An ON-OFF Multi-Rate Loss Model with a Mixture of Service-Classes of Finite and Infinite Number of Sources

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Abstract—We consider an ON-OFF traffic model of a single link which accommodates service-classes of finite population. Calls arrive according to a quasi-random process and, if accepted, enter the system via state ON; then calls may alternate between ON-OFF states. When a call is transferred to state OFF, it releases the bandwidth held in state ON, while when it tries to return to state ON it re-requests its bandwidth. If it is available a new ON-period (burst) begins; otherwise burst blocking occurs and the call remains in state OFF. We prove that the proposed finite source ON-OFF model (f-ON-OFF) has a product form solution and provide an accurate recursive formula for the call blocking probabilities calculation. For the burst blocking probabilities calculation we propose an approximate formula. Finally, we generalize the f-ON-OFF model to include a mixture of service-classes of finite and infinite number of sources. Simulation results validate our analytical methodology.

I. INTRODUCTION

The call-level quality of service (QoS) assessment in the multi-service environment of modern communications networks remains an open issue, due to the bursty behavior of traffic of many applications. The easiest way to describe that behavior is by the ON-OFF traffic model, where Poisson arriving calls (consisting of a random number of bursts), when accepted in the system, alternate between transmission (ON) periods and no transmission (OFF) periods. Despite the difference between the ON-OFF and Constant Bit Rate (CBR) traffic, the basis of call-level modelling in both cases is a CBR traffic model. This is the Erlang Multirate Loss Model (EMLM) [1], where Poisson arriving calls of CBR service-classes compete for the available link bandwidth under the complete sharing (CS) policy. The EMLM has a Product Form Solution (PFS) which leads to an accurate Call Blocking Probabilities (CBP) calculation [2], [3]. In [4], the EMLM is extended to a model in which each service-class has a finite population of traffic sources. We characterize this model “Engset Multirate Loss Model (EnMLM)”, since for a single service-class, it gives the same results with the Engset formula for the time congestion probability [5]. The EnMLM has a PFS, whereas the CBP calculation is based on an accurate recursive formula. As in the EMLM, the CBP formula in the EnMLM requires the determination of link occupancy distribution. Although this is done recursively, it is complex in EnMLM; the reason is that prior to that determination, the system state-space needs enumeration and processing [4]. The models mentioned above cope with the bursty traffic through the notion of equivalent bandwidth [6]. However, a more sophisticated model exists for bursty traffic and especially for the ON-OFF traffic model, in which the notion of both Call and Burst Blocking (CB, BB) is introduced [7]. In [7], Poisson arriving calls enter the system via state ON only, and then alternate between the ON-OFF states, or remain always in state ON. When an in-service call passes from state ON to state OFF, it releases the bandwidth held in state ON. When it needs to return to state ON, it re-requests its bandwidth and, if available, a new burst begins; otherwise, the burst is blocked (BB Probability - BBP) and the call remains in state OFF.

In this paper, we combine the infinite ON-OFF (inf-ON-OFF) model with the EnMLM, to form the finite source ON-OFF model (f-ON-OFF). We prove that the f-ON-OFF model has a PFS, and provide an accurate recursive formula for calculating the CBP. This formula is based on the determination of the link occupancy distribution, which is of higher complexity compared to that of [7]. This is due to the enumeration and processing of the state space, prior to the calculation of the link occupancy distribution. As far as the BBP calculation is concerned, we propose an approximate formula. To complete this study, we present a generalized f-ON-OFF model that describes a mixture of service-classes with finite and infinite population of sources. We propose formulas for the CBP/BBP calculations, and evaluate them by comparing the analytical with the simulation results. Applications of the proposed f-ON-OFF model can be the same with those of the inf-ON-OFF model and the EnMLM. The inf-ON-OFF model is applied in ATM networks, by using the fast buffer reservation scheme [7]. The EnMLM has been applied in a radio ATM LAN [8]; this is a basic application of the f-ON-OFF model, because it exploits the finite and bursty features of the model.

In section II we present the f-ON-OFF model; firstly we describe the model, secondly we derive the formulas for the link occupancy distribution and the CBP calculation, and thirdly we derive the BBP formula. In section III, the generalized f-ON-OFF model is presented. In section IV, we present a numerical example whereby we evaluate the accuracy of the analytical formulas. Section V concludes.
II. THE PROPOSED f-ON-OFF MODEL

A. Description of the model

We consider a link with a pair of capacities $C$ and $C^*$, accommodating $K$ service-classes of ON-OFF-type calls under the CS policy. The first capacity, named real (real link), corresponds to state ON while the second one, named fictitious (fictitious link), corresponds to state OFF. Each call of a service-class $k$ ($k=1,\ldots,K$) comes from a finite source population $N_k$, and requires $b_k$ bandwidth units (b.u.). If this bandwidth is available then the call enters the system via state ON, otherwise the call is blocked and lost. The mean arrival rate of the idle sources is $\lambda_k=(N_k-n'^k_k-n''_k)\nu_k$, where $n'^k_k$ is the number of in-service sources of service-class $k$ in the $i$th state ($i=1 \Rightarrow$ state ON, $i=2 \Rightarrow$ state OFF), and $\nu_k$ is the arrival rate per idle source (constant). This call arrival process is characterized quasi-random [5]. As far as the call holding time is concerned, it is exponentially distributed. At the end of an ON-period a call of service-class $k$ releases the $b_k$ b.u. which become available to calls of all service-classes and may begin an OFF-period with probability $\alpha_k$, or depart from the system with probability $1-\alpha_k$. While the call is in state OFF, it seizes fictitious bandwidth ($b_k$ b.u.) of the fictitious link. At the end of an OFF-period the call returns to state ON (with probability 1) by re-requesting $b_k$ b.u. When $C=C^*$, there are always available $b_k$ b.u. in state ON, i.e. no BB occurs. When $C<C^*$, then if there is available bandwidth in the real link, i.e. if $j_1+b_2 \leq C$, (where $j_1$ is the occupied real link bandwidth), the call returns to state ON and a new burst begins, otherwise the BB occurs and the call remains in state OFF for another OFF-period (Fig. 1). A new service-class $k$ call is accepted in the system with $b_k$ b.u. if it meets two constraints: 1) $j_1+b_2 \leq C$ and 2) $j_1+j_2+b_2 \leq C$ where $j_2$ is the occupied fictitious link bandwidth. The first constraint ensures that the occupied bandwidth of all existing ON calls together with the new call does not exceed the capacity of the real link. The second constraint prevents the system from accepting new calls when “most” of the system capacity are in state OFF. This is done in order to ensure that the BBP will remain in an acceptable level.

B. CBP determination

To describe the analytical model for the CBP determination the following notation is necessary:

$$n'^k_k=(n'^k_1,\ldots,n'^k_{2},n'^k_K)$$

$$\bar{n}=(\bar{n}_1,\bar{n}_2)$$

$$n'^k_k=n'^k_1+\ldots,n'^k_2+1,\ldots,n'^k_K$$

$$n''_k=n''_1,\ldots,n''_2,\ldots,n''_K$$

$$\bar{n}_1=\min(n'^k_1,n'^k_2),\bar{n}_2=\min(n'^k_3,n'^k_4),\ldots,\bar{n}_K=\min(n'^k_K,n'^k_K)$$

$$\tilde{n}'_1=\min(n''_1,n''_2),\tilde{n}'_2=\min(n''_3,n''_4),\ldots,\tilde{n}'_K=\min(n''_K,n''_K)$$

$P_{ik,fin}$ is the utilization of the $i$th type link by service-class $k$.

$$P_{ik,fin} = \frac{e_{ik,fin}}{\mu_k} \quad \text{for } i = 1$$

$$P_{ik,fin} = \frac{\nu_k}{(1-\sigma_k)\mu_{2k}} \quad \text{for } i = 2$$

(1)

where: $e_{ik,fin}$ is the total arrival rate of service-class $k$ calls to the $i$th state and $\mu_{1k}$ is the mean holding time of service-class $k$ calls to the $i$th state.

![Figure 1. Call and burst blocking in the f-ON-OFF model.](image)

Fig. 2 shows the state transition rates of the f-ON-OFF model. Assuming the existence of Local Balance (LB) between adjacent states, the following LB equations exist (Fig. 2):

$$\mu_{1k} n'^k_1 \sigma_k P(\bar{n}) = \mu_{2k} (n''_1+1) P(\bar{n}^{<}_{k-2})$$

(3a)

$$\mu_{1k} n'^k_1 (1-\sigma_k) P(\bar{n}) = (N_k-n''_1-n''_2+1) \nu_k P(\bar{n}^{>}_{k-1})$$

(3b)

$$\mu_{2k} n''_k P(\bar{\nu}) = \mu_{1k} (n''_1+1)(1-\sigma_k) P(\bar{n}^{<}_{k+})$$

(3c)

(3d)

where: $P(\bar{n}),P(\bar{n}^{<}_{k-2}),P(\bar{n}^{>}_{k-1}),P(\bar{n}^{<}_{k+})$ are the probability distributions of the states: $\bar{n},\bar{n}^{<}_{k-2},\bar{n}^{>}_{k-1},\bar{n}^{<}_{k+}$, respectively.

Both (3b) and (3c) describe the balance between the rates of a new arrival of service-class $k$ call and the corresponding departure from the system, while (3a) and (3d) the ON-OFF alternations of a call. Based on the LB assumption, $P(\bar{n})$, is given by a product form formula which satisfies (3a – 3d):

$$P(\bar{n}) = \frac{1}{G} \prod_{k=1}^{K} \left( \frac{N_k}{n''_k+1} \right) \prod_{l=1}^{n''_k} l \prod_{i=1}^{n''_K} \frac{2}{n''_k^i}$$

(4)

where: $G=G(\Omega)$ is a normalization constant chosen so that the sum of the probabilities of all states is unity, while

$$\bar{j} \in \Omega \Leftrightarrow \left\{(j_1 \leq C \land (\sum_{s=1}^{2} j_s \leq C^*)\right\}$$

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Based on $P(\bar{n})$, we express $P(\bar{n}^i_{k^-})$ by:

$$(N_k - n_k^i - n_k^1 + 1)P_{ik,fin} P(\bar{n}^i_{k^-}) = n_k^i P(\bar{n})$$  \hspace{1cm} (5)$$

where $i = 1, 2, \ldots, rE = 1, 2$ and $tei$. Since the CS policy is a coordinate convex policy [2], (5) can be written as:

$$(N_k - n_k^i - n_k^1 + 1)P_{ik,fin} \gamma_{ik}(\bar{n}) P(\bar{n}^i_{k^-}) = n_k^i P(\bar{n})$$  \hspace{1cm} (6)$$

where $\gamma_{ik}(\bar{n}) = 1$ if $n_k^i \geq 1$, while $\gamma_{ik}(\bar{n}) = 0$ if $n_k^i = 0$. Consider the set: $\Omega_j = \{\bar{n} \in \Omega : \bar{n}B = \bar{j}, n_k^i \geq 0, k=1, \ldots, K, i=1,2\}$

where: $B$ is a $(2K \times 2)$ matrix with entries $b_{i,k}$. The $(i, k)^{th}$ row of $B$ is denoted by $B_{ik} = (b_{i,k1}, b_{i,k2})$;

Summing (6) over $\Omega_j$ we have:

$$N_k P_{ik,fin} \sum_{\bar{n} \in \Omega_j} \gamma_{ik}(\bar{n}) P(\bar{n}^i_{k^-}) = n_k^i P(\bar{n})$$  \hspace{1cm} (7)$$

The left hand side (LHS) of (7) can be written as:

$$\text{LHS} = N_k P_{ik,fin} \sum_{\bar{n} \in \Omega_j \cap \{ n_k^i \geq 1 \} } P(\bar{n}^i_{k^-})$$

$$+ P_{ik,fin} \sum_{\bar{n} \in \Omega_j \cap \{ n_k^i < 1 \} } (n_k^i + n_k^1 - 1) \gamma_{ik}(\bar{n}) P(\bar{n}^i_{k^-})$$

Since $\Omega_j \cap \{ n_k^i \geq 1 \} = \{ \bar{n} : n_k^i - B = \bar{j} - B_{i,k} \}$ we use the

following change of variables: $n_{m}^{*i} = n_{m}^{i} - 1$ for $m = k$, otherwise $n_{m}^{*i} = n_{m}^{i}$ for $m \neq k$. Thus:

$$\text{LHS} = N_k P_{ik,fin} \sum_{\bar{n} \notin \Omega_k} P(\bar{n}^i_{k^-}) - P_{ik,fin} \sum_{\bar{n} \notin \Omega_k} (n_k^i + n_k^1) P(\bar{n}^i_{k^-})$$  \hspace{1cm} (8)$$

where $\bar{n}^i_{k^-} = (n_k^i, n_k^1)$, $\bar{n}^{*i} = (n^{i}, n^{1})$, and $n^{i} = (n^{1}, ..., n_{k}^{i}, ..., n_{K}^{1})$ and $n^{1} = (n_{1}^{i}, n_{2}^{1}, ..., n_{K}^{1})$

The first term of (8) is equal to:

$$N_k P_{ik,fin} \sum_{\bar{n} \notin \Omega_k} P(\bar{n}^i_{k^-}) = N_k P_{ik,fin} G(\bar{j} - B_{i,k})$$

where $G(\bar{j} - B_{i,k})$ denotes the distribution of $\bar{j} = (j_1, j_2)$. The second term of (8) is equal to:

$$P_{ik,fin} \sum_{\bar{n} \notin \Omega_k} (n_k^i + n_k^1) P(\bar{n}^i_{k^-}) =$$

$$= P_{ik,fin} \sum_{\bar{n} \notin \Omega_k} (n_k^i + n_k^1) \frac{P(\bar{n}^i_{k^-})}{G(\bar{j} - B_{i,k})} G(\bar{j} - B_{i,k}) =$$

$$= P_{ik,fin} \sum_{\bar{n} \notin \Omega_k} (n_k^i + n_k^1) P(\bar{n}^i_{k^-}) G(\bar{j} - B_{i,k}) +$$

$$= P_{ik,fin} E((n_k^i + n_k^1) G(\bar{j} - B_{i,k})$$

where $E(n_k^i + n_k^1 G(\bar{j} - B_{i,k})$ is the estimator of $n_k^i + n_k^1$ given $\bar{j}$. By substituting the “new” terms in (8), we have:

$$\text{LHS} = (N_k - E((n_k^i + n_k^1) G(\bar{j} - B_{i,k})) P_{ik,fin} G(\bar{j} - B_{i,k})$$  \hspace{1cm} (9)$$

The right hand side (RHS) of (7) can be written as:

$$\text{RHS} = \sum_{\bar{n} \in \Omega_j} n_k^i P(\bar{n} | \bar{j}) G(\bar{j}) = \sum_{\bar{n} \in \Omega_j} n_k^i \frac{P(\bar{n})}{G(\bar{j})} G(\bar{j})$$  \hspace{1cm} (10)$$

for $\bar{j} \in \Omega$ and $k = 1, \ldots, K$. Combining (9),(10) it is proved that:

$$(N_k - E((n_k^i + n_k^1) G(\bar{j} - B_{i,k})) P_{ik,fin} G(\bar{j} - B_{i,k}) = E(n_k^i | \bar{j}) G(\bar{j})$$  \hspace{1cm} (11)$$

Multiplying both sides of (11) by $b_{i,k}$ and summing over $k = 1, \ldots, K$ and $i = 1, 2$ we have:

$$\sum_{i=1}^{2} \sum_{k=1}^{K} (N_k - E((n_k^i + n_k^1) G(\bar{j} - B_{i,k})) b_{i,k} G(\bar{j} - B_{i,k}) =$$

$$= \sum_{i=1}^{2} E(n_k^i b_{i,k} | \bar{j}) G(\bar{j})$$

$\sum_{i=1}^{2} E(n_k^i b_{i,k} | \bar{j}) G(\bar{j})$
TABLE I
STATE SPACE, RESPECTIVE REAL AND FICTITIOUS OCCUPIED LINK BANDWIDTH, EQUIVALENT REAL AND FICTITIOUS LINK OCCUPANCIES AND CB STATES ($C=C^*$)

<table>
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<th>$n_1^*$</th>
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with $\bar{j} \in \Omega \iff \{j_k \leq C \cap (\sum_{j=1}^{2} j_k \leq C^*)\}$.

To determine $E((n_k^* + n_k^{'i})j - B_{1,k})$ we use a lemma of [4]. According to it, two stochastic systems with: (a) the same traffic parameters ($K, N_k, p_{d,k,u}$), and (b) the same set of states, are equivalent since they result in the same CBP. We find a new stochastic system, so as to determine $E((n_k^* + n_k^{'i})j - B_{1,k})$ by choosing the bandwidth requirements of the service-classes and $C, C'$ in the new stochastic system according to two criteria: 1) conditions (a) and (b) are valid, 2) each state has a unique occupancy $\bar{j} = (j_1, j_2)$ then a state $\bar{j}$ is reached only via state $\bar{j} - B_{1,k};$ thus the estimator

$$E((n_k^* + n_k^{'i})j - B_{1,k}) = n_k^* - 1 + n_k^{'i}$$

(13)

According to (13), we can write (12) as follows:

$$\sum_{i=1}^{2} \sum_{k=1}^{K} (N_k - n_k^{'i} - n_k^* + 1)b_{l,k,i}p_{d,k,u}G(j - B_{1,k}) = j_sG(\bar{j})$$

(14)

Equation (14) is the recursive formula used for the determination of $G(\bar{j})$. If $\bar{j} \notin \Omega$ then $G(\bar{j}) = 0$, while $G(\bar{j})$’s can be calculated in terms of an arbitrary $G(\bar{0})$ and the normalization condition $\sum_{j \in \Omega} G(\bar{j}) = 1$.

Although (14) is simple, it cannot be used straightforward. In order to calculate the $G(\bar{j})$’s, the system state-space needs enumeration and processing. The following example reveals the problems that can arise when one tries to use (14) straightforward. Consider two service-classes which require $b_1 = 3$ and $b_2 = 1$ b.u. per call, respectively. The number of sources of each service-class is $N_1 = N_2 = 5$. The real and fictitious capacities of the link are $C = C^* = 3$ b.u. The traffic description parameters of the two service-classes are:

1st service-class: $v_1 = 0.002$, $\mu_{11} = 0.5$, $\mu_{21} = 0.6$, $\sigma_1 = 0.75$, $p_{11,fin} = 0.004$, $p_{21,fin} = 0.0036$

2nd service-class: $v_2 = 0.011$, $\mu_{12} = 1.0$, $\mu_{22} = 1.0$, $\sigma_2 = 0.75$, $p_{12,fin} = 0.04$, $p_{22,fin} = 0.03$

In Table I we present the state-space of this example, the respective $(j_1, j_2)$, the real and fictitious link occupancies of the equivalent system ($j_{1(equiv)}, j_{2(equiv)}$) and the CB states for each service-class. Grey cells correspond to CB states for both service-classes, while non grey cells to CB states of the 1st service-class. The state space consists of 12 discrete states ($n_1^{'}, n_1^{*i}, n_2^{'}, n_2^{*i}$).

However, the link occupancies are not discrete, since the same link occupancy may correspond to two or more states; e.g. $(j_1, j_2) = (0,3)$ appears twice: for $(n_1^{'}, n_1^{*i}) = (0,0,3)$ and for $(n_1^{'}, n_1^{*i}) = (0,1,1,0).$ Thus, we cannot use (14) for the calculation of $G(\bar{j})$’s (which vector should be used?) unless we find an equivalent stochastic system whose states have unique link occupancy. Such a system (heuristically found) is the following: $b_1 = 5$, $b_2 = 2$, $C = C^* = 6$. In the columns entitled $j_{1(equiv)}$ and $j_{2(equiv)}$ of Table I we present the link occupancies of the equivalent system. In both systems, initial and equivalent, the CB states of each service-class are the same. Having found an equivalent system, we calculate $G(\bar{j})$’s via (14), and then determine the CBP, $P_{b_1} = 30.58\%$ and $P_{b_2} = 2.88\%$, by using the formula:

$$p_{b_h} = \sum_{j_s \in \Gamma_h} G(\bar{j})$$

(15)

where $G = \sum_{j_s \in \Omega} G(\bar{j})$ is a normalization constant.

C. BBP determination

Consider the previous example but increase $C^*$ to $C^* = 4$ b.u so as BBP to appear. Table II presents the system state-space, the respective $j_1, j_2, j_{1(equiv)}$ and $j_{2(equiv)}$ the CB and the BB states for each service class. The state space consists of 19 states. An equivalent stochastic system is the following: $b_1 = 5$, $b_2 = 2$, $C = 6$, $C^* = 8$. Both systems, initial and equivalent, have the same CB and BB states (Table II). Grey cells correspond to
CB, BB states for both service-classes, while non-grey cells to CB, BB states of the 1st service-class. In order to determine the BBP of calls of a service-class k, \(p^*_b_k\), we are interested in the number of service-class k-calls in state OFF, \(n^*_k\), when the system is in a BB state; e.g., according to Table II, a BB state of the 2nd service-class, is \((n^1, n^2, n_1^2, n_2^2) = (1,0,0,1)\) and the corresponding value of \(\tilde{j} = (j_{1\text{eqv}}, j_{2\text{eqv}}) = (5,2)\). Multiplying \(n^*_k\) by the corresponding \(G(\tilde{j})\) and the service rate in state OFF \(\mu_{2k}\), we obtain the rate whereby service-class k OFF-calls would depart from the BB state if it were possible. By summing up these rates over the BB state-space \(\Omega^*\) of each service-class k, we obtain the following summation:

\[
\sum_{j \in \Omega^*} n^2_j G(\tilde{j}) \mu_{2k}
\]

where \(G(\tilde{j})\)'s are calculated by (14).

Thus, (16) can be seen as the normalized rate of the service-class k OFF-calls by which OFF-calls would depart from the BB states if it were possible. For the record, the BBP of this example are: \(P^*_{b_1} = 14.81\%\), \(P^*_{b_2} = 1.50\%\), while the corresponding simulation results (95% confidence interval) are: \(P^*_{b_1} = 14.32 \pm 0.31\%, P^*_{b_2} = 1.48 \pm 0.05\%\).

### III. Generalization of the F-ON-OFF Model

Consider a transmission link with a pair of capacities \(C\) and \(C^*\), accommodating \(K_{\text{fin}}\) service-classes of quasi-random input and \(K_{\text{inf}}\) service-classes of random–Poisson input. Then the \(G(\tilde{j})\)'s calculation can be done by the following formula:

\[
\frac{2}{\sum_{b_{1k}} \sum_{k=1}^{K_{\text{fin}}} (N_k - n_{1k} + 1)b_{1k, n_{1k}} p_{1k, n_{1k}} G(\tilde{j} - B_{1k}) + 2 \sum_{b_{1k}} b_{1k} p_{1k} G(\tilde{j} - B_{1k})} = j_1 G(\tilde{j})
\]

where \(\tilde{j} \in \Omega \Leftrightarrow \{j_1 \leq C \land (\sum_{s=1}^{2} j_s \leq C^*)\}\)

The second part of (17):

\[
\sum_{b_{1k}} b_{1k} p_{1k} G(\tilde{j} - B_{1k}) = j_1 G(\tilde{j})
\]

corresponds to the Poisson input and for its proof see [7]. Similarly to \(p_{1k, n_{1k}}\) we denote by:

\[
p_{ik} = \frac{\lambda_k}{\mu_{ik}} = \begin{cases} \frac{\lambda_k}{(1 - \sigma_k)\mu_{1k}} & \text{for } i = 1 \\ \frac{\lambda_k}{(1 - \sigma_k)\mu_{2k}} & \text{for } i = 2 \end{cases}
\]

where: \(e_k\) is the total arrival rate of service-class \(k\) calls to the \(i^{th}\) state.

Such a mixture of service-classes does not destroy the accuracy of the model. The BBP calculation can be done via (15), while the BBP calculation via (16) for the service-classes of infinite population and via the following formula for the service-classes of finite population:

\[
P^*_{b_k} = \frac{\sum_{j \in \Omega^*} y_{2k}(\tilde{j}) G(\tilde{j}) \mu_{2k}}{\sum_{j \in \Omega} y_{2k}(\tilde{j}) G(\tilde{j}) \mu_{2k}}
\]

where: \(y_{ik}(\tilde{j})\) is the average number of service-class \(k\) calls in state \(i\) of the system state \(\tilde{j}\), calculated by:

\[
y_{ik}(\tilde{j}) = \frac{p_{ik} G(\tilde{j} - B_{ik})}{G(\tilde{j})}
\]

while \(G(\tilde{j})\)'s are calculated by (19). The logic behind (19) is similar to that of (16); For the proof of (20) see [9].

### IV. Numerical Example - Evaluation

We present analytical CBP and BBP results of the generalized f-ON-OFF model. For the BBP we also provide simulation results (mean values of 10 runs with 95% confidence interval), in order to evaluate the accuracy of the proposed BBP formula. Since the accuracy of the CBP formula has been proved (the f-ON-OFF model has a PFS), simulation results are not presented.

Consider two service-classes which require \(b_1 = 12\) and \(b_2 = 2\) b.u. per call, respectively, and a link with \(C = C^* = 46\) b.u. Calls of the 1st service-class arrive according to a quasi-random process \((N_1 = 5)\), while calls of the 2nd service-class arrive according to a Poisson process. The traffic description parameters of the two service-classes are:

1st service-class: \(v_1 = 0.02, \mu^1_{11} = 0.5, \mu^1_{21} = 0.6, \sigma_1 = 0.9, p_{11, \text{fin}} = 0.1, p_{21, \text{fin}} = 0.108\).

2nd service-class: \(v_2 = 0.1, \mu^2_{12} = 1.0, \mu^2_{22} = 1.0, \sigma_2 = 0.9, p_{12} = 1.0, p_{22} = 0.9\).

The equivalent system used is the following: \(b_1 = 25, b_2 = 4\) and \(C = C^* = 95\). When \(C = C^*\) no BB occurs. We increase \(C^*\) from \(C^* = 46\) to \(C^* = 51\), in order for BB to occur; then the equivalent system used is: \(b_1 = 29, b_2 = 5, C = 115\) and \(C^* = 125\). We further increase \(C^*\) to 56 b.u.; then, the equivalent system is: \(b_1 = 29, b_2 = 5, C = 115\) and \(C^* = 140\). Fig. 3 and 4 show the analytical CBP results for both service-classes, for:

a) \(C = C^* = 46\), b) \(C = 46, C^* = 51\) and c) \(C = 46, C^* = 56\), whereas, Fig. 5 shows the analytical and simulation BBP results for
C=46, C*=51 and for C=46, C*=56. At each point in the horizontal axis entitled “arrival rate” v1 is constant, while λ2 increases in steps of 0.1, i.e. point 1 is (v1, λ2) = (0.02, 0.1), point 2: (v1, λ2) = (0.02, 0.2), ..., point 7: (v1, λ2) = (0.02, 0.7).

As Fig. 3 and 4 show, there is a significant CBP decrease of both service-classes, when we increase C* from 46 to 51, or to 56. This CBP decrease was anticipated, because, when C* increases, more calls can pass to state OFF, releasing bandwidth (in state ON) for new calls. However, the increase of C* results in BB increase (Fig. 5).

V. CONCLUSION

We propose the f-ON-OFF traffic model for a single link, which accommodates service-classes of quasi-random input. The proposed model has a PFS, which help us derive an accurate and recursive formula for the CBP calculation. The BBP calculation is based on an approximate formula. Furthermore we generalize the f-ON-OFF model to include a mixture of service-classes with finite and infinite population of sources. Numerical results validate our methodology.

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REFERENCES