Reasoning with Data-Centric Business Processes

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Abstract. We describe an approach to modelling and reasoning about data-centric business processes and present a form of general model checking. Our technique extends existing approaches, which explore systems only from concrete initial states.
Specifically, we model business processes in terms of smaller fragments, whose possible interactions are constrained by first-order logic formulae. In turn, process fragments are connected graphs annotated with instructions to modify data. Correctness properties concerning the evolution of data with respect to processes can be stated in a first-order branching-time logic over built-in theories, such as linear integer arithmetic, records and arrays.
Solving general model checking problems over this logic is considerably harder than model checking when a concrete initial state is given. To this end, we present a tableau procedure that reduces these model checking problems to first-order logic over arithmetic. The resulting proof obligations are passed on to appropriate “off-the-shelf” theorem provers. We also detail our modelling approach, describe the reasoning components and report on first experiments.

1 Introduction

Data is becoming increasingly important to large organisations, both private enterprises and large government departments. Recent headlines on “big data” (cf. [7]) suggest that many organisations manage unprecedented amounts of structured data, and that worldwide, the volume of information processed by machines and humans doubles approximately every two years. Organisations need to be able to organise and process data according to their defined business processes, and according to business rules that may further specify properties of the processed data.

Unfortunately, most approaches to business process modelling do not adequately support the analysis of the complex interactions and dependencies that exist between an organisation’s processes and data. Although they may support process analysis, helping users find and remove errors in their models, most fall short when the processes are closely tied to structured data. The reasons for this are specific to the concrete formalism used for the analyses, but can normally be traced back to the fact that classical propositional logic or discrete Petri-nets are used. Neither of these can adequately represent structured data and the operations on it. In other words, these tools’ analyses make coarse abstractions of the data, and instead focus mostly on the correctness of workflows.

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The business artifact approach, initially outlined in [8], was one of the first to tackle this issue. It systematically elevates data to be a “first-class citizen”, while still offering automated support for process analysis. Its cornerstones are artifacts, which are records of data values that can change over time due to the modifications performed by services, which are formalised using first-order logic. Process analysis is provided, essentially, by means of model checking. That is, the following question is answered automatically: given some artifact model, a database providing initial values, and a correctness property in terms of a first-order linear-time temporal logic formula (called LTL-FO), do all possible artifact changes over time satisfy the correctness property? For the constraints given in [5], this problem is always decidable.

In this paper, we present an approach to modelling and reasoning about data-centric business processes, which is similar to this work, but which offers reasoning support that goes beyond that work’s “concrete model checking”. Our approach is based on process fragments that describe specific tasks of a larger process, as well as constraints for limiting the interactions between the fragments. As such it is also inspired by what is known as declarative business process modelling [9], meaning that users do not have to create a single, large transition system containing all possible task interleavings. Instead, users can create many small process fragments whose interconnections are governed by rules that determine which executions are permitted.

In our framework, those rules are given by first-order temporal logic. Unlike [5], we choose to extend CTL*, i.e., a branching time logic, rather than LTL, since process fragments are essentially annotated graphs and CTL* is, arguably, an appropriate formalism to express its properties (cf. [3]). Our database is given in terms of JSON objects [4], enriched by a custom, static type system which models and preserves the type information of any input data. Process fragments may modify data, and one can easily state and answer the concrete model checking problem as outlined above.

However, our approach also works if one does not start with an initial concrete database; that is, we intend to not only check whether it is possible to, reach a bad state (e.g., a set of data for which no process fragment is applicable) from some given state (i.e., the initial set of data), but also to determine whether for any set of data a bad state can be reached. In other words, we support what we call generic model checking. As the domains of many data items are infinite (e.g., any item of type integer), this problem is considerably harder, in fact, generally undecidable.

Informally, the two reasoning problems we are interested in are:

**Concrete data model checking problem:** Given a specification \( S \), a database \( s_0 \), and a CTL*(FO) formula \( \Phi \). Does \((s_0, S) \models \Phi \) hold?

**Unrestricted model checking problem:** Given a specification \( S \) and a CTL*(FO) formula \( \Phi \). For every database \( s_0 \), does \((s_0, S) \models \Phi \) hold?

As will become clear below, a specification is comprised of a process model, logical definitions, and constraints to combine process fragments. The relation \((s_0, S) \models \Phi \) means that the pair \((s_0, S)\) satisfies the query \( \Phi \). See Section 4 for the precise semantics.

Without any further restrictions, both problems are not even semi-decidable. This can be seen, e.g., by reduction from the domain-emptyness problem of 2-register machines. Hence, practical approaches need to work with restrictions to recover more pleasant complexity properties.
Fig. 1. Model of a purchase order system as process fragments and definitions.

2 A Running Example: Purchase Order

In this section, we introduce a simplified model of a purchase order system using process fragments. The purpose of the modelled system is to accept incoming purchase orders and process them further (packing, shipping, etc.), or to decline them straight away if there are problems. The whole model is depicted as a graph in Fig. 1 where the biggest process fragment is on the left, with further atomic fragments beside it (labelled Paid, Shipped, and Completed, respectively). Both process tasks, represented as nodes in the graph, and connections are typically annotated with extra information.

Node annotations determine whether or not a node is an initial and/or a final node, an entry and/or an exit node. This information is used to constrain the ways in fragments
can connect. Edges can carry a guard given as a formula and a simple program written in the programming language Groovy. The purpose of the program (given in the field “script”) is to modify the underlying database, which is referred to by the variable db.

The depicted system model has one initial node, Init, where it waits for a purchase order to arrive. Then, the system can either start to pack (i.e., enter node Pack), or decline the order (i.e., enter node Declined). An order can be declined if the guard \( \neg \text{acceptable}(\text{db}) \) in the annotation of edge \( e_2 \) is satisfied. The predicate \text{acceptable} is defined in the Definitions section of our input specification. In a nutshell, the sections Definitions and Constraints contain domain-knowledge, encoded as logical rules. (The constraint named “nongold” states that non-gold customers must pay before shipment; \( W \) is the “weak until” operator.)

If the order is not declined, an attempt will be made to pack its constituents. If all are in stock, the process will continue to the node Packed. However, if one or more items are missing, they need to be ordered in, which is expressed in the loop between the nodes Pack and Stocktake.

Informally, process fragments are linked together as follows. Starting from a state comprised of an init node and a given initial database, an outgoing transition from the current state can only be executed if it satisfies the transition’s guard. If it is satisfied, the associated program is executed to determine the new value of the database, and the edge’s target node becomes the new current state. The entry and exit annotations impose implicit constraints on how fragments can be combined: the execution of a new process fragment must always start with its entry node\(^1\) coming from an exit node. In other words, there are implicit transitions between all exit and all entry nodes. However, if a guard is associated to an entry node, this guard sits on all its implicit incoming transitions. The computation stops if from the current state no successor can be reached, either because there is no outgoing edge, the guards of all outgoing edges are not satisfied by the current state, or a depth limit has been reached.

In our example, two possible sequences are Init \( \rightarrow \) Declined, or Init \( \rightarrow \) Pack \( \rightarrow \ldots \rightarrow \) Invoice \( \rightarrow \) Paid \( \rightarrow \) Shipped \( \rightarrow \) Completed. It is not required to cover all fragments, as illustrated by the first run.

The database which can be modified by the programs given in the “script” annotations, is represented as a JSON object. See, for example, left hand side of Fig. 2. (The right hand side contains type definitions for the JSON data, see also Sec. 3.) The program annotated on edge \( e_2 \), which leads into node Declined, simply sets the field \text{final} inside \text{status} to \text{true}. Crucial for our example is the list of open items, under \text{status}, which has to be empty to be able to ship a purchase order. If it is not, constituents of the order are missing and need to be ordered until the list is empty.

As for sample queries consider the CTL* (FO) formula \( \neg (\exists F \text{ db.status.final} = \text{true}) \), which can be seen as a planning goal. The runs on the model above that falsify it lead to a database \text{db} that has reached a “final” state, with \text{status.final} being set to \text{true}. Planning queries are useful, e.g., for flexible process configuration from fragments during runtime. Another interesting query is \( \text{AG}(\forall s: \text{Stock} . (s \in \text{db.stock} \Rightarrow s.\text{available} \geq 0)) \). It is a safety property, saying that at all stages in the process run, and for all possible stock

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\(^1\) For simplicity we assume every fragment contains exactly one entry node.
items, the number of available items is non-negative. Such queries are typical during design time, and pose an unrestricted model checking problem.

3 Modelling Data With JSON Logic

Faithful modelling of business processes requires being able to model the objects (or data) manipulated by the processes and, of course, their evolution over time. In this section we focus on data modelling, which is based on JSON extended with a type system.

JSON [4] is simple, standardised, textual data representation format. In addition to a standard set of atomic values such as integers and strings, JSON supports two structuring techniques: sequencing (“arrays”) and arbitrarily nested hierarchies (through “objects”). Our choice of JSON (rather than XML, say), is based on the ease with which it can be written and understood by humans. JSON is sufficiently rich to be a plausible format for representing the data used in business processes, and its human ease-of-use is extremely helpful.

Other than simply being the medium in which data is represented, there are two important functions that JSON must support. Firstly, it must be possible to manipulate JSON values in the course of executing a specification. This functionality is realised through the use of the Groovy programming notation.

Secondly, it must be possible to express logical predicates over JSON values, both to guard process transitions and to pick out certain forms of value that are of interest. In particular, if a specification is to achieve a particular end-goal, with a database being in a particular configuration, we need to be able to describe how the various values in that database inter-relate. It is this that motivates our choice of the logically expressive capabilities of first order logic, together with sorts such as lists and numbers.

In addition to first-order predicates, we also use a simple type system over JSON values. This provides a simple mapping into the sorts of our underlying first-order logic.
We note that the type system is indispensable for unrestricted model checking, in order to derive from it logical axioms for object and array manipulations.

### 3.1 A Type System for JSON

First we briefly summarise the syntax that is fully described in the IETF RFC [4]:

JSON values can be numbers, booleans (true and false), strings (written between double-quotes, e.g., "a string") and a special value null. JSON’s arrays are written as comma-separated values between square-brackets, e.g., [1, "string", [true]]. JSON objects are similar to records or structures in languages such as Pascal and C. They are written as lists of field-name/value pairs between braces. Both forms are illustrated in Figure 2

Sibling field-names within an object should be unique, and are considered unordered. Therefore, an object can be thought of as a finite map from field-names to further JSON values. Following this conception, we write Obj{vf} to denote an object whose field names are the domain of finite map vf, with field s’s value being vf(s).

JSON does not impose any restrictions on the structure of values. For example, a list may contain both strings and integers. However, we choose to restrict this freedom with a simple type system comparable to those in third-generation languages such as C.

Let JSON types be denoted by \( \tau \), \( \tau' \), \( \tau_1 \) etc., then

\[
\tau ::= \text{Integer} \mid \text{Bool} \mid \text{String} \mid \text{List}[\tau] \mid \text{Option}[\tau] \mid \text{ObjTy}[tf] \mid \text{EnumTy}[sl]
\]

where tf is a finite map from strings to types, and sl is a list of strings.

The Option and EnumTy types are the only ones that do not have a obvious connection back to a set of JSON values. The Option type is used to allow for values that are not necessarily always initialised, but which come to acquire values as a process progresses. We do not expect to see the option-constructor occur with multiple nestings, e.g., a type such as Option[Option[String]]. The EnumTy type is used to model finite enumerated types, where each value is represented by one of the strings in the provided list. This flexibility in the type system allows for more natural modeling.

Values are assigned types with the following inductive relation, where we write \( v : \tau \) to indicate that JSON value \( v \) has type \( \tau \), where the meta-variables \( i \) and \( s \) correspond to all possible integer and string values respectively, and where we use \( e \in \ell \) to mean that element \( e \) is a member of list \( \ell \):

\[
\begin{align*}
\text{true} : \text{Bool} & \quad \text{false} : \text{Bool} & \quad i : \text{Integer} & \quad s : \text{String} \\
\forall v \in \text{els}, v : \tau & \quad \forall v \in \text{el}, v : \tau & \quad [\text{els}] : \text{List}[\tau] \\
\text{null} : \text{Option}[\tau] & \quad v : \text{Option}[\tau] & \quad s \in \text{sl} \\
\forall v \in \text{dom}(vf), v : \tau & \quad \forall s \in \text{dom}(vf), vf(s) : tf(s) \\
\text{Obj{vf}} : \text{ObjTy}[tf] & \quad \text{dom}(vf) = \text{dom}(tf)
\end{align*}
\]

This type system is simple and designed to be pragmatic. Meta-theoretically, it is not particularly elegant. In particular, values may have multiple types: if a value \( v \) is of type \( \tau \), then it is also of type Option[\tau]; string values are not just of type String, but also have an arbitrary number of possible enumeration types.
3.2 From JSON to First-Order Logic

When a user develops a business specification, we expect them to name the various types of interest with the type system above. When concrete initial values are given for a concrete model-checking problem, we use that type system to check that these values really do have the appropriate type. The same system is used to ensure that logical guards and goal-conditions are sensible, as discussed below. It also plays a pivotal role in our reasoning procedure for the unrestricted model checking problem, which requires to reflect the semantics of a JSON type model in many-sorted first-order logic. We are going to describe that now.

We fix a non-empty set $S$ of sorts and a first-order logic signature $\Sigma$ comprised of function and predicate symbols of given arities over $S$. We assume infinite supplies of variables, one for every sort in $S$. A constant is a 0-ary function symbol. The (well-sorted $\Sigma$-)terms and atoms are defined as usual. We assume $\Sigma$ contains a predicate symbol $\approx_s$ (equality) of arity $s \times s$, for every sort $s \in S$. Equational atoms, or just equations, are written infix, usually without the subscript $s$, as in $1 + 1 \approx 2$. We write $\phi[x]$ to indicate that every free variable in the formula $\phi$ is among the list $x$ of variables, and we write $\phi[t]$ for the formula obtained from $\phi[x]$ by replacing all its free variables $x$ by the corresponding terms in the list $t$.

We assume a sufficiently rich set of Boolean connectives (such as $\neg$, $\wedge$) and the quantifiers $\forall$ and $\exists$. The well-sorted $\Sigma$-formulas, or just (FO) formulas are defined as usual. We are particularly interested in signatures containing (linear) integer arithmetic. For that, we reserve the sort symbol $\mathbb{Z}$, the constants $0, \pm 1, \pm 2, \ldots$, the function symbols $+$ and $-$, and the predicate symbol $>$, each of the expected arity over $\mathbb{Z}$.

The semantics of our logic is the usual one: a $\Sigma$-interpretation $I$ consists of non-empty, disjoint sets, called domains, one for each sort in $S$. We require that the domain for $\mathbb{Z}$ is the set of integers, and that every arithmetic function and predicate symbol is mapped to its obvious function over the integers. A (variable) assignment $\alpha$ is a mapping from the variables into their corresponding domains. Given a formula $\phi$ and a pair $(I, \alpha)$ we say that $(I, \alpha)$ satisfies $\phi$, and write $(I, \alpha) \models \phi$, iff $\phi$ evaluates to true under $I$ and $\alpha$ in the usual sense (the component $\alpha$ is needed to evaluate the free variables in $\phi$). If $\phi$ is closed then $\alpha$ is irrelevant and we can write $I \models \phi$ instead of $(I, \alpha) \models \phi$. We say that a closed sentence $\phi$ is valid (satisfiable) iff $I \models \phi$ for all (some) interpretations $I$.

In order to map our JSON modelling framework to FOL we let the sorts $S$ contain all the defined type names in the JSON type model of the given specification. In the example in Section 2 these are $\text{DB}$, $\text{Stock}$ and $\text{Status}$. Without loss of generality we assume that the top-level type in a JSON type model is always called $\text{DB}$\footnote{We need additional sorts, e.g., for truth values and integers, as mentioned. The sorts in $S$ are written in italics, as in $\text{DB}$.}. We call any JSON term of type $\text{DB}$ a database. See again Section 2 for an example. We fix a dedicated variable $db$ of sort $\text{DB}$. Informally, $db$ will be used to hold the database at the current time point.

Furthermore, we must provide mappings into FOL from terms that are specific to JSON. In some sense, both JSON’s arrays and its objects are generic “arrays”, values
that can be seen as collections of independently addressable components. The JSON syntax for that is a usual one: \( a[i] \), denotes the value of the \( i \)th element of array \( a \); and \( \text{obj}.\text{fld} \), denotes the value of \( \text{obj} \)'s field called \( \text{fld} \). These are the accessor operations. Their FOL representation (as terms) is \( \text{index}(a, i) \) and \( \text{fld}(\text{obj}) \), respectively.

This mapping allows to formulate predicates on JSON data in FOL. For example, the guard \( \text{db}.\text{status.paid} \neq \text{true} \) in Sec. 2's example maps to the formula \( \text{paid}((\text{status}(\text{db}))) \neq \text{true} \). We also support updator operations for both arrays and objects. For arrays, we have \( \text{update}(a, i, v) \), which denotes an array that is everywhere the same as \( a \) except that at index \( i \) it has value \( v \). For objects, we have analogous updator functions per field. If an object type had fields \( \text{fld1}, \text{fld2} \) etc., we would then have the term \( \text{upd fld1(obj, v)} \), denoting an object everywhere the same as \( \text{obj} \) except with value \( v \) for its field \( \text{fld1} \). We note that these mappings can be automated without effort. With field and array updators to hand, we can translate a model’s scripts (Groovy fragments on graph-edges) into a logical form. This translation is to a term of one free variable \( db \), denoting the effect of that script on \( db \).

Because standard FOL theorem provers do not natively support the theory of arrays and objects, we generate suitable FOL axioms from the given JSON type model. For arrays, the appropriate axioms are well-known and for objects, there are analogous axioms. For example, \( \text{fld1}(\text{upd fld1(obj, v)}) = v \), and \( \text{fld2}(\text{upd fld1(obj, v)}) = \text{fld2(obj)} \).

In addition, we have concrete syntax for writing complete values (e.g., \([2, 4, 6]\) for a list of three elements), though this is actually just syntactic sugar for a chain of updates over some underlying base object. In particular, any database has a (FOL) term representation, called “database as a term” below. Moreover, this same term language allows us to give partial specifications of filled databases. For example, the term \( \text{upd gold(db, true)} \) stands for a (any) database represented by the constant \( db \) whose \( \text{gold} \) field holds the value \( \text{true} \), with the other fields arbitrary. Indeed, analysing such partially filled databases is one of the main goals of our research agenda.

4 Modelling Processes

In this section we describe our framework for modelling processes. As said earlier, it is centered around the notion of process fragments that manipulate databases over time. The cooperation of the fragments is described by (temporal) constraints. All constraints and guards in state transitions may refer to user-specified predicates on (components of) the database, which we call (logical) definitions here. We will introduce these components now.

4.1 Process Fragments

A guard \( \mu \) is a FOL formula with free variables at most \( \{db\} \); an update term \( u \) is a FOL term with free variables at most \( \{db\} \). By Guard (Update) we denote the set of all guards (update terms); \( \text{GProg} \) is the set of all Groovy programs. Without further formalization we assume the Groovy programs are “sensible” and describe database updates that can be characterized as update terms.
A process fragment $F$ is directed labeled graph $(N, E, λ^N, λ^E)$, where $N$ is a set of nodes, $E \subseteq N \times N$ is a set of edges, $λ^E : E \mapsto \text{Guard} \times \text{GProg} \times \text{Update}$ is an edge labeling function, and $λ^N : N \mapsto 2^{\text{init}, \text{entry}, \text{exit}} \cup \{\text{Guard}\}$ is an edge labeling function.

The informal semantics of process fragments has been given in Section 2 already. The precise semantics of a set of process fragments is given by first translating it into one single process model $\mathcal{P}$ and then defining the semantics of $\mathcal{P}$ in terms of its runs.

More formally, a process (model) $\mathcal{P}$ is a quadruple $(N, n_0, E, λ^E)$ where $N$, $E$ and $λ^E$ are as above and $n_0 \in N$ is the initial node. Suppose as given a set $\mathcal{F} = \{F_1, \ldots, F_k\}$ of process fragments, for some $k \geq 1$, where $F_i = (N_i, E_i, λ^N_i, λ^E_i)$ and $N_i$ and $N_j$ are disjoint, for all $i \neq j$. Suppose further, without loss of generality, that exactly one node in $\bigcup_{1 \leq i \leq k} N_i$ is labeled as an init node. Let $n_0$ be that node. The process model $\mathcal{P} = (s_0, n_0, N, E, λ^E)$ associated to $\mathcal{F}$ is defined as follows:

$$\begin{align*}
N &= \bigcup_{1 \leq i \leq k} N_i \\
E &= (\bigcup_{1 \leq i \leq k} E_i) \cup E^+ \\
λ^E &= (\bigcup_{1 \leq i \leq k} λ^E_i) \cup λ^+
\end{align*}$$

where ($ε$ denotes the empty Groovy program)

$$E^+ = \{(m, n) \mid m \in N_i, n \in N_j, \text{exit} \in λ^N_i(m) \text{ and } \text{entry} \in λ^N_j(n), \text{ for some } 1 \leq i, j \leq k\}$$

$$λ^+ = \{(m, n) \mapsto (γ, ε, \text{db}) \mid (m, n) \in E^+ \text{ and } \{\text{entry}, γ\} \subseteq λ^N_i(n), \text{ for some } 1 \leq j \leq k\}$$

For the above construction to be well-defined we require that every entry node in every fragment $F_i$ is also labeled with a guard $γ$ (which could be $T$).

### 4.2 Definitions and Constraints

Definitions are logical abbreviations. As such, they are not semantically necessary. Nonetheless, just as in mathematics, they are a crucial aid in the construction and comprehensibility of useful models. Formally, a definition (for $p$) is a closed formula of the form $∀x:s . p(x) ⇔ φ[x]$ where $x$ is list of variables of sorts $s \subseteq S$, $p$ is a predicate symbol of the proper arity, and $φ$ is a formula.

Constraints specify how process fragments can be combined. The idea has been pursued before, e.g., in the Declare system [9] which uses propositional (linear) temporal logic for that. In order to take data into account, we work with a fragment of CTL$^∗$ over first-logic, which we refer to as CTL$^∗$(FO). The syntax of our CTL$^∗$(FO) state formulae is given by $Φ ::= ζ | ¬Φ | Φ ∧ Φ | λψ | Eψ$, where $ζ$ is a FO formula with free variables at most $\{\text{db}\}$, and $ψ$ a path formula defined via $ψ ::= Φ | ¬ψ | ψ \land ψ | Xψ | Xψψ | ψ \land ψ$. (The operator $X$ is “weak next”.) A constraint then is simply a state formula. Notice that because constraints may contain the free variable $\text{db}$, our logic is not obtained from propositional CTL$^∗$ by replacing propositional variables by closed formulas.

Figure 1 contains some examples of definitions and constraints.

### 4.3 Specifications and Semantics

The modelling components describing so far are combined into specifications. Formally, a specification $S$ is a tuple $(\mathcal{P}, \mathcal{D}, \mathcal{C})$ where $\mathcal{P}$ is a process, $\mathcal{D}$ is a set of definitions and $\mathcal{C}$ is a set of constraints. An instance $\mathcal{I}$ (of $S$) is a pair $(s_0, S)$, where $s_0$ is a database (as a term) and $S$ is a specification.
We are now in the position to provide a formal definition for the model checking problems stated in the introduction. Let $S = (P, D, C)$ be as above, where $P$ is of the form $(N, n_0, E, A)$ and $\phi$ a state formula with free variables at most $\{db\}$, the query.

As a first step to define the satisfaction relation $(s_0, S) \models \phi$ between an instance and a query we make the constraints $C$ part of the query. Assume $\phi$ is given in negation normal form (this is always possible) and that it starts with a path quantifier $(E$ or $A$). The expanded query $\phi_C$ is the formula $A(C \Rightarrow \psi)$ if $\phi = A\psi$, for some formula $\psi$, and it is $E(C \land \psi)$ if $\phi = E\psi$. Here, $C$ is read as a conjunction of its elements. (The rationale for this definition is that the desired treatments of constraints is indicated by the path quantifier in the query.) Notice that with $\phi$ also $\phi_C$ is a query. Now define $(s_0, S) \models \phi$ iff $(s_0, P, D) \models \phi_C$, i.e., the triple $(s_0, P, D)$ satisfies $\phi_C$. It remains to define the latter satisfaction relation, which we turn to now.

As a convenience, we say that $P$ contains a transition $m \xrightarrow{\gamma} n$ if $(m, n) \in E$ and $\lambda^P(m, n) = (\gamma, u)$, for some guard $\gamma$ and Groovy program $u$ as an update term.

A run $r$ (of $(P, D)$) from $s_0$ is a possibly infinite sequence $(n_0, s_0)(n_1, s_1)(n_2, s_2)\cdots$ of pairs of nodes and databases, also called states, such that (i) $P$ contains transitions of the form $(n_i \xrightarrow{\gamma_{i,n_i}} n_{i+1})$, (ii) $D \models \gamma_i[s_i]$ and (iii) $s_{i+1} = u_i[s_i]$. In item (i) in case $i = 0$ the node $n_0$ is meant to be the initial node $n_0$ in $P$. Notice that in item (ii) the definitions $D$ play the role of axioms from which the instantiated guard $\gamma_i[s_i]$ is to follow. Occasionally the nodes in a run are not important. and we confuse a run with its projection on the states $s_0, s_1, s_2, \ldots$.

For a run $r = (n_0, s_0)(n_1, s_1)(n_2, s_2)\cdots$ and $i \geq 0$ we define $r[i] = (n_i, s_i)$, sometimes also $r[i] = s_i$. By $r'$ we denote the truncated run $(r_i, s_i)(r_{i+1}, s_{i+1})\cdots$, by $|r|$ the number of elements in the run or $\infty$, if $r$ is, in fact, infinite. Obviously, $r^0 = r$.

For any formula $\phi \in \text{CTL}^*(\text{FO})$ with free variables at most $\{db\}$ we define $(s_0, P, D) \models \phi$ as follows:

\[
\begin{align*}
(s_0, P, D) \models \zeta & \iff (D \Rightarrow \zeta[s_0]) \\
(s_0, P, D) \models \neg \psi & \iff (s_0, P, D) \not\models \psi \text{ is not true} \\
(s_0, P, D) \models \psi_1 \land \psi_2 & \iff (s_0, P, D) \models \psi_1 \text{ and } (s_0, P, D) \models \psi_2 \\
(s_0, P, D) \models A\psi & \iff (P, D, r) \models \psi \text{ for all runs } r \text{ starting in } s_0 \\
(s_0, P, D) \models E\psi & \iff (P, D, r) \models \psi \text{ for some run } r \text{ starting in } s_0,
\end{align*}
\]

where the relation $(P, D, r) \models \psi$ is defined as

\[
\begin{align*}
(P, D, r) \models \phi & \iff (s_0, P, D) \models \phi \\
(P, D, r) \models \neg \psi' & \iff (P, D, r) \models \psi' \text{ is not true} \\
(P, D, r) \models \psi_1' \land \psi_2' & \iff (P, D, r) \models \psi_1' \text{ and } (P, D, r) \models \psi_2' \\
(P, D, r) \models X\psi' & \iff |r| > 1 \text{ and } (P, D, r') \models \psi' \\
(P, D, r) \models \neg \psi' & \iff |r| \leq 1, \text{ or } |r| > 1 \text{ and } (P, D, r') \models \psi' \\
(P, D, r) \models \psi_1' \land \psi_2' & \iff \text{there exists a } j \geq 0, \text{ such that } |r| > j \text{ and } (P, D, r') \models \psi_2', \\
& \quad \text{and } (P, D, r') \models \psi_1' \text{ for all } 0 \leq i < j \\
(P, D, r) \models \psi_1' \land \psi_2' & \iff (P, D, r') \models \psi_1' \text{ for all } i \leq |r|, \text{ or there exists a } j \geq 0, \text{ such that } |r| > j \text{ and } (P, D, r') \models \psi_2', \\
& \quad \text{and } (P, D, r') \models \psi_1' \text{ for all } 0 \leq i \leq j.
\end{align*}
\]

We further assume the usual “syntactic sugar”, such as $\forall, \Rightarrow$ (implies), $G$ (always), $F$ (eventually), or $W$ (weak until) operators, which can easily be defined in terms of the above set of operators in the expected way. Note that we distinguish a strong next
operator, $X$, from a weak next operator, $\bar{X}$ as described in [11]. This gives rise to the following equivalences: $\psi R \Phi = \Phi \land (\psi \lor \bar{X}\psi R \Phi)$ and $\psi U \Phi = \Phi \lor \psi \land X\psi U \Phi$ as one can easily verify by using the above semantics. This choice is motivated by our bounded model checking algorithm, which has to evaluate CTL'(FO) formulae over finite traces as opposed to infinite ones. For example, when evaluating a safety formula, such as $G\psi$, we want a trace of length $n$ that satisfies $\psi$ in all positions $i \leq n$ to be a model of said formula. On the other hand, if there is no position $i \leq n$, such that $\psi'$ is satisfied, we don’t want this trace to be a model for $F\psi'$. This is achieved in our logic as $G\psi = \psi \land \bar{X}G\psi$ and $F\psi = \psi \lor XF\psi$ hold. Note also that $\neg X\psi \neq X\neg\psi$, but $\neg X\psi = \bar{X}\neg\psi$.

5 Reasoning with Tableaux for CTL'(FO)

Tableau calculi for temporal logics have been considered for a long time [6] e.g. as an appropriate and natural reasoning procedure. There is also a version for propositional CTL' [11]. However, we are not aware of a first-order logic tableaux calculus that accommodates our requirements, hence we devise one, see below. We note that we circumvent the difficult problem of loop detection by working in a bounded model checking setting, where runs are artificially terminated when they become too long.

Suppose we want to solve an unrestricted model checking problem, i.e., to show that $(s_0, P, D) \models \phi_C$ holds, for every database $s_0$. As usual with tableau calculi, this is done by attempting to construct a countermodel for the negation of this statement. The universally quantified database $s_0$ then becomes a Skolem constant, say, $db$, representing an (unknown) initial database. A state then is a pair of the form $(n, u[db])$ where $n \in N$ and $u[db]$ is an update term instantiated with that initial database. We find it convenient to formulate the calculi’ inference rules as operators on (sets of) sequents. A sequent is an expression of the form $s \vdash Q \Phi$ where $s$ is a state, $Q \in \{E,A\}$ is a path quantifier, and $\Phi[db]$ is a (possible empty) set of CTL'(FO) formulas in negation normal form with free variables at most $\{db\}$. When we write $s \vdash Q \Phi$ we mean $s \vdash Q (\emptyset) \cup \Phi$.

The informal semantics of a sequent $(n, u[db]) \vdash Q \Phi[db]$ is “some run of the instance $(db, P, D)$ has reached the state $(n, u[db])$ and $(n, u[db]) \vdash Q \Phi[u[db]]”.

A tableau calculus, the calculus below derives trees that represent disjunctions of conjunctions of formulas. More precisely, the nodes are labeled with sets of sequents that are read conjunctively, and sibling nodes are connected disjunctively. The purpose of the calculi’ inference rules is to analyse a given sequent by breaking up the formulas in the sequent according to their boolean operators, path quantifiers and temporal operators. An additional implicit and/or structure is given by reading the formulas $\Phi$ in $s \vdash E \Phi$ conjunctively, and reading the formulas $\Phi$ in $s \vdash A \Phi$ disjunctively. The reason is that $A$ does not distribute over “or” and $E$ does not distribute over “and”.

We need some more definitions to formulate the calculus. A formula is classical iff it contains no path quantifier and no temporal operator. A formula is a modal atom iff its top-level operator is a path quantifier or a temporal operator. A sequent $s \vdash Q \Phi$ is classical if all formulas in $\Phi$ are classical.

A tableau node is a (possibly empty) set of sequents, denoted by the letter $\Sigma$. We often write $\sigma; \Sigma$ instead of $\{\sigma\} \cup \Sigma$. We simply speak of “nodes” instead of “tableau nodes” if confusion with the nodes in graphs is unlikely.
Let $\phi_C$ be a given expanded query and $S$ a specification as introduced before. The *initial sequent* is the sequent $s_0 \vdash_\rightarrow \neg \phi_C$, where $s_0 = (n_0, \emptyset)$ is the *initial state*, for some fresh constant $\emptyset$. Notice that the expanded query is negated, corresponding to the intuition of attempting to compute a countermodel for the negation of the expanded query.

Because we are adopting a standard notion of tableau derivations it suffices to define the inference rules. (The root node contains the initial sequent only.) The components $P$ and $D$ are left implicit below.

**Boolean rules.** The implicit reading of $\phi$ as disjunctions/conjunctions in a $\vdash_{A}/\vdash_{E}$ sequent sanction the following rules.

\[
\begin{align*}
\text{E-}\wedge & \quad s \vdash_\rightarrow \phi \land \psi, \Phi; \Sigma \\
& \quad s \vdash_\rightarrow \phi, \psi, \Phi; \Sigma \\
\text{E-}\lor & \quad s \vdash_\rightarrow \phi, \psi, \Phi; \Sigma \\
& \quad s \vdash_\rightarrow \phi \lor \psi, \Phi; \Sigma \\
\text{A-}\lor & \quad s \vdash_{A} \phi \lor \psi, \Phi; \Sigma \\
& \quad s \vdash_{A} \phi, \psi, \Phi; \Sigma \\
\text{A-}\land & \quad s \vdash_{A} \phi \land \psi, \Phi; \Sigma \\
& \quad s \vdash_{A} \phi, \psi, \Phi; \Sigma
\end{align*}
\]

if $\phi$ is not classical or $\psi$ is not classical (no need to break classical formulas apart).

**Rules to separate classical sequents.** The following rules separate away the classical formulas from the modal atoms in $\phi$. Every classical sequent can be passed on to a first-order theorem prover; if the result is “unsatisfiable” then the node is closed.

\[
\begin{align*}
\text{E-Split} & \quad s \vdash_\rightarrow \Phi; \Sigma \\
& \quad s \vdash_{E} \Gamma[u]; s \vdash_{E} \Phi \setminus \Gamma; \Sigma \\
\text{A-Split} & \quad s \vdash_{A} \Phi; \Sigma \\
& \quad s \vdash_{A} \Gamma[u]; s \vdash_{A} \Phi \setminus \Gamma; \Sigma
\end{align*}
\]

if $s = (n, u[\emptyset])$ for some $n$, $\Gamma$ consists of all classical formulas in $\phi$. $\Gamma[u[\emptyset]]$ is obtained from $\Gamma$ by replacing every free occurrence of the variable $\emptyset$ in all its formulas by $u[\emptyset]$, and $\Gamma \neq \emptyset$ and $\Gamma[u[\emptyset]] \neq \emptyset$.

The left rule exploits the equivalence $E(\phi \land \Phi) \equiv E\phi \land E\Phi$ if $\phi$ is classical, and the right rule exploits the equivalence $A(\phi \lor \Phi) \equiv A\phi \lor A\Phi$ if $\phi$ is classical.

**Rules for path quantifiers.** The next rules eliminate path quantifiers, where $Q \in \{E, A\}$.

\[
\begin{align*}
\text{E-Elim} & \quad s \vdash_{E} Q \phi, \Phi; \Sigma \\
& \quad s \vdash_{Q} \phi, s \vdash_{E} \Phi; \Sigma \\
\text{A-Elim} & \quad s \vdash_{A} Q \phi, \Phi; \Sigma \\
& \quad s \vdash_{Q} \phi, s \vdash_{A} \Phi; \Sigma
\end{align*}
\]

The soundness of the left rule follows from the equivalences $E(Q \phi \land \Phi) \equiv E \phi \land E \Phi \equiv Q \phi \land E \Phi$, and the soundness of the right rule follows from the equivalences $A(Q \phi \lor \Phi) \equiv A \phi \lor A \Phi \equiv Q \phi \lor A \Phi$.

The above rules apply also if $\Phi$ is empty. Notice that in this case $\Phi$ represents the empty conjunction in $s \vdash_{E} \Phi$, which is equivalent to $\top$, and the empty disjunction in $s \vdash_{A} \Phi$, which is equivalent to $\bot$.

When applied exhaustively, the rules so far lead to sequents that all have the form $s \vdash_{Q} \Phi$ such that (a) $\Phi$ consists of classical formulas only, or (b) $\Phi$ consists of modal atoms only with top-level operators from $\{U, R, X, \overline{X}\}$. 

12
**Rules to expand \( \text{U} \) and \( \text{R} \) formulas.** The following rules perform one-step expansions of modal atoms with \( \text{U} \) and \( \text{R} \) operators.

\[
\begin{align*}
\text{U-Exp} & \quad s \vdash_{Q} (\phi \text{U} \psi), \Phi; \Sigma \\
\quad s \vdash_{Q} \phi \lor (\psi \land \text{X}(\phi \text{U} \psi)), \Phi; \Sigma \\
\text{R-Exp} & \quad s \vdash_{Q} (\phi \text{R} \psi), \Phi; \Sigma \\
\quad s \vdash_{Q} (\psi \land \text{X}(\phi \text{R} \psi)), \Phi; \Sigma
\end{align*}
\]

When applied exhaustively, the rules so far lead to sequents that all have the form \( s \vdash_{Q} \phi \) such that (a) \( \Phi \) consists of classical formulas only, or \( \Phi \) consists of modal atoms only with top-level operators from \( \{\text{X}, \overline{\text{X}}\} \).

**Rules to simplify \( \text{X} \) and \( \overline{\text{X}} \) formulas.** Below we define inference rules for one-step expansions of sequents of the form \( s \vdash_{Q} \text{X} \phi \) and \( s \vdash_{Q} \overline{\text{X}} \phi \). The following inference rules prepare their application.

\[
\begin{align*}
\text{E-X-Simp} & \quad s \vdash_{E} \text{X} \phi_{1}, \ldots, \text{X} \phi_{n}, \overline{\text{X}} \psi_{1}, \ldots, \overline{\text{X}} \psi_{m}; \Sigma \\
& \quad s \vdash_{E} \text{X}(\phi_{1} \land \cdots \land \phi_{n} \land \psi_{1} \land \cdots \land \psi_{m}); \Sigma
\end{align*}
\]

if \( n + m > 0 \), where \( Y = \overline{\text{X}} \) if \( n = 0 \) else \( Y = \text{X} \). Intuitively, if just one of the modal atoms in the premise is an X-formula then a successor state must exist to satisfy it, hence the X-formula in the conclusion. Similarly:

\[
\begin{align*}
\text{A-X-Simp} & \quad s \vdash_{A} \text{X} \phi_{1}, \ldots, \text{X} \phi_{n}, \overline{\text{X}} \psi_{1}, \ldots, \overline{\text{X}} \psi_{m}; \Sigma \\
& \quad s \vdash_{A} \text{X}(\phi_{1} \lor \cdots \lor \phi_{n} \lor \psi_{1} \lor \cdots \lor \psi_{m}); \Sigma
\end{align*}
\]

if \( n + m > 0 \), where \( Y = \text{X} \) if \( m = 0 \) else \( Y = \overline{\text{X}} \).

The correctness of this rule follows from the equivalences \( \text{A}(\text{X} \phi \lor \overline{\text{X}} \psi) \equiv \text{A}(\overline{\text{X}} \phi \lor \text{X} \psi) \equiv \text{A}(\phi \lor \psi) \).

To summarize, with the rules so far, all sequents can be brought into one of the following forms: (a) \( s \vdash_{Q} \Gamma \), where \( \Gamma \) consists of classical formulas only, (b) \( s \vdash_{Q} \text{X} \phi \), or (c) \( s \vdash_{Q} \overline{\text{X}} \phi \).

**Rule to close branches.** The following rule derives no conclusions and this way indicates that a branch in a tableau is “closed”.

\[
\begin{align*}
\text{Unsat} & \quad s_{1} \vdash_{Q_{1}} \phi_{1}; \cdots; s_{n} \vdash_{Q_{n}} \phi_{n}
\end{align*}
\]

if every \( \Phi_{i} \) consists of closed classical formulas, and \( \land (\mathcal{D} \cup \Phi_{1} \cup \cdots \cup \Phi_{n}) \) is unsatisfiable (not satisfiable).

**Rules to expand \( \text{X} \) and \( \overline{\text{X}} \) formulas.**

\[
\begin{align*}
\text{E-\overline{X}-Exp} & \quad (m, t) \vdash_{E} \overline{\text{X}} \phi; \Sigma \\
& \quad (n_{1}, u_{1}[t]) \vdash_{E} \gamma_{1}[t] \land \phi; \Sigma \\
& \quad \cdots \\
& \quad (n_{k}, u_{k}[t]) \vdash_{E} \gamma_{k}[t] \land \phi; \Sigma \\
& \quad (m, t) \vdash_{E} \neg \gamma_{k}[t] \land \cdots \land \neg \gamma_{1}[t]; \Sigma
\end{align*}
\]

if there is a \( k \geq 0 \) such that \( m \xrightarrow{u_{i}} n_{i} \) are all transitions in \( \mathcal{P} \) emerging from \( m \), where \( 1 \leq i \leq k \).

This rule binds the variable \( db \) in the guards to the term \( t \), which represents the current database, while it leaves the formula \( \phi \) untouched. The variable \( db \) in \( \overline{\text{X}} \phi \) refers
The rules to separate classical sequents above will bind $db$ in $\Phi$ correctly. There is also a rule $E\cdot X\cdot Exp$ whose premise sequent is made with the $X$ operator instead of $\mathcal{X}$. It differs from the $E\cdot X\cdot Exp$ rule only by leaving away the rightmost conclusion. We do not display it here for space reasons. We note that both rules are defined also if $k = 0$.

If there is a $k \geq 0$ such that $m \xrightarrow{\gamma_i^m} n_i$ are all transitions in $P$ emerging from $m$, where $1 \leq i \leq k$.

This rule will for each of the conclusion sequent lead to a case distinction (via branching) whether the guard of a transition is true or not. Only if the guard is true the transition must be taken. The conclusion sequent $(m, t) \vdash E\gamma_1[t] \lor \cdots \lor \gamma_k[t]; \Sigma$ forces that at least one guard is true. Analogously to above, there is also a rule $A\cdot X\cdot Exp$ for the $X$ case, which does not include this sequent. This reflects that $X$ formulas are true in states without successor.

Both rules also work as expected if $k = 0$: for $A\cdot X\cdot Exp$ the formula in the sequent $(m, t) \vdash \gamma_1[t] \lor \cdots \lor \gamma_k[t]$ is equivalent to $\bot$ (false); for $A\cdot \mathcal{X}\cdot Exp$ the premise sequent is deleted. If additionally $\Sigma$ is empty then the result is a node with the empty set of sequents. This does not indicate branch closure; branch closure is indicated by deriving no conclusions, not a unit-conclusion, even if empty.

This concludes the presentation of the tableau calculus. As said above, we enforce derivations to be finite by imposing a user-specified maximal length on the number of state transitions it executes. This is realized as a check in the rules to expand $X$ and $\mathcal{X}$ formulas by pretending a value $k = 0$ of transitions emerging from the node of the considered state, if the run to that state becomes too long. (This is not formalized above.)

For this bounded model checking setting we obtain a formal soundness and completeness result for the (hence, bounded) unrestricted model checking problem. More precisely, given a specification $S = (P, D, C)$, $(s_0, S) \models \Phi$ holds for every database $s_0$ relative to all runs of maximal length shorter than a given finite length $l$ if and only if the fully expanded tableau with initial node $(n_0, db) \vdash \phi_C$ is closed. (A tableau is closed if each of its leafs is closed as determined by the $\text{Unsat}$ rule or the $E\cdot X\cdot Exp$ rule.)

The $\text{Unsat}$ tableau rule requires a call to a (sound) first-order theorem prover. Depending on the underlying syntactic fragment of FOL these calls may not always terminate. However, if a classical sequent is provably satisfiable then it is possible to extract from the tableaux branch a run that constitutes a counterexample to the given problem. Moreover, this formula will often represent general conditions on the initial database $s_0$ under which the query $\Phi$ is not satisfied by $(s_0, S)$ and this way provide more valuable feedback than a fully concrete database.

6 Practice and Experiments

In this section, we provide some notes on the implementations of the theory presented in the preceding sections.
Satisfiability Checking on the Nodes. Before we can model-check the truth of formulas over the graph structure of a full specification, we must be able to evaluate first-order formulas with respect to nodes within that graph. When performing checking with a concrete initial state, all subsequent states will be concrete as well, and evaluating quantified formulas is straightforward as long as quantification is over finite domains, as is typical. On the other hand, if the initial state is only characterised with a formula, then checking satisfiability of formulas with respect to that node and all its successors becomes a full-blown theorem-proving problem.

We solve this problem by translating to the standard TPTP format \[13\], which has recently been extended to include arithmetic \[12\], and then using off-the-shelf first-order provers. Our current backend is SPASS+T \[10\], which has good support for arithmetic in addition to sorted first-order logic.

Model Checking. For concrete model checking, we assume that there are no two definitions for same predicate symbol, that definitions are not recursive, and that all quantifications inside the bodies $\phi$ range over concrete data items. With these assumptions, definitions can be expanded as necessary, and we can efficiently decide if formulas (edges’ guards and the classical sub-formulas of the model checking problem) are satisfied with respect to concrete database values. In theory, SPASS+T should do the same, but we have found that our own custom guard evaluator performs better, and is also guaranteed to terminate. When performing concrete model checking, we can also execute scripts directly as Groovy programs rather than needing to manipulate them as first order terms.

We have fully implemented the preceding section’s generic tableau system for concrete model checking, giving us an efficient procedure that is guaranteed to terminate on problems given a depth-bound. In our practical experiments on the example in Section 2 we could (dis)prove queries like the ones mentioned there in very short time.

Our implementation is also capable of generating proof obligations in the TPTP format for unbounded model checking. It also emits the necessary axioms to reflect the semantics of objects and arrays, as explained in Section 3. We have experimented with smaller examples and found that SPASS+T is capable of handling them. At the current stage, however, the implementation is not mature enough yet, and so our experiments are too premature to report on. We also plan to consider alternatives to SPASS+T by implementing the calculus in \[2\] and by linking in SMT-solvers.

7 Conclusions and Future Work

We described a novel approach to modelling and reasoning about data-centric business processes. Our modelling language treats data, process fragments, constraints and logical definitions of business rules on a par. Our research plan focuses on providing strong analytical capabilities on the corresponding models by taking all these components into account. The main ambition is to go beyond model checking from concrete initial states. To this end we have devised a novel tableau calculus that reduces what we called unrestricted model checking problems to first-order logic over arithmetic.

Our main contributions so far are conceptual in nature. Our main theoretical result is the soundness and completeness of the tableau calculus, as explained at the end of Section 4. Our implementation is already fully functional for concrete model checking.
Much remains to be done, at various levels. The tableau implementation needs to be completed and improved for efficiency, and more experiments need to be carried out.

The main motivation for using JSON and Groovy is their widespread acceptance in practice and available tool support, which we exploit in our implementation. For the same reason we want to extend our modelling language by front-ends for established business process modeling techniques, in particular BPMN. This raises (also) some non-trivial interesting theoretical issues. For example, how to map BPMN’s parallel-And construct into our framework. We expect that by using process fragments and constraints on them an isomorphic mapping is possible.

References