for mean cur-
vature indicates a convex tending profile, which is more typical 
of an accreting profile with a berm, whereas a negative value 
indicates a concave tending profile more typical of an eroding 
beach with scarps. Deviation-from-trend is another feature that 
was created based on its suspected potential as an indicator of 
shoreline erosion in the area. Other features, such as beach 
slope, volume-per-width, and width, are traditional measures 
of sediment dynamics along a beach [2]. Near-shoreline slope 
was selected for comparison in performance to the slope of 
the entire subaerial profile (beach slope). Orientation is another 
potentially useful feature. Changes in shoreline orientation can 
422 alter the longshore sediment transport in the region by creating 
relative differences in breaking wave angle [41].

As an example, Fig. 7 shows the high alongshore variation in 425 
beach width, beach slope, and mean curvature extracted from 
426 the August 2003 DEM. Referring to the beach width plot, 427 
notice the rise, sharp decrease and then subsequent increase 428 
in width over the range of 3000 to 5000 m alongshore. This 429 
is an effect of the nourishment project on either side of the 430 
pier earlier that year. The negative curvature values in the pier 
431 vicinity are indicative of a scarp formation in the berm due to 432 
erosion.

IV. ANALYSIS AND RESULTS

A. Feature Performance Metrics

As outlined in our framework (see Fig. 1), the discriminative 
436 potential of the morphologic features $f_j$ are evaluated based on 
the functional differences (separability) in their estimated 
class-conditional pdfs, i.e., $p(f_j|C_1)$ and $p(f_j|C_2)$, for $j = 1$ to 9 
(see Figs. 8 and 9). For this analysis, two symmetric and 440 
bounded forms of Kullback–Leibler divergence ($D_{kl}$), i.e., the 441 
Jensen–Shannon divergence (JSD) and a normalized form of 442 
JSD, are used as performance measures of interclass separabil-
ity. $D_{kl}$ between two pdfs $P$ and $Q$ of a continuous random 
444
variable \( x \) can be approximated by a discrete sum using the 446 sampled distributions as follows:

\[
D_{\text{kl}}(P, Q) = \sum_x P(x) \Delta x \log \frac{P(x)}{Q(x)} \tag{5}
\]

where \( \Delta x \) is the interval between samples of the quantized random variable [16]. In this work, \( \Delta x \) was scaled by the range of values for each feature to reduce effects stemming from inconsistent binning on the resultant divergence calculations. \( D_{\text{kl}} \) is measured in bits of information, nonnegative, and equal to zero if and only if \( P = Q [22], [23]. \)

**JSD**: JSD [42] can be expressed as

\[
JSD(P, Q) = \pi_1 D_{\text{kl}}(P, M) + \pi_2 D_{\text{kl}}(Q, M) \tag{6}
\]

where \( \pi_1 + \pi_2 = 1 \), and \( M = \pi_1 P + \pi_2 Q \). JSD is a nonnegative symmetric bounded form of \( D_{\text{kl}} \) between two pdfs and their weighted average, \( M \). It is equal to zero if and only if \( P = Q \) [43]. The form of JSD in (6) naturally fits to our two-class decision problem by letting \( P = p(f_j|C_1) \) and \( Q = p(f_j|C_2) \), and \( \pi_1 \) and \( \pi_2 \) are the binary class priors, respectively.

By applying JSD within a decision-theoretic framework as we have done here, it is equivalent to the well-known mutual information measure \( I(f_j; C) \) between the random feature \( f_j \) and the random class variable \( C \) (see Appendix). We are explicitly quantifying the amount of information gained from feature \( f_j \) on knowing the class label \( C \) through the JSD measure. In this case, the question being asked is whether the profile being queried is of class “erosion” tending or “accretion” tending. For this binary question, the maximum amount of information possible is 1 bit. \( JSD(P, Q) \in [0, 1] \), which means we expect good features (more interclass separability) to yield \( 0 \ll JSD < 1 \).

**Normalized JSD**: Separability based on \( D_{\text{kl}} \) and, subsequently, JSD does not directly consider the inherent complexity of each feature [44]. To account for feature complexity in our rankings, we construct a normalized information divergence [45]. We select a feature’s probabilistic entropy, \( I(f_j) \), as the complexity measure

\[
C_{P,Q} = H(f_j) = H(\pi_1 p(f_j|C_1) + \pi_2 p(f_j|C_2)) = H(\pi_1 P + \pi_2 Q)
\]

where \( H \) is the information-theoretic entropy function [22], and JSD of (6) as the information term, \( I_{P,Q} \), to construct an information divergence as follows:

\[
NJSD(P, Q) = \frac{I_{P,Q}}{C_{P,Q}} = \frac{JSD(P, Q)}{H(f_j)} = \frac{JSD(P, Q)}{H(\pi_1 P + \pi_2 Q)} \tag{7}
\]

The measure is referred to as Normalized JSD (NJSD). From (7), we see that NJSD is bounded by 1 and equal to 0 if and only if \( P = Q \), which implies values close to 0 have poor class separation. More tightly bounded, \( NJSD(P, Q) \in [0, 1/H(f_j)] \). Therefore, we expect good features to yield larger NJSD values, \( 0 \ll NJSD < 1/H(f_j) \).

To better understand the behavior of (7), take, for example, the following two cases given two features, i.e., \( f_1 \) and \( f_2 \):

**Case 1**: \( f_1 \) and \( f_2 \) have equal entropy, i.e., \( H(f_1) = H(f_2) \), but \( JSD(f_1) < JSD(f_2) \). \( f_2 \) is selected as the more optimal feature because \( f_2 \) provides more separation between classes based on JSD.

**Case 2**: \( f_1 \) and \( f_2 \) have equal divergence, i.e., \( JSD(f_1) = JSD(f_2) \), but \( H(f_1) < H(f_2) \). \( f_1 \) is selected as the more optimal feature because it requires less complexity to describe its behavior.

NJSD selects the feature that provides a compromise between least uncertainty and most information gained on knowing the class label. This behavior is desirable in feature selection problems. Features with lower relative entropy (less uncertainty) are more stable and, in return, can potentially enable more reliable classification outside of the training data.

**Median Metric**: For comparison to divergence-based rankings, a simple median-based measure originally constructed for LiDAR-derived (non-Gaussian) samples was selected to assess...
Fig. 10. Ranking of each feature’s interclass separation between erosion and accretion tending profiles by epoch and for each of the four measures. For NJSD, JSD, and median, 1 indicates the feature that is most separable and 9 least separable. For \( R^2 \), 1 indicates the feature most correlated to shoreline change and 9 least correlated.

1-D interclass separation. The median-based measure is unitless and has the following form [16]:

\[
d_j = \left| \frac{\text{median}(f_{\text{erosion}}) - \text{median}(f_{\text{accretion}})}{\sqrt{\text{mad}(f_{\text{erosion}})^2 + \text{mad}(f_{\text{accretion}})^2}} \right|
\]  

(8)

where \( f_{\text{erosion}} \) represents feature \( j \) for all points in class erosion. Index \( j \) runs from 1 to 9 for all nine features, and mad is the median absolute deviation about the cluster median value. Equation (8) is a measure of the separation of the cluster centers in feature space relative to their spread that is robust to outliers and computationally fast. There is no theoretical maximum value for (8) in the limit as cluster spreads vanish. In general, the larger the value in (8), the more separable the classes for a given feature [16].

B. Feature Performance

Each feature’s separability between “erosion” and “accretion” tending profiles (\( C_1 \) or \( C_2 \)) was ranked for the six data epochs using the two divergence measures, namely, JSD and NJSD, and the median metric. Those features that rank highest yield the most interclass separation according to that measure. In addition, Pearson’s squared correlation coefficient (\( R^2 \)) was computed to evaluate the strength of a linear relationship between the features and shoreline change [23]. In general, a feature that is well correlated with measured shoreline change (i.e., change viewed as a continuous random variable rather than a binary class label) would be expected to also exhibit a significant divergence in the class-conditional pdfs. Instances in which the feature’s ranking from the divergences is different from those suggested by correlation indicate potential cases where using the full pdf instead of merely second-order descriptions is important.

Individual Epochs: Fig. 10 shows the rankings of each feature’s interclass separation based on JSD, NJSD, and median metric, and based on correlation, by epoch. From the rankings, we can assess the relative importance of the morphologic features for a given epoch as well as assess temporal stability of the features and ranking stability of the measures. Instances in which a feature was ranked lower by NJSD compared with JSD indicate cases where a feature’s performance was penalized for its inherent entropy.

Take, for example, Epoch 1, the longest epoch of \( \sim 1.6 \) years. As shown in Fig. 10, deviation-from-trend (DT) and width (W) ranked highest for all measures indicating good agreement. In contrast, for Epoch 2, the shortest epoch, only JSD and NJSD ranked DT high, whereas all four measures ranked W high. This indicates that DT is performing quite different as a feature. There is also a pattern of decreased performance (lower rankings) of W for Epochs 3, 4, and 6 due to exclusion of profiles in all or part of the nourishment region of the beach. Spreading of postnourishment sediment away from the region results in a strong retreat of the shoreline saturating the W relationship. However, DT maintained a strong performance across the epochs, indicating it is useful as a feature both within the nourishment zone and outside the region.

Overall Rankings: Table III shows the overall rankings for each measure computed by averaging the rankings across all 585 successive epochs (see table for feature abbreviations). Divergences, i.e., JSD and NJSD, closely agreed and ranked DT highest followed by S. On average, only the rankings for G and V differed between the two measures. This suggests that G provided a better balance between feature entropy and class divergence across the six epochs. Median metric ranked S highest followed by G. Correlation ranked DT highest followed by V. These results suggest that inclusion of the entire pdf shape is important in determining the relative importance of S and G.
Deviations from trend performed best overall as a feature and was ranked high by all measures over the longest survey epochs. This is noteworthy in that the pier region’s deviation from the natural trend of the beach is believed by coastal researchers to be a strong contributing factor to it being a long-term erosion zone [46]. Other strong performers included beach width and volume, whose values are more directly linked to shoreline position. Beach slope also performed well, except as measured by correlation, and outperformed near-shoreline slope. Orientation performed worst overall. A potential explanation is that orientation effects on shoreline response patterns are scale dependent, manifesting at much larger distances alongshore compared with the finer scales considered in this work.

Those features that rank highest do not imply a causal relationship to shoreline erosion. Their utility as a morphologic indicator is analogous to how a physical symptom manifests as a result of a particular disease, providing the doctor an indication of its origin. In this case, the examined morphologies are governed by physical processes in the region, some of which also drive shoreline dynamics. This connection can result in morphologic indicators (symptoms) of shoreline change patterns in the region, which is what our method seeks to detect. Such features can provide important metrics for characterizing a beach and be incorporated for segmentation of erosion-prone zones, as shown next.

C. Classification

To assess the potential utility of the morphologic features for segmenting zones of beach more likely to erode, the top two ranked features selected by each measure (labeled $f_1$ and $f_2$) are used to implement a two-class naive Bayes classifier [23], [47], i.e.,

$$P(C_i|f_1, f_2) = \frac{p(f_1|C_i)p(f_2|C_i)P(C_i)}{\sum_{i=1}^{2} p(f_1|C_i)p(f_2|C_i)P(C_i)} \quad (9)$$

for $i = 1$ to 2. $C_i$ represents an “erosion” or “accretion” tending profile as previously defined. In spite of its naive design, such a classifier often works quite well in practice and suits our objective. We are concerned only with each feature’s individual influence (not joint effect) on class occurrence learned from the data.

The classifier is trained by estimating class-conditional pdfs $p(f_j|C_i)$ using the nonparametric Parzen approach developed in Section II. The prior probabilities $P(C_i)$ are simply the ratio of the number of class erosion or class accretion profiles to the total number of training profiles. In this example, we test the classifier on each epoch having trained it using only the 611 profiles from the other survey epochs. For instance, if we are testing on Epoch 1 (August 2003 to March 2005), we train the classifier using profiles from all other epochs, then classify the August 2003 profiles using the extracted features. The class label with the maximum a posteriori (MAP) probability is then selected for each profile, and results are compared with the actual class occurrence for each profile in Epoch 1. Classification is, therefore, based on the beach’s current morphologic state given prior training data regardless of their occurrence in time. This differs from classic time series prediction, which is not predictive given the limited temporal coverage and deviations from our objective.

Due to differences in temporal length and physical conditions experienced between epochs, the training data were normalized by linear scaling to ensure correspondence in the range of feature values while preserving the distribution. There are 2010 627 profiles for each period to test except for Epoch 6, which had 852 profiles, and Epochs 3 and 4, which had approximately 1600 and 1850 profiles because of the nourishment exclusion.

Classification Results: The top two ranked features selected 631 by divergence (DT, S) had an average success rate (number 632 of correctly classified profiles divided by the total number 633 of profiles for a given epoch) of 74%. Success rates of 80% were achieved for Epochs 4 and 6 and a maximum of 84% for 635 Epoch 1. In comparison, classification results based on the top 636 two features selected by the median metric (S, G) and correlation (DT, V) had an average success rate of 61% and 69%, respectively, supporting the utility of the divergence method.

To better assess classifier performance, Fig. 11 shows the variation in classification results alongshore for each epoch using DT, S selected by divergence. The plots are grayscale color-coded by the four possible binary classification outcomes as found in a confusion matrix: true positive (correctly classified as “erosion” tending), true negative (correctly classified as “accretion” tending), false positive (Type I error), and false negative (Type II error). The classifier does well within the highly erosive pier zone (star), correctly segmenting the majority of the region as erosion tending for each epoch. Outside the pier...
Fig. 11. Classification results alongshore for each epoch using DT and S. True Positive = erosion tending, True Negative = accretion tending. Strong performance is evident in the erosive pier zone (star). The rectangles outline the region of beach excluded in the analysis.

860 region, the classifier tends to bias toward “accretion” tending profiles (see Fig. 11), resulting in a higher false negative rate for the majority of epochs (38%, on average, compared with 16% false positive rate). This is a desirable performance trait where the concern is with segmenting zones of high erosion potential. For most of the epochs, the majority of the beach experienced relatively subtle changes in shoreline position. Only the pier region exhibited consistent loss of shoreline for the majority of the epochs, and DT enabled the classifier to correctly segment the region. Additional information provided by S helped the classifier, in some instances, discern the more subtle zones of erosion occurrence outside the region.

Example of Classifier Utility: Fig. 12 shows the posteriori probability of erosion occurrence alongshore for the August 2003 and March 2005 beach using DT and S. Notice the difference in probability of erosion within the pier region between the two dates. The August 2003 survey followed previous nourishment; hence, the beach was very wide with high deviation in the pier zone. This resulted in high probabilities of erosion occurrence in the region. In contrast, the March 2005 beach was much narrower and had a reduced deviation in the pier region. This terrain configuration was unique in the data and resulted in a much lower certainty of erosion occurrence.

Although presented as examples, such classifiers offer many possibilities as coastal LiDAR data coverage continues to grow. For example, trained classifiers can be used to simulate the probabilistic response of shoreline for different beach modifications or magnitudes of erosion occurrence or to generate probability plots to detect potential coastal hazards and direct mitigation efforts directly after a prestorm LiDAR acquisition.

Fig. 12. Probability of erosion occurrence alongshore for August 2003 and March 2005 beach surface using DT and S. The zoomed-in view of the pier region shows the difference in erosion probability between the beaches due to differing terrain states.
We have developed a framework to exploit information from multitemporal LiDAR data acquired over a beach with the objective of systematically extracting and detecting morphologies that are indicative of observed patterns in shoreline change. By extracting features along cross-shore profile lines, the beach surface can be parameterized into several meaningful 1-D morphometrics and partitioned into binary erosion or accretion zones (classes) dependent on shoreline change between LiDAR acquisitions. This provides a mechanism for examining the relationship between morphology and shoreline change patterns alongshore. By incorporating full pdf information, the developed method proved to be more powerful in revealing indicative morphologies compared with first- and second-order relational measures alone.

The entire time period spanned by the LiDAR observations considered in this paper is only about 3.5 years. This is, arguably, too short a period from which to derive strong conclusions regarding the long-term underlying coastal processes in the study region. Therefore, we focused our analysis on evaluating the ability of the extracted features to discern short-term patterns in shoreline change based on the time periods spanned by the successive LiDAR acquisitions. The portability of those features found most effective in our study region to another beach will depend on the similarity in physical characteristics and coastal processes. The nine features examined represent only a subset of the many that could potentially be extracted from the LiDAR data. The developed framework can be easily applied to examine additional features, such as mean grain size or distance from an inlet, extended to multiple shoreline change classes, and is applicable to any sandy beach area with multitemporal LiDAR data coverage.

Of the four performance measures examined, the NJSD is the single most meaningful measure to assess the relative importance of the features because it uses the entire pdf and normalizes on the basis of entropy. JSD can be also equally considered as meaningful in cases where the entropy of the features is similar across the feature space. Correlation and the median-based measure sometimes fail to capture strong relationships or overemphasize weak relationships when geometric measures of the shape and spread do not adequately describe the distribution. It should be noted, however, that these measures can also provide insights. The median-based measure can serve as a reasonable tool for ranking features when computational limitations preclude the estimation of the pdfs and the underlying pdfs are expected to be highly non-Gaussian. High divergence values do not guarantee that a simple binary decision boundary will yield accurate classification. High rankings from the correlation measure can, however, suggest features that are amenable to simple binary decision rules, which is useful if such rules are to be used in a subsequent classification.

The classification results that were presented evaluate the ability to predict solely based on subtle variations in beach morphology without explicitly accounting for spatial correlation in the binary class variables. Overall, the results are promising; however, they must be understood in the context for which they were intended. The predictions are based on probabilities of erosion occurrence for a given morphologic feature learned from the LiDAR data with no direct inclusion of time, bathymetry, or the governing physics of sediment transport. Therefore, the classification results are data dependent and not intended as a stand-alone tool for predicting shoreline change. Rather, the success of the results demonstrates that certain morphologies can be systematically extracted from LiDAR data sets and incorporated to discern patterns in beach change, supporting the notion of morphologic change indicators and their potential utility for beach characterization.

Finally, the focus here was on assessing individual morphologies rather than their joint effect. Methods such as mutual information or other approaches could be applied to examine the utility of feature pairs for discerning erosion and accretion zones. Additionally, the effect of scale dependence on a feature’s performance is an additional area of further investigation.

\[ JSD = \sum_x \pi_1 P_1 \log \left( \frac{P_1}{\pi_1 P_1 + \pi_2 P_2} \right) + \sum_x \pi_2 P_2 \log \left( \frac{P_2}{\pi_1 P_1 + \pi_2 P_2} \right) \]

\[ = \sum_x \pi_1 P_1 \log(P_1) - \sum_x \pi_1 P_1 \log(\pi_1 P_1 + \pi_2 P_2) + \sum_x \pi_2 P_2 \log(P_2) - \sum_x \pi_2 P_2 \log(\pi_1 P_1 + \pi_2 P_2) \]

\[ = H(\pi_1 P_1 + \pi_2 P_2) - \pi_1 H(P_1) - \pi_2 H(P_2) \]

where \( H \) is the information-theoretic entropy function. From the law of total probability, the first term in (10) is equal to

\[ H(\pi_1 P_1 + \pi_2 P_2) = H(\pi_1 P(x|C_1) + \pi_2 P(x|C_2)) = H(x), \]

Furthermore, we get the following result for the second terms in (10):

\[ -\pi_1 H(P_1) - \pi_2 H(P_2) = \sum_{c=1}^2 \pi_c \sum_x P(x|C) \log(\pi(x|C)) \]

\[ = -H(x|C). \]

Replacing the terms in (10) with (11) and (12), we get the mutual information equivalency, i.e.,

\[ JSD = H(x) - H(x|C) = I(x;C). \]
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Dr. Slatton was named the 2006 winner of the Presidential Early Career Award for Scientists and Engineers for work on predicting signal propagation in highly cluttered environments using remotely sensed geometry from ALSM. On March 30, 2010, he passed away from cancer, and the world lost a brilliant researcher, talented teacher, and loving father.
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