On the Handoff-Call Blocking Probability Calculation in W-CDMA Cellular Networks

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Abstract

In this paper, we investigate the call blocking probability calculations in the uplink of W-CDMA mobile cellular networks. In these networks, we distinguish two types of blocking, the new-call blocking, which concerns the initial call establishment request, and the handoff-call blocking, due to the users’ mobility from one cell to the other. To analyze the system, we describe it by a Discrete Time Markov Chain, and based on it, we derive a recursive formula for the calculation of the state probabilities. Consequently, the new-call blocking probabilities and the handoff-call blocking probabilities are determined. To evaluate the proposed formulas, the analytical results are compared with simulation results. This comparison shows that the accuracy of the proposed formulas is very satisfactory.

1. Introduction

The third generation mobile cellular networks (3G networks) offer users a wide range of advanced services, while they service a great number of users through efficient use of the available spectrum and other network resources [1]. Services include wide-area wireless voice telephony and broadband wireless data, all in the mobile environment. Typical service rates are of 5-10 Mbps. The most common 3G networks use the Wideband Code Division Multiple Access (W-CDMA) technology for the underlying air interface [2].

In mobile cellular networks, the geographical area is divided to cells. Each cell is controlled by a base station (BS), which is named NodeB in W-CDMA networks. Different BSs communicate to each other through a core network (usually fixed and wired). If a mobile user (MU) which is located in a specific cell communicates with another MU (of the same or different cell), this communication is performed through the corresponding BSs. The communication link from the MUs to the BS is referred to as uplink, whereas the communication link from the BS to the MUs is referred to as downlink. These communication links are divided into channels; this division allows more than one MUs to communicate simultaneously with the BS. For this communication, a MU seizes one or more channels depending on the service. If no channels are available in the cell, the call is blocked and lost. We distinguish two types of call blocking: new-call blocking and handoff-call blocking. The first type refers to the failure of the initial call-connection establishment, whereas the second type refers to the blocking of in-service calls when they move from one cell to another. The procedure of moving from one cell to another, while a call is in progress, is called handoff. A call admission control (CAC) policy must be deployed to guarantee that the handoff-call blocking probability will be significantly lower than that of the new-call blocking.

Several teletraffic models have been proposed for the new-call blocking probabilities calculation in CDMA networks [3]-[6]. However, in these papers the handoff-call blocking probabilities are not particularly studied. Most of the teletraffic models are based on the well-known Kaufman-Roberts recursion (K-R recursion) for multiple service-classes [7], [8]. In [3], the new-call blocking calculation in the uplink of a W-CDMA cell is based on an extension of the K-R recursion, where Poisson traffic (i.e. infinite number of traffic sources) is assumed. This work was extended in [4], while assuming finite number of traffic sources for each service-class. In [5], another teletraffic model is presented for CDMA cellular networks supporting elastic traffic (in service calls can alter the assigned bandwidth). The new-call blocking calculation is not efficient because the provided formulas are not recurrent. A more general call arrival process is assumed in [6]; it is based on the Delbrouck’s model...
[9]. However, the derived model cannot be directly compared with the above-mentioned teletraffic models, because a different CAC is applied. Similar teletraffic models have been developed for other types of wireless networks as WLAN (standard IEEE 802.11) and WMAN (standard IEEE 802.16) [10], [11].

In this paper, we extend the model of [3] by including in our study the calculation of handoff-call blocking probabilities of Poisson traffic. We also aim at proposing a CAC policy that results in a much lower handoff-call blocking probabilities compared to new-call blocking probabilities. Firstly, we present our analysis for a single service. Next, we generalize to multiple services.

This paper is organized as follows. In section II the W-CDMA system under consideration is described. In section III we present a method for the calculation of the so called local blocking probabilities – i.e. the probabilities of call blocking in a particular system state (the existence of local blocking differentiates the study of CDMA networks from wire networks). In section IV we calculate analytically the system state probabilities and call blocking probabilities. In section V we generalize to multiple services. In section VI we present some numerical examples in order to validate the accuracy of our analytical results. Finally, we conclude in section VII.

2. System Description

Assume a W-CDMA cellular system that supports a single service (more precisely, single service-class). This system consists of a reference cell surrounded by neighbouring cells. We focus on the uplink direction only – i.e. we consider calls from the MUs to the NodeB.

The main parameters of a call are:

- \( R \): Transmission bit rate.
- \( E_s/N_0 \): Signal energy per bit divided by noise spectral density, required to meet a predefined Block Error Rate.
- \( v \): Activity factor at physical layer.

The new-call arrival rate is denoted by \( \lambda_N \), whereas the handoff-call arrival rate is denoted by \( \lambda_H \). We assume that calls of both types arrive according to the Poisson process. Note that this assumption is equivalent to the assumption of infinite number of traffic sources that generate calls. The new-call holding time is exponentially distributed with mean \( \mu_N^{-1} \), whereas the handoff-call holding time is exponentially distributed with mean \( \mu_H^{-1} \). It is reasonable to assume that \( \mu_N^{-1} > \mu_H^{-1} \), since the calls that are handed off to the NodeB of the reference cell, have already spent some time in-service in another NodeB. Finally, the offered traffic-load of new and handoff calls is defined as: \( a_N = \lambda_N \mu_N^{-1} \) and \( a_H = \lambda_H \mu_H^{-1} \), respectively.

2.1 Interference and Call Admission Control

The uplink capacity of a W-CDMA cell is interference limited [2]. Therefore, the inclusion of all types of interferences in the model is necessary. Three types of interferences are of main importance: a) the intra-cell interference, \( I_{\text{intra}} \), caused by MUs of the reference cell, b) the inter-cell interference, \( I_{\text{inter}} \), caused by MUs of the neighboring cells, c) interference of the thermal noise, \( P_N \), which corresponds to the interference of an empty system. A typical value of the thermal noise power density is -174dBm/Hz [2]. Throughout the paper we assume perfect power control – i.e. the received at the Node B power from each call is the same [2].

The CAC in the W-CDMA system under consideration is performed by measuring the noise rise, \( NR \), which is defined as the ratio of the total received power at the NodeB, \( I_{\text{total}} \), to the thermal noise power, \( P_N \):

\[
NR = \frac{I_{\text{total}}}{P_N} = \frac{I_{\text{intra}} + I_{\text{inter}} + P_N}{P_N} \tag{1}
\]

When a call arrives to the cell, the CAC estimates the noise rise and if it exceeds a predefined threshold value, the call is blocked and lost. Since in our system we have two types of calls, we define two threshold values: \( NR_N \) and \( NR_H \), for new calls and handoff calls, respectively. In order to privilege the handoff calls, the two threshold values must be chosen in such a way that:

\[
NR_H > NR_N \tag{2}
\]

2.2 User Activity

A MU, during his call’s duration, alternates between transmitting and silent periods. This behavior is characterized by the activity factor, \( v \), which represents the fraction of the call’s holding time during which the user is occupying system resources. MUs that at a time instant occupy system resources are referred to as active users. The rest of the users (passive users) are in silent period and do not occupy any system resources. The user activity can be modelled by a Bernoulli random variable with probability of success equal to the activity factor, \( v \) (0 \(< v \leq 1 \)). Typical values of \( v \) are 0.67 for voice services and 1.0 for data services.
2.3 Load Factor and Cell Load

The cell load, \( n \), is defined as the ratio of the received (at the NodeB) power from all active users to the total received power 0:

\[
\frac{I_{\text{intra}} + I_{\text{inter}}}{I_{\text{intra}} + I_{\text{inter}} + P_N}
\]

Hence from (1) and (3) we can derive the relation between the noise rise and the cell load:

\[
n = \frac{NR - 1}{NR}
\]

Due to (4), from the two noise rise thresholds, \( NR_N \) and \( NR_H \), we can derive two cell load thresholds, \( n_N \) and \( n_H \), which can be used for CAC. Typical the thresholds \( n_N \) and \( n_H \) must be lower than or equal to a maximum value, \( n_{\text{max}} = 0.8 \), which can be considered as the shared system resource.

The load factor, \( L \), of (5) can be considered as the resource requirement of a call [3]:

\[
E[R] = \frac{E_b}{N_0 W} + \frac{0.8}{N_0 W} = \frac{E_b}{N_0 W}
\]

where \( W \) is the chip rate of the W-CDMA carrier (3.84 Mcps).

The cell load can be written as the sum of the intra-cell load, \( n_{\text{intra}} \) (cell load that derives from the users of the reference cell) and the inter-cell load, \( n_{\text{inter}} \) (cell load that derives from the users of the neighboring cells). They are defined in (5) and (6), respectively:

\[
n_{\text{intra}} = mL
\]

\[
n_{\text{inter}} = (1 - n_{\text{max}}) \frac{I_{\text{inter}}}{P_N}
\]

According to the adopted CAC policy, the following condition is used at the NodeB in order to decide whether to accept a new call or not:

\[
n + L \leq n_N
\]

Similarly, the condition of acceptance of a handoff call is shown in (9):

\[
n + L \leq n_{\text{H}}
\]

3. Local Blocking Probabilities

The probability that a call is blocked when arriving at an instant with intra-cell load, \( n_{\text{intra}} \), is called local blocking probability (LBP) [3]. Due to conditions (8) and (9) we have different LBPs for new and handoff calls. These probabilities are shown in (10) and (11) for new calls and for handoff calls, respectively.

\[
\beta_N(n_{\text{intra}}) = P_n(n_{\text{intra}} + n_{\text{inter}} + L > n_N)
\]

\[
\beta_H(n_{\text{intra}}) = P_n(n_{\text{intra}} + n_{\text{inter}} + L > n_{\text{H}})
\]

In order to calculate the LBPs, \( \beta_N \) and \( \beta_H \), we can use (6) and (7). The only unknown parameter is the inter-cell interference, \( I_{\text{inter}} \). Similarly to [3] and [4], \( I_{\text{inter}} \) can be modelled as a lognormal random variable (with parameters \( \mu_I \) and \( \sigma_I \)), that is independent of the intra-cell interference, \( I_{\text{intra}} \). Hence, the mean, \( E[I_{\text{inter}}] \) and the variance, \( \text{Var}[I_{\text{inter}}] \) of \( I_{\text{inter}} \) are given by (12) and (13):

\[
E[I_{\text{inter}}] = e^{\mu_I + \frac{\sigma_I^2}{2}}
\]

\[
\text{Var}[I_{\text{inter}}] = (e^{\sigma_I^2} - 1)e^{2\mu_I + \sigma_I^2}
\]

Consequently, because of (6), the inter-cell load, \( n_{\text{inter}} \), will also be a lognormal random variable. Its mean value, \( E[n_{\text{inter}}] \), and the variance, \( \text{Var}[n_{\text{inter}}] \), are calculated by:

\[
E[n_{\text{inter}}] = e^{\mu_n + \frac{\sigma_n^2}{2}} = \frac{1 - n_{\text{max}}}{P_N} E[I_{\text{inter}}]
\]

\[
\text{Var}[n_{\text{inter}}] = (e^{\sigma_n^2} - 1)e^{2\mu_n + \sigma_n^2} = \left(\frac{1 - n_{\text{max}}}{P_N}\right)^2 \text{Var}[I_{\text{inter}}]
\]

The parameters \( \mu_n \) and \( \sigma_n \) are of \( n_{\text{inter}} \) can be found by solving (14) and (15):

\[
\mu_n = \ln(E[I_{\text{inter}}]) - \ln\left(\frac{1}{2}\right) - \ln\left(\frac{1 - n_{\text{max}}}{P_N}\right)
\]

\[
\sigma_n = \sqrt{\ln(1 + CV[I_{\text{inter}}]^2)}
\]

The coefficient of variation, \( CV[I_{\text{inter}}] \), is defined as:
From (10) and (11) it follows that the probabilities of non-blocking of new and handoff calls, are given by (19) and (20), respectively.

\[ 1 - \beta_N(n_{intra}) = P(n_{inter} \leq n_N - n_{intra} - L) \]  

(19)

\[ 1 - \beta_H(n_{intra}) = P(n_{inter} \leq n_H - n_{intra} - L_k) \]  

(20)

The right hand side of (19) and (20), is the cumulative distribution function of \( n_{inter} \). It is denoted by \( F_{n_{inter}}(x) = \Phi_{n_{inter}}(x) \) and can be calculated from [4]:

\[ F_{n_{inter}}(x) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{\ln x - \mu_{n_{inter}}}{\sigma_{n_{inter}}} \right) \right] \]  

(21)

where \( \text{erf}(\ast) \) is the well-known error function.

Hence, if we substitute \( x = n_N - n_{intra} - L \) into (21), from (19) we obtain:

\[ \beta_N(n_{intra}) = \begin{cases} 1 - F_{n_{inter}}(x_N), & x_N \geq 0 \\ 1, & x_N < 0 \end{cases} \]  

(22)

Similarly, if we substitute \( x = n_H - n_{intra} - L \) into (21), from (20) we obtain:

\[ \beta_H(n_{intra}) = \begin{cases} 1 - F_{n_{inter}}(x_H), & x_H \geq 0 \\ 1, & x_H < 0 \end{cases} \]  

(23)

### 4. Call Blocking Probabilities

In what follows we describe the system as a Discrete Time Markov Chain (DTMC), determine the state probabilities and consequently, the blocking probabilities for calls of both types.

To this end, similarly to [3] and [4], we discretize the cell load, \( n \), the intra-cell load, \( n_{intra} \), and the load factor, \( L \), by the use of a basic cell load unit, \( g \):

\[ j = \frac{n}{g}, \quad c = \frac{n_{intra}}{g} \quad \text{and} \quad b = \text{round} \left( \frac{L}{g} \right) \]  

(24)

The unit \( g \) must be chosen to be small enough to avoid high discretization errors and big enough to avoid a large state space.

The resultant discrete values \( j, c \) and \( b \) of (24) can be considered as the number of occupied resources in the system, the number of occupied resources by the active users and the calls resource requirement, respectively. Based on the above, we can describe the system as a two-dimensional DTMC, where the system state can be defined as a two-dimensional vector \( \bar{s} = (s_N, s_H) \), with \( s_N \) the number of new calls and \( s_H \) the number of handoff calls in the system.

An example of such a DTMC for a small system is shown in Fig. 1. We see that in the transitions from lower to higher states the call arrival rates, \( \lambda_N \) and \( \lambda_H \), are multiplied by the state-dependent passage factors, \( PF_N(\bar{s}) \) and \( PF_H(\bar{s}) \). These factors express the probability of passing from a state \( \bar{s} \) to higher states. Intuitively, the passage factors are smaller in higher states, since the probability of a call being blocked is higher in higher states due to the presence of more calls in the system. Consequently, at some states we expect that the passage factors are very small (practically zero) and that the system never passes to higher states. This implies that there is a maximum number of new calls, \( s_{N,\text{max}} \), and a maximum number of handoff calls, \( s_{H,\text{max}} \), in the system. Note that due to (2) it is expected that: \( s_{H,\text{max}} > s_{N,\text{max}} \). In the example of Fig. 1 we have \( b=1 \), \( s_{N,\text{max}} = 2 \) and \( s_{H,\text{max}} = 3 \).

The passage factors are related to the blocking factors as follows:

\[ BF_N(\bar{s}) = 1 - PF_N(\bar{s}) \]  

(25)

\[ BF_H(\bar{s}) = 1 - PF_H(\bar{s}) \]  

(26)
From the above definitions we see that the blocking factors express the probability that a call is blocked in state $s_r$.

Note that the number of occupied resources, $j$, can be calculated by adding the resources allocated to all in-service calls:

$$j = s_N b + s_H b = (s_N + s_H) b$$  \hspace{1cm} (27)

The highest reachable system state (maximum number of occupied resources), $j_{\text{max}}$, can be calculated by:

$$j_{\text{max}} = (s_{N,\text{max}} + s_{H,\text{max}}) b$$  \hspace{1cm} (28)

Based on the above, the two-dimensional DTMC can be simplified to the one-dimensional DTMC with (macro-)state $j$. Hence, the system of Fig. 1 can be also shown as in Fig. 2. The state-dependent transition rates, $\lambda_N(j)$ and $\lambda_H(j)$, shown in Fig. 2, are given by (29) and (30), respectively.

$$\lambda_N(j) = \lambda_N(1 - BF_N(j))$$  \hspace{1cm} (29)

$$\lambda_H(j) = \lambda_H(1 - BF_H(j))$$  \hspace{1cm} (30)

The average number of new and handoff calls in state $j$ is denoted by $Y_N(j)$ and $Y_H(j)$, respectively. The blocking factors, similarly to [3] and [4], can be calculated by (31) and (32):

$$BF_N(j) = \sum_{c=0}^{\max} \beta_N(c) A(c | j)$$  \hspace{1cm} (31)

$$BF_H(j) = \sum_{c=0}^{\max} \beta_H(c) A(c | j)$$  \hspace{1cm} (32)

The resource occupancy, $A(c|j)$, is defined as the conditional probability that $c$ resources are occupied by active users in state $j$ and, similarly to [3], can be calculated from (33):

$$A(c|j) = vA(c-b | j-b) + (1-v)A(c | j-b)$$

for $j = 1, \ldots, j_{\text{max}}$ and $c \leq j$

where $A(0|0)=1$ and $A(c|j)=0$ for $c>j$.

Let us denote by $q(j)$ the probability that the system is in state $j$. By solving the one-dimensional DTMC, we can calculate the state probabilities, $q(j)$, by the following recursion:

$$q(j) = \frac{1}{j} a_N (1 - BF_N(j-b)) b q(j-b) +$$

$$\frac{1}{j} a_H (1 - BF_H(j-b)) b q(j-b)$$

for $j=1, \ldots, j_{\text{max}}$ and $q(j)=0$ for $j<0$.

Due to space limitations we do not present in this paper the derivation of (34). This derivation is similar to the method used also in [3], [7] for the determination of the state probabilities.

With the aid of (34) the call blocking probabilities can be calculated by adding all the state probabilities multiplied by the corresponding blocking factors for all possible system states. Therefore, the new-call blocking probabilities and the handoff-call blocking probabilities, are given by (35) and (36), respectively:

$$B_N = \sum_{j=0}^{\max} q(j) BF_N(j)$$  \hspace{1cm} (35)

$$B_H = \sum_{j=0}^{\max} q(j) BF_H(j)$$  \hspace{1cm} (36)

5. Generalization to Multiple Services

The generalization to multiple services can be performed based on the previous analysis of the single service. Due to space limitations we do not get into details of this generalization and present only the necessary formulas.

Assume a W-CDMA system supporting $K$ services. The state probabilities, $q(j)$, can be calculated by the following recursion:
calculated by (38) and (39), respectively:

\[ q(j) = \sum_{k=1}^{K} a_{k,N} (1 - BF_{k,N}(j - b_k)) b_k q(j - b_k) + \]
\[ \sum_{k=1}^{K} a_{k,H} (1 - BF_{k,H}(j - b_k)) b_k q(j - b_k) \]

for \( j = 1, \ldots, j_{\text{max}} \) and \( q(j) = 0 \) for \( j < 0 \),

where \( \sum_{j=0}^{C} q(j) = 1 \)

where \( a_{k,N} \) is the offered traffic-load of new calls of service \( k \) \((k = 1, \ldots, K)\), \( a_{k,H} \) is the offered traffic-load of handoff calls of service \( k \), \( BF_{k,N} \) is the blocking factor of new calls of service \( k \), \( BF_{k,H} \) is the blocking factor of handoff calls of service \( k \), and \( b_k \) is the resource requirement of a service \( k \) call (either new or handoff).

If \( K = 1 \), (37) becomes (34). The new-call and the handoff-call blocking probabilities of service \( k \), can be calculated by (38) and (39), respectively:

\[ B_{k,N} = \sum_{j=0}^{j_{\text{max}}} q(j) BF_{k,N}(j) \]  
(38)

\[ B_{k,H} = \sum_{j=0}^{j_{\text{max}}} q(j) BF_{k,H}(j) \]  
(39)

6. Evaluation

In this section we compare the analytical versus simulation results in respect of the call blocking probabilities, for both types of calls, new and handoff. We evaluate two different W-CDMA services with the following parameters:

i) \( R = 144 \) Kbps, \( E_b/N_0 = 3 \) dB and \( \nu = 0.7 \).

ii) \( R = 384 \) Kbps, \( E_b/N_0 = 4 \) dB and \( \nu = 0.6 \).

The inter-cell interference is assumed to be lognormally distributed with mean value \( E[L_{\text{near}}] = 2*10^{-18} \) mW and with coefficient of variation \( CV[L_{\text{near}}]=1 \). The thermal noise power density is taken to be -174dBm/Hz. For discretization we used \( g = 0.001 \).

First, we determine the call blocking probabilities for nine different traffic-load points (points 1 to 9 in the \( x \)-axis of Figs. 3-5) and for the following cell load thresholds: \( n_H = n_{\text{max}} = 0.8 \) and \( n_N = 0.7 \). Each traffic-load point corresponds to some values of the offered traffic-load for the new and handoff calls, as it is shown in Tables I and II. That is, in the case of the 144 Kbps service, according to Table I the point 1 corresponds to \( \alpha_N = 1.0 \) and \( \alpha_H = 0.2 \), the point 2 to \( \alpha_N = 1.5 \) and \( \alpha_H = 0.3 \) and so on. In Fig. 3 we present the analytical and simulation call blocking probabilities for the 144 Kbps service, whereas in Fig. 4 the corresponding results of the 384 Kbps service are presented. In both figures the results show that the accuracy of the analytical calculations is absolutely satisfactory. We also observe that, as it was expected, by increasing the offered traffic-load the call blocking probabilities are also increased.

Second, we consider only the 144 Kbps service. We vary the value of \( n_N \), while keeping constant the value of \( n_H \), and for the same offered traffic-load (Table I) calculate the handoff-call blocking probabilities. Four different values of \( n_N \) were used: 0.75, 0.70, 0.65 and 0.60. The corresponding analytical results are presented in Fig. 4. We observe that by reducing the value of \( n_N \) we can significantly reduce the handoff-call blocking probabilities. This is especially true for the high offered traffic-load (traffic-load points 8 and 9).

7. Conclusion

In this paper, first we described the uplink of a W-CDMA system as a two-dimensional Markov Chain and then, by aggregating the processes, as a one-dimensional Markov Chain. Based on the latter, we proposed a recursive formula for the efficient calculation of the state probabilities, when a single service is assumed. Consequently, we determined analytically the blocking probabilities of new and handoff calls. A generalization to multiple services was also presented. The analytical call blocking probabilities results were evaluated through simulation. This evaluation showed that the accuracy of the proposed formulas is very satisfactory.

8. References


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