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WORKING PAPER

Testing for a rational bubble under long memory

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Abstract

We analyze the time series properties of the S&P500 dividend-price ratio in the light of long memory, structural breaks and rational bubbles. We find an increase in the long memory parameter in the early 1990s by applying a recently proposed test by Sibbertsen and Kruse (2009). An application of the unit root test against long memory by Demetrescu et al. (2008) suggests that the pre-break data can be characterized by long memory, while the post-break sample contains a unit root. These results reconcile two empirical findings which were seen as contradictory so far: on the one hand they confirm the existence of fractional integration in the S&P500 log dividend-price ratio and on the other hand they are consistent with the existence of a rational bubble. The result of a changing memory parameter in the dividend-price ratio has an important implication for the literature on return predictability: the shift from a stationary dividend-price ratio to a unit root process in 1991 is likely to have caused the well-documented failure of conventional return prediction models since the 1990s.

JEL-number: C12, C22, G12.

Keywords: Rational bubbles, dividend-price ratio, fractional integration, changing persistence.

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1 Introduction

The existence of bubbles in stock market prices is both, highly important and controversial. A natural starting point for any paper on bubbles is the question what exactly a bubble is. This question seems to be intuitively easy to answer, it turns out to be difficult to give an exact definition. Garber (2000, p. 7) calls bubble a "fuzzy word filled with import but lacking any solid operational definition" What is common in most definitions is that a bubble is a long-lasting deviation from a fundamentally justified level. Thus share prices rise to "unrealistically high levels" (Kindleberger 1989) and are "unexplained based on what we call fundamentals" (Garber 2000). A reasonable definition is given by Rosser (2000), who argues that a speculative bubble exists "when the price of something does not equal its market fundamentals for some period of time for reasons other than random shocks; [Fundamental] is often argued to be a long-run equilibrium consistent with a general equilibrium."

Testing for a bubble is challenging (for a survey see Gurkaynak 2008). Early tests based on variance bounds (Shiller 1981, LeRoy and Porter 1981) and West's two step test (West 1987) have been followed by cointegration-based tests. These tests are derived from the relation between share prices and dividends (Campbell and Shiller 1987) and basically test for cointegration between prices and dividends, or similarly, test the dividend-price ratio or dividend yield for stationarity. Since cointegration means that two series move together in the sense that deviations from the equilibrium lead to an adjustment, cointegration of prices and dividends or stationarity of the dividend-price ratio is inconsistent with a rational bubble. Rational bubbles require that prices are continuously moving away from the equilibrium. Otherwise they will immediately collapse: No rational investor who knows that the price exceeds the fundamental value is ready to buy a stock, if the share price is expected to adjust to the equilibrium over the investment horizon. On the other hand, the presence of a unit root in the dividend-price ratio is consistent with the presence of a rational bubble.

Empirical evidence, however, is ambiguous. While some studies conclude that the S&P500 stock market index (Diba and Grossmann 1987, Taylor and Peel 1998) does not contain a bubble, others find evidence for its existence (Campbell and Shiller 1987, Froot and Obstfeld 1991, Craine 1993, Lamont 1998). Whereas early research relied on integer degrees of integration, recently a class of long-memory models known in the literature as autoregressive fractionally integrated moving average (ARFIMA) processes has been utilized. They allow for persistence and mean reversion. The long memory inherent in ARFIMA processes may therefore explain the lack of clear empirical evidence on rational bubbles.

Applying the fractional integration technique, Koustas and Serletis (2005) find strong evidence for the existence of long memory in the S&P 500 log dividend-price ratio and their results support the hypothesis of no rational bubble. However, the authors do not take a potential change in the fractional degree of integration into account. A similar approach is used by Cuñado et al. (2005) to detect a bubble in the Nasdaq index during the late 1990s, who find evidence for the existence of a rational bubble in this market. Philipps et al. (2011) propose sequential unit root tests for detecting switches from nonstationarity to explosive behaviour in the Nasdaq. Breitung and Homm (2011) further investigate different tests for the unit root hypothesis against an explosive alternative and find evidence for a bubble in the Nasdaq index at the end of the 1990s. Moreover, they suggest real-time monitoring procedures for the detection of bubbles. Another strand of the literature extending previous research, tests for a change from an $I(0)$ to an $I(1)$ process. Sollis (2006) applies tests for a change in persistence in the $I(0)/I(1)$ framework. Beside the limitation to integer integration that rules out long memory dynamics, the tests used by Sollis (2006) have the undesirable property that they tend to reject the null hypothesis of a constant $I(0)$ (or constant $I(1)$) process with probability one if the true data generating process is indeed constant $I(1)$ (or constant $I(0)$), see Leybourne et al. (2007).

We contribute to the literature by merging the approaches described above and applying a test for changing persistence under fractional integration that has been recently proposed by Sibbertsen and Kruse (2009). As a result we find a significant break in the memory of the S&P 500 log dividend-price ratio in July 1991. By applying the unit root test against long memory proposed by Demetrescu et al. (2008) we find strong evidence for long memory before the break in 1991 and a unit root afterwards. These results confirm on the one hand the previous result of fractional integration in this time series and on the other hand they are in line with other empirical studies that find evidence for a unit root which is consistent with existence of a rational bubble in the S&P500 stock market.

The paper proceeds as follows: Section 2 introduces the notation of rational bubbles in stock prices. Section 3 outlines the econometric methodology and the test for changing persistence. Section 4 describes the empirical results and section 5 summarizes and concludes.

2 Rational bubbles

Following the fundamental equation of asset pricing, see Blanchard and Watson (1982) and Campbell et al. (1997), the present price P_t of a share is the future cash flow, consisting of the next period's price P_{t+1} and dividend payments D_{t+1}, discounted with the rate R_{t+1}, the expected return that a marginal rational trader requires in order to hold the asset under consideration:

$$P_t = E_t\left[(D_{t+1} + P_{t+1})/(1 + R_{t+1})\right] \ . \tag{1}$$

For tractability the expected discount rate is assumed to be constant. While few people would agree that this assumption is realistic, the basic idea also holds in the case of a time-varying discount rate, see the huge literature on stochastic discount factors, particularly Cochrane (2005). Furthermore Craine's (1993) derivation of cointegration-based testing for rational bubbles does not require a constant discount factor. Under the assumption that $R_t = R =$ constant, equation (1) evolves to:

$$P_t = E_t\left[D_{t+1} + P_{t+1}\right]/(1 + R) \ . \tag{2}$$

Since equation (2) also applies to P_{t+1}, we may solve equation (2) k periods forward which yields

$$P_t = \sum_{i=1}^{k} E_t\left[D_{t+i}\right]/(1 + R)^i + E_t\left[P_{t+k}\right]/(1 + R)^k \ . \tag{3}$$

A unique solution is only obtained if $\lim_{k\to\infty} E_t\left[P_{t+k}\right]/(1 + R)^k = 0$. In this case, we obtain the fundamental value of the stock (F_t) which is given by the infinite sum of the expected present value of future dividends

$$F_t = \sum_{i=1}^{\infty} E_t \left[D_{t+i} \right] / (1+R)^i \ . \tag{4}$$

If the transversality condition $\lim_{k \to \infty} E_t \left[P_{t+k} \right] / (1+R)^k = 0$ does not hold, an infinite number of solutions exists. Any of these can be written as

$$P_t = F_t + B_t \ , \tag{5}$$

where $B_t = E_t \left[B_{t+1} \right] / (1+R)$ and B_t is the rational bubble. The bubble component captures the part of the share price that is due to expected future price changes. Thus, the price contains a rational bubble, if investors are ready to pay more for the share, than they know is justified by the discounted stream of future dividends. Since they expect to be able to sell the share even at a higher price, the current price, although exceeding the fundamental value, is an equilibrium price. The model therefore allows the development of a rational bubble, in the sense that a bubble is fully consistent with rational expectations. In the rational bubble model, investors are fully cognizant of the fundamental value, but nevertheless they may be willing to pay more than this amount.[1] This is the case if expectations of future price appreciation are large enough to satisfy the rational investor's required rate of return. To sustain a rational bubble, the stock price must grow faster than dividends (or cash flows) in perpetuity and therefore a rational bubble implies a lack of cointegration between the stock price and fundamentals, i.e. dividends, see Craine (1993).

Under the more realistic assumption of a time-varying discount rate, Campbell and Shiller (1988) suggest a loglinear approximation of returns $R_{t+1} = (P_{t+1} - P_t + D_{t+1})/P_t$. It is given by

$$r_{t+1} = p_{t+1} - p_t + \ln(1 + \exp(d_{t+1} - p_{t+1})) \ , \tag{6}$$

where r, p and d are the natural logarithm of $1+R$, P and D, respectively. This nonlinear relationship can be linearized by applying a first-order Taylor approximation:

$$r_{t+1} \approx \alpha + \lambda p_{t+1} + (1-\lambda) d_{t+1} - p_t \ . \tag{7}$$

[1] If they are not aware of this fact, the bubble is irrational, see O'Hara (2008) for a brief discussion on rationality.

Solving this equation forward and imposing the no-bubble condition $\lim_{k\to\infty} \lambda^k(d_{t+k} - p_{t+k}) = 0$ yields

$$p_t = \alpha/(1-\lambda) + \sum_{k=0}^{\infty} \lambda^k \left[(1-\lambda)d_{t+1+k} - r_{t+1+k}\right] \; . \tag{8}$$

Finally, by taking the expectation of equation (8) based on information available at time t, this leads to an expression for the log dividend-price ratio:

$$d_t - p_t = -\alpha/(1-\lambda) + \sum_{k=0}^{\infty} \lambda^k \left[-E_t(\Delta d_{t+1+k} + r_{t+1+k})\right] \; . \tag{9}$$

Following Craine (1993), the log dividend yield is stationary under the no-bubble restriction if dividend growth and logarithmic stock returns are stationary. Importantly, the presence of a unit root in $d_t - p_t$ is consistent with a rational bubble in the stock price.

As some economists find the idea of a permanent rational bubble unrealistic, such as Froot and Obstfeld (1991, p. 1190), who state that "it is difficult to believe that the market is literally stuck for all time on a path along with price-dividend ratios eventually explode", in a more elaborated model Blanchard and Watson (1982) describe a bubble that collapses with probability one in the long-run, but continues in the next period with probability π and bursts with probability $(1-\pi)$. However, if it grows in expectation at a rate $(1+R)/\pi$, which is sufficient to compensate for the risk, the net present value of the bubble is positive and holding the bubble is rational. The idea of emerging and collapsing bubbles is closely related to the empirical literature testing for changes in the degree of integration in the dividend yield.

3 Testing for changing memory

This section describes the testing procedure for a structural change in the long memory parameter. We apply a CUSUM of squares-based test proposed by Sibbertsen and Kruse (2009). The authors assume that the data generating process follows an ARFIMA(p, d, q) process as proposed by Granger and Joyeux (1980):

$$\Phi(L)(1-L)^d y_t = \Psi(L)\varepsilon_t \; ,$$

where ε_t are iid random variables with mean zero and variance σ^2. The autoregressive- and moving average- polynomials $\Phi(L)$ and $\Psi(L)$ are assumed to have all roots outside the unit circle. The degree of integration of y_t is therefore solely determined by the memory parameter d. This process is said to be fractionally integrated of order d. The test proposed by Sibbertsen and Kruse (2009) considers the following pair of hypotheses,

$$H_0 \quad : \quad d = d_0 \quad \text{for all } t \tag{10}$$

$$H_1 \quad : \quad \begin{cases} d = d_1 & \text{for } \ t = 1, \ldots, [\tau T] \\ d = d_2 & \text{for } \ t = [\tau T] + 1, \ldots, T \end{cases} \tag{11}$$

where $[x]$ denotes the biggest integer smaller than x. The differencing parameter is restricted to $1/2 < d_0 < 3/2$ under H_0, while $0 \leq d_1 < 1/2$ and $1/2 < d_2 < 3/2$. Please note that d_1 and d_2 can be interchanged. We test the null hypothesis of constant memory ($1/2 < d_0 < 3/2$) against a change from stationary ($0 \leq d_1 < 1/2$) to non-stationary ($1/2 < d_2 < 3/2$) long memory at $[\tau T]$ or vice versa. In case that $0 \leq d_0 < 1/2$, the time series y_t is integrated once, i.e., $z_t \equiv \sum_{i=1}^{t} y_i$. The fractional degree of integration of z_t is then given by $d_0 + 1 \in [1, 3/2)$. The test is then carried out for z_t instead of y_t. Sibbertsen and Kruse (2009) show that this simple approach works well in practice. The test statistic, originally proposed by Leybourne et al. (2007), is given by

$$R_{CS} = \frac{\inf_{\tau \in \Lambda} K^f(\tau)}{\inf_{\tau \in \Lambda} K^r(\tau)}, \tag{12}$$

where $K^f(\tau)$ and $K^r(\tau)$ are CUSUM of squares-based statistics based on the forward and reversed residuals of the data generating process as given below. The relative breakpoint $\tau \in [\underline{\tau}, \bar{\tau}] \equiv \Lambda$ is assumed to be unknown and a simple estimator is given at the end of this section. In detail, $K^f(\tau)$ and $K^r(\tau)$ are given by

$$K^f(\tau) = \frac{1}{[\tau T]^2} \sum_{t=1}^{[\tau T]} \hat{v}_{t,\tau}^2$$

and

$$K^r(\tau) = \frac{1}{(T - [\tau T])^2} \sum_{t=1}^{T-[\tau T]} \tilde{v}_{t,\tau}^2 \ .$$

Here, $\hat{v}_{t,\tau}$ are the residuals from the OLS regression of y_t on a constant based on the observations up to $[\tau T]$. This is

$$\hat{v}_{t,\tau} = y_t - \bar{y}(\tau)$$

with $\bar{y}(\tau) = [\tau T]^{-1} \sum_{t=1}^{[\tau T]} y_t$. Similarly $\tilde{v}_{t,\tau}$ is defined for the reversed time series. Sibbertsen and Kruse (2009) show that the limiting distribution ($T \to \infty$) of R is given by

$$T^{-2d_0} R_{CS} \Rightarrow \frac{\inf\limits_{\tau \in \Lambda} L^f_{d_0}(\tau)}{\inf\limits_{\tau \in \Lambda} L^r_{d_0}(\tau)} \tag{13}$$

with

$$\begin{aligned}
L^f_{d_0}(\tau) &= \int_0^{\tau} W^*_{d_0}(r,\tau)^2 dr \\
L^r_{d_0}(\tau) &= \int_0^{1-\tau} V^*_{d_0}(r,\tau)^2 dr \\
W^*_{d_0}(r,\tau)^2 &= \left(W_{d_0}(r) - \tau^{-1} \int_0^{\tau} W_{d_0}(r) dr \right)^2 \\
V^*_{d_0}(r,\tau)^2 &= \left(W_{d_0}(1-r) - (1-\tau)^{-1} \int_{\tau}^{1} W_{d_0}(r) dr \right)^2 .
\end{aligned}$$

The fractional Brownian motion W_{d_0} (type II, see Marinucci and Robinson 1999) is given by

$$W_{d_0}(r) = \frac{1}{\Gamma(d_0+1)} \int_0^r (r-s)^{d_0} dW(s) \ , r > 0 \ ,$$

where Γ denotes the Gamma function and W is the regular Brownian motion. It is worthwhile to note that it depends on the fractional degree of integration under the null hypothesis, d_0. Sibbertsen and Kruse (2009) provide response curves to compute critical values which works well in practice. The simulation results therein show that the size properties of the test are satisfying even though the unknown parameter d_0 is estimated. The estimation of d_0 has little impact on the empirical size properties of the test. It introduces a small size distortion which gets smaller with an increasing sample size. Their results are obtained for samples sizes which are much smaller than in our application. Therefore, it is expected that the estimation uncertainty of d_0 has a negligible impact, if any, on our results. However, as explained in Section 4 in detail, we treat this issue carefully by considering different estimators for d_0. It is important to note that the CUSUM of squares-based test maintains satisfactory size properties when the data generating process exhibits GARCH effects.[2]

[2]Unreported simulation results for $T = 1500$, three different settings for GARCH parameters and six different values for d_0, confirm this claim. Full results are available upon request from the authors. See

The null hypothesis $H_0 : d = d_0$ is rejected for small values of R in favor of the alternative H_1 stated in (8). Theorem 2 in Sibbertsen and Kruse (2009) shows that the test is consistent if a change from $0 \leq d_1 < 1/2$ to $1/2 < d_2 < 3/2$, or vice versa, occurs. In addition, the simulation results reported in Sibbertsen and Kruse (2009) suggest that the test has substantial power against changes of the long memory parameter within the (non-)stationary region. This means in particular that the test is powerful in detecting changes in the non-stationary region, say from $d = 0.6$ to $d = 1$.

Finally, we note that the unknown breakpoint τ is estimated by minimizing the objective function $\mathcal{S}(\tau)$ over $\tau \in \Lambda$:

$$\hat{\tau} = \arg \inf_{\tau \in \Lambda} \frac{1}{[\tau T]^2} \sum_{t=1}^{[\tau T]} \hat{v}_{t,\tau}^2 \equiv \arg \inf_{\tau \in \Lambda} \mathcal{S}(\tau) \ . \tag{14}$$

This estimator is consistent as shown in Sibbertsen and Kruse (2009), Theorem 3. The simulation results therein show that the estimator performs well in finite samples. The later the breakpoint is located in the sample, the more accurate is the breakpoint estimator in terms of small-sample bias and variance. In addition, we note that the performance of the breakpoint estimator is also not substantially affected by GARCH disturbances, see Heinen et al. (2009).

4 Empirical evidence

The monthly data set can be downloaded from Robert Shiller's web site[3]. The sample spans from January 1871 to March 2009 which results in 1659 monthly observations. We consider the log dividend-price ratio $y_t = \ln(D_t/P_t)$. This time series is depicted in Figure 1. The graph shows a clear change in the behaviour in the last part of the sample. The vertical line illustrates the estimated breakpoint of changing persistence in July 1991. A detailed description of the application is given below.

also Heinen et al. (2009) for further results.

[3]http://www.econ.yale.edu/ shiller/data.htm

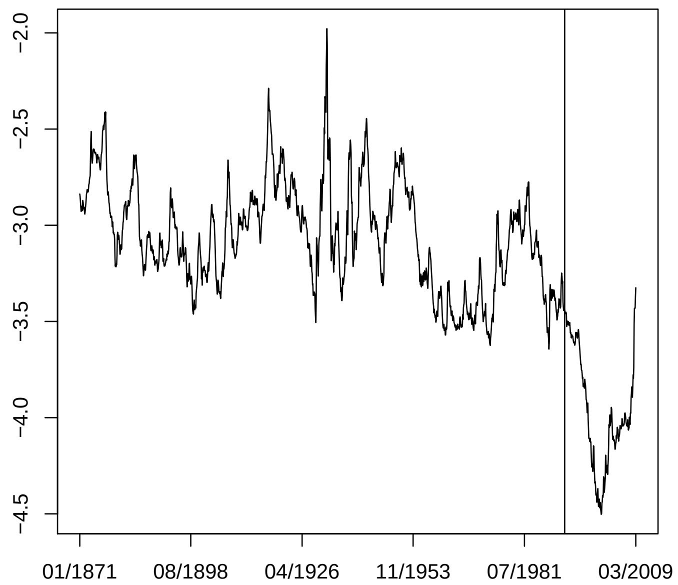

Figure 1: S&P 500 log dividend-price ratio (January 1871 to March 2009).

The set of potential breakpoints is specified as $\Lambda = [0.1T, 0.9T]$ which is a common choice in the breakpoint literature, see Perron (2006). This setting corresponds to potential break dates between September 1884 and May 1995. When carrying out the CUSUM of squares-based test for a change in the fractional degree of integration we obtain a test statistic which equals 0.424, see Table 1. Recall that the critical values depend on d_0 which is an unknown parameter and hence estimated. We consider (i) semi-parametric log-periodogram estimation and (ii) full parametric maximum likelihood estimation of properly specified ARFIMA-GARCH models. Results in Koustas and Serletis (2005) suggest that conditional heteroscedasticity plays an important role for the S&P500 log dividend-price ratio. Regarding log-periodogram estimation of d_0 we apply the method proposed by Geweke and Porter-Hudak [GPH] (1983). While this estimator is to some extent robust against GARCH effects, see Hauser (1997), the second approach takes GARCH effects directly into account by specifying a full model for y_t.

Table 1: CUSUM of squares-based test for changing memory

	Full sample	Pre-break sample	Post-break sample
	Test statistics		
R_{CS}	0.424	0.891	0.827
	Critical values: Log-periodogram regression		
$cv_{1\%}$	0.221	0.476	0.058
$cv_{5\%}$	0.354	0.612	0.131
$cv_{10\%}$	0.452	0.695	0.204
$cv_{90\%}$	2.242	1.466	4.863
$cv_{95\%}$	2.869	1.698	7.626
$cv_{99\%}$	4.570	2.049	17.333
	Critical values: ARFIMA-GARCH model		
$cv_{1\%}$	0.289	0.569	0.077
$cv_{5\%}$	0.430	0.701	0.160
$cv_{10\%}$	0.529	0.776	0.242
$cv_{90\%}$	1.874	1.258	4.127
$cv_{95\%}$	2.271	1.347	6.179
$cv_{99\%}$	3.550	2.016	13.342

Notes: R_{CS} is the CUSUM of squares-based test statistic for constant memory. The limiting distribution of R_{CS} depends on d which is estimated, see Table 2 and 3. $cv_{\alpha\%}$ is the critical value at the α% level of significance based on these point estimates. If R_{CS} is lower than a certain critical value, the null of constant memory is rejected in favor of increasing memory and vice versa.

The GPH estimator is based on an approximation of the spectral density of y_t near the origin. In more detail, the following regression is considered

$$\log(I_j) = \log c_f - 2dX_j + \log \xi_j \ , \quad j = 1, 2, \ldots, m \tag{15}$$

where $I_j = \frac{1}{2\pi n} \left| \sum_{t=0}^{T-1} y_t \exp\left(\frac{i2\pi jt}{T}\right) \right|^2$ is the j-th periodogram ordinate, c_f is the spectral density of the short-run component at frequency zero, X_j denotes the j-th Fourier frequency and ξ_j are assumed to be i.i.d. with $-E(\log \xi_j) = 0.577216...$ which is known as the Euler constant. The GPH estimator for d equals the -1/2 times the OLS estimator of the slope parameter in the log-peridogram regression (12). Further details can be found in Geweke and Porter-Hudak (1983). As the choice of number of frequencies

Table 2: Log-periodogram regression

	Full sample		Pre-break sample		Post-break sample	
Test of Bias	5.094	(0.000)	3.507	(0.000)	1.113	(0.133)
GPH	0.848	(0.000)	0.614	(0.000)	1.170	(0.000)
$[m]$	$T^{1/2} = 40$		$T_1^{1/2} = 38$		$T_2^{4/5} = 72$	

Notes: Test of Bias is the test statistic for the null hypothesis that no bias occurs in the log-periodogram regression (12). GPH is the semi-parametric estimator for the long memory parameter d with bandwidth $[m]$. $T_1 = 1447$ and $T_2 = T - T_1 = 212$ are the number of observations in the first and the second sub-sample, respectively. P-values are reported in brackets beside the corresponding estimate or test statistic.

m that are used in (12) is a crucial issue, we apply the bias test proposed by Davidson and Sibbertsen (2009) to choose between the MSE-optimal rate $m = [T^{4/5}]$ and the one suggested by GPH, i.e., $m = [T^{1/2}]$. This test checks the null hypothesis of no bias in the log-periodogram regression by using a Hausman-testing principle. An application of this test leads to a rejection of the null hypothesis at conventional levels of significance (p-value $= 0.000$), see Table 2. Therefore, we proceed with the smaller bandwidth choice $m = [T^{1/2}]$. The corresponding GPH estimate of d_0 is 0.848 which indicates non-stationary long memory. This estimate is significantly different from zero as the corresponding p-value equals 0.000, see Table 2. The critical values for R based on this estimate are reported in Table 1 below the values of the CUSUM of squares-based test statistics. They lead to a rejection of the null hypothesis at the ten percent level of significance.

Second, we build an ARFIMA$(p, d_0, 0)$–GARCH$(1, 1)$ model where all parameters are estimated jointly by maximum likelihood. As a first step, we select the autoregressive lag length p according to usual information criteria with a maximal lag length of $p_4 = [4(T/100)^{1/4}] = 7$, cf. Schwert (1989). We start by considering the pure ARFIMA process, i.e., excluding the GARCH process,

$$\Phi(L)(1-L)^{d_0} y_t = \mu + \varepsilon_t \quad (16)$$

$$\Phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \ldots - \phi_p L^p \; . \quad (17)$$

The lag polynomial $\Phi(L)$ is assumed to have all roots outside the unit circle. The results for different criteria are Schwarz ($\widehat{p} = 1$), Hannan-Quinn ($\widehat{p} = 4$) and Akaike ($\widehat{p} = 4$).

Table 3: Estimation results for ARFIMA(p,d,0) models

	Full sample		Pre-break sample		Post-break sample	
			ARFIMA			
d	0.769	(0.000)	0.552	(0.000)	1.113	(0.000)
μ	-2.840	(0.000)	-2.839	(0.000)	16.556	(0.767)
ϕ_1	0.529	(0.000)	0.767	(0.000)	–	–
ϕ_2	-0.050	(0.083)	-0.093	(0.034)	–	–
ϕ_3	0.052	(0.092)	0.055	(0.103)	–	–
ϕ_4	0.066	(0.015)	0.101	(0.000)	–	–
ϕ_5	0.065	(0.006)	–	–	–	–
LAR	0.816	–	0.881	–	–	–
			GARCH			
ω	0.001	–	0.001	–	>0.000	–
α	0.103	(0.000)	0.100	(0.001)	0.148	(0.008)
β	0.839	(0.000)	0.832	(0.000)	0.839	(0.000)
			Distribution			
df	5.840	–	5.855	–	5.936	–
$\ln(\theta)$	0.172	(0.000)	0.170	(0.000)	0.214	(0.054)
			Diagnostics			
$Q(15)$	18.220	(0.251)	18.652	(0.230)	13.304	(0.579)
LM(15)	16.520	(0.348)	18.218	(0.251)	13.800	(0.541)
$Q^2(15)$	8.643	(0.895)	7.191	(0.952)	9.824	(0.831)
ARCH(5)	2.835	(0.725)	1.627	(0.898)	1.764	(0.881)

Notes: The estimated model is given by equations (18)–(20). P-values are reported in brackets beside the corresponding estimate or test statistic if available. LAR denotes the largest autoregressive root of the AR polynomial $\Phi(L)$. df is the degree of freedom of the skewed Student-t distribution, while θ is the asymmetry parameter. $Q(15)$ ($Q^2(15)$) is the Ljung-Box statistic applied to the (squared) residuals with 15 lags, LM(15) is a serial correlation LM test statistic with 15 lags and ARCH(5) is Engle's ARCH-LM test with 5 lags.

Unfortunately, none of these choices lead to satisfying results of diagnostic tests regarding remaining autocorrelation in the residuals $\widehat{\varepsilon}_t$.[4] Therefore, we increase the lag length chosen via Akaike and Hannan-Quinn Information Criterion by one and obtain better

[4]These results are not reported but available from the authors upon request.

results. Table 3 shows that the null hypothesis of no remaining serial correlation in the residuals up to lag 15 cannot be rejected at conventional significance levels when using a Ljung-Box statistic Q and an LM test for serial correlation. The selection of 15 lags is of course arbitrary, but the conclusions do not change when 10 or 20 lags are used.

In the next step of our model building procedure, we add a GARCH$(1,1)$ variance equation to the ARFIMA$(5, d_0, 0)$ model. Hence, the specified model is given by

$$\Phi(L)(1-L)^{d_0} y_t = \mu + \varepsilon_t \tag{18}$$

$$\varepsilon_t \sim (0, h_t) \tag{19}$$

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1} . \tag{20}$$

Diagnostic tests (Q^2 and ARCH-LM) confirm the ad-hoc choice of the lag length $(1,1)$ for the GARCH process. We consider four different possible distributions for the innovations ε_t: Normal, Student-t, skewed Student-t and Generalized Error Distribution (GED). The skewed Student-t distribution provides the best fit to the data when the same information criteria are questioned as before. Moreover, estimation results for d_0 are quite stable across different specifications for the innovation distribution. In Table 1 we only report results for the model with a skewed Student-t distribution.[5] The estimate for the main parameter of interest, i.e., d_0, is 0.769 and therefore slightly lower than the GPH estimate. Koustas and Serletis (2005) find a similar value based on a shorter sample. It should be noted that this estimate is also significantly different from zero. All of the diagnostic tests (Ljung-Box for the level and squared residuals, LM autocorrelation test and the test for neglected ARCH effects) indicate that the estimated model is correctly specified. The critical values for the CUSUM of squares-based test statistic $R = 0.424$ based on the ARFIMA-GARCH model are reported in the lower panel of Table 1. These critical values imply a rejection of the null hypothesis at the five percent level of significance.

In sum, we find a rejection of the null hypothesis that the long memory parameter is constant in favor of the alterative that the memory parameter is larger during the post-break sample than before. The estimated breakpoint is July 1991. The empirical objec-

[5] All other results are available from the authors upon request.

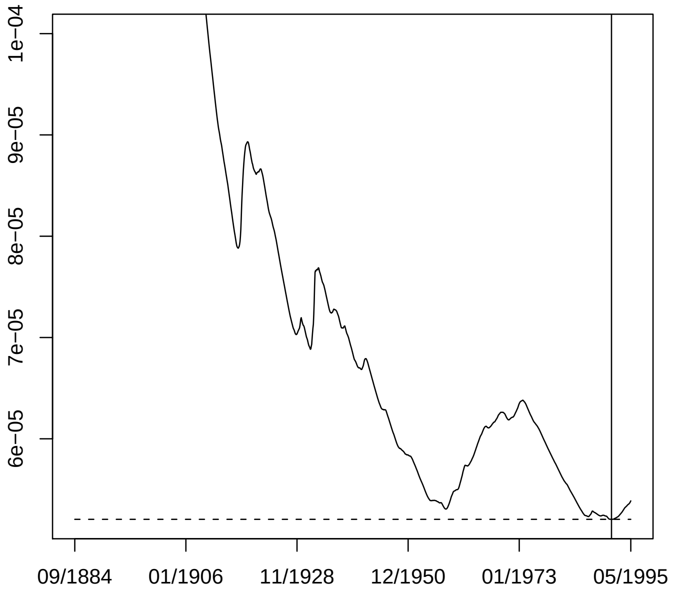

Figure 2: Empirical objective function $\mathcal{S}(\tau)$.

tive function $\mathcal{S}(\tau)$, which has to be minimized for breakpoint estimation is depicted in Figure 2, see also equation (14). It can be seen that the function has a minima close to the end but not at the border. There is, however, another region with very low values of S. If the interval of potential breakpoints would be tighter than specified here, the relatively late break date in 1991 would be excluded and hence undetectable. Interestingly, within this area the minimum is located in August 1958 which is close to the breakpoint estimations reported in Sollis (2006).

While most empirical studies do not include the 1990s and are therefore not comparable, the evidence of a structural change towards a bubble in the 1990s is confirmed by several recent studies. For instance, Bohl (2003) finds evidence for a bubble in the US stock market only for the sample period 1871 to 2001, but not if the sample ends in 1995. Similarly, Nasseh and Strauss (2004) find a structural change in the relation between prices and dividends in the mid 1990s. Cunado et al. (2005) focus on the comparatively short sample period from 1994 to 2003 and the Nasdaq index. They find evidence for the existence of a bubble, but cannot determine when the bubble has started.

Finally, our results strongly corroborate the results by Lettau and and Van Nieuwerburgh (2008), who also identify a structural break in the dividend-price ratio in 1991. The timing of our break is also in line with recent empirical work on other financial time series (see for instance Pastor and Stambaugh 2001 for the equity risk premium). The estimated break date coincides with the recovery of the stock market that followed the recession of 1991 and led to the stable increase of share prices and the subsequent explosive behaviour from the mid-1990s on, that ended with the collapse at the turn of the century. The structural break found in our analysis reflects the change in valuation ratios that could be observed in the bull markets of the 1990s, which marked the transition to a non-stationary process.[6] At this time the forecasting power of traditional models broke down (Lettau and Van Nieuwerburgh 2008). While some analysts saw the rapid rise in US stock markets justified by low inflation, a decline of the equity risk premium and increased productivity growth (Nasseh and Strauss 2004) and in the 1990s the business press coined the notation of the "new economy", benefitting from globalization and the revolution in information and communication technology, some economists suggested the existence of a stock market bubble (see inter alia Shiller 2001, Stiglitz 2003). In a recent contribution, Park (2010) provides international evidence for the importance of changes of persistence in the dividend yield for its predictive power for stock returns.

The next steps of our analysis include an analysis of the two sub-samples which are generated according to the breakpoint estimate. That is, we consider the pre- and the post-break sample separately from each other. We repeat all steps of the testing and estimation exercise. This is done in order to test whether the break in July 1991 is the only one which occurred in our sample. The outcome of the sub-sample analysis reveals clearly that no additional breaks in the long memory parameter are present. Detailed results on estimation and inference can be found in the middle and the last column of

[6] "The extraordinary valuation ratios in the late 1990s represent a significant challenge for the benchmark model. Given the historical record of returns, fundamentals, and prices, it is exceedingly unlikely that persistent stationary shocks to expected returns are capable of explaining price multiples like those seen in 1999 or 2000." (Lettau and Van Nieuwerburgh 2008, p. 1608).

Tables 1, 2 and 3.

In the light of testing for rational bubbles it is necessary to compare pre- and post-break degrees of integration. From a first sight, the estimates suggest that the pre-break degree of integration d_1 is significantly different from zero and close to the limiting value of 0.5 for stationarity processes. It is, however, unclear whether d_1 is significantly different from unity. The post-break estimate d_2 is in contrast to d_1 quite close to unity. In order to carry out a formal and suitable test of the unit root hypothesis $H_0 : d_{1,2} = 1$ against long memory $H_1 : d_{1,2} < 1$, we apply the lag augmented Lagrange multiplier test proposed by Demetrescu et al. (2008). Their procedure builds upon the test regression ($x_t = \Delta y_t$)

$$x_t = \phi x^*_{t-1} + \sum_{i=1}^{k} \psi_i x_{t-i} + \varepsilon_t \tag{21}$$

with $x^*_t = \sum_{j=1}^{t-1} \frac{x_{t-j}}{j}$. The unit root hypothesis can be expressed as $H_0 : \phi = 0$, while the alternative of long memory is given by $H_1 : \phi < 0$. Demetrescu et al. (2008) suggest to use a t-statistic for H_0 versus H_1 which has a standard normal limiting distribution. Furthermore, they recommend to specify the lag length k deterministically, i.e., $k_4 = [4(T/100)^{1/4}]$ or $k_{12} = [12(T/100)^{1/4}]$. When computing the t-statistics for H_0, we choose OLS based and White's heteroscedasticity-robust standard errors. The corresponding t-statistics are denoted as t and t^*, respectively. Regarding deterministic terms, Demetrescu et al. (2008) suggest to regress x_t on the first difference of the deterministic terms in a first step prior to estimation of equation (21). In the case of a linear trend, x_t is de-meaned, while it is not modified if a constant is specified as the first difference of a constant simply disappears.

Table 4: Unit Root Test against Long Memory

	Constant					
	k_4			k_{12}		
	Full sample	Pre-break sample	Post-break sample	Full sample	Pre-break sample	Post-break sample
t	-3.738^r	-2.478^r	2.011	-1.709^r	-2.111^r	-0.040
t^*	-2.754^r	-4.117^r	1.991	-1.565	-2.509^r	-0.034
	Linear trend					
	k_4			k_{12}		
	Full sample	Pre-break sample	Post-break sample	Full sample	Pre-break sample	Post-break sample
t	-3.892^r	-4.165^r	1.952	-1.882^r	-2.627^r	-0.087
t^*	-2.748^r	-2.446^r	1.928	-1.667^r	-2.108^r	-0.074

Notes: The t-statistic based on the OLS variance estimator is labeled as t, while White's heteroscedasticity-robust standard errors are used in t^*. The lag length k is selected according to $k_4 = [4(T/100)^{1/4}]$ and $k_{12} = [12(T/100)^{1/4}]$. Significant statistics at the nominal level of five percent are marked with an r-superscript.

Table 5: Standard Unit Root Tests

	Constant					
	Modified AIC			Modified SIC		
	Full sample	Pre-break sample	Post-break sample	Full sample	Pre-break sample	Post-break sample
Ng-Perron	-1.663	-2.196^r	-1.473	-2.038^r	-2.883^r	-0.588
DF-GLS	-1.539	-2.088^r	-0.957	-2.037^r	-2.037^r	-0.587
	Linear Trend					
	Modified AIC			Modified SIC		
	Full sample	Pre-break sample	Post-break sample	Full sample	Pre-break sample	Post-break sample
Ng-Perron	-3.084^r	-3.833^r	-2.143	-3.453^r	-4.292^r	0.356
DF-GLS	-2.960^r	-3.555^r	-0.402	-3.439^r	-4.274^r	0.388

Notes: Reported number are the MZ_t statistics of the Ng-Perron (2001) unit root test and the unit root statistics of the Elliott et al. (1996) test. Asymptotic 5% critical values are -1.98 and -2.91 for the Ng-Perron (2001) test in the case of a constant and a linear trend, respectively. The analogous critical values for the DF-GLS test are -1.94 and -2.89. Significant statistics at the nominal level of five percent are marked with an r-superscript.

Results are reported in Table 4. It can be seen that the unit root hypothesis has to be clearly rejected for the full sample and the pre-break period but not for the post-break period. The p-values are either zero or very close to unity. The inclusion of heteroscedasticity-robust standard errors does not have an impact on the outcomes although the test statistics are generally somewhat higher. The specification of deterministic terms has only little impact on the test results. Moreover, the test decision is not influenced by the choice of the lag length k via k_4 or k_{12}.

As a robustness check we consider two standard unit root tests with good size and power properties. We apply the MZ_t test proposed by Ng and Perron (2001) and the Dickey-Fuller GLS test by Elliott et al. (1996). Empirical results can be found in Table 5. We report test statistics for two types of deterministic terms as before: constant and a linear trend. The lag length is selected by applying the modified AIC, HQIC and SIC. Since the results for the modified HQIC and SIC are the same, we only report the latter ones. It can be seen that the results are in line with the outcomes of the test against long memory, see Table 4. The null hypothesis of a unit root cannot be rejected in the second sub-sample. This result holds true regardless of the specification of deterministic terms and the choice of the information criterion.[7] Therefore, the evidence for non-stationarity of the dividend-price ratio in the post-break period is particularly strong. On the contrary, the results suggest the opposite for the pre-break sample.

We conclude that the log dividend-price ratio of the S&P500 exhibits significant long memory before the break takes places and that it contains a unit root afterwards. The non-stationarity in the second sub-sample invalidates the no-bubble condition. Hence, we find statistical evidence which is consistent with the existence of a rational asset price bubble. The bubble starts in July 1991 and it does not seem to collapse during our sample since we do not find any evidence for a significant decline in persistence in the post-break period. A reduction in persistence would have indicated the collapse of the bubble.

[7] The application of the KPSS test (Kwiatkowski et al. 1992) with both types of deterministic terms and different kernels for the spectrum estimation at frequency zero suggests strong evidence for non-stationarity as well. Full results are available from the authors upon request.

Finally, we consider the earnings-price ratio as a further robustness check.[8] The earnings-price ratio may share dynamic properties similar to the dividend-price ratio. In the following, we provide a brief summary of results.[9] Indeed, our results suggest a high degree of similarity: The CUSUM of squares-based test statistic R_{CS} equals 0.709. Full sample estimates for d_0 are 0.548 (based on GPH estimator with $m = [T^{1/2}]$) and 0.515 (ARFIMA $(4, d, 0)$-GARCH(1,1)). The corresponding critical values at the five percent level are given by 0.707 and 0.760. In the latter case, the test for constancy of the memory parameter rejects the null of no structural break, while the decision based on the GPH estimate is borderline. The breakpoint estimate corresponds to June 1991 which is very similar to the one for the dividend-price ratio (July 1991). Another similarity is found in the estimated fractional degree of integration: for the earnings-price ratio we obtain $\widehat{d_1} = 0.477$ (GPH estimate with $m = [T^{1/2}]$) and $\widehat{d_1} = 0.503$ (ARFIMA-GARCH). For the post-break sample, we also find estimates of d_2 slightly above one for the earnings-price ratio. The results for the Demetrescu et al. (2008) test and the two standard unit root tests indicate that the earnings-price ratio is non-stationary during the post-break sample. This conclusion does not change with respect to the specification of deterministic terms and the choice of the lag length. In sum, these findings underline the robustness of our previous findings.

5 Conclusions

Previous research on the time series properties of the S&P500 dividend price-ratio led to conflicting results. Especially, the existence of rational bubbles is highly controversial and at the same time of great importance. In a recent study, Koustas and Serletis (2005) apply fractionally integrated time series models, thereby finding evidence for long memory, but at the same time providing evidence against a rational bubble. However, the authors do not account for the possibility of structural breaks in the memory parameter and therefore, their study is potentially flawed. Furthermore, Sollis (2006) considers tests for changing persistence in the $I(0)/I(1)$ framework which rules out long memory dynamics. This author finds a break from stationarity to non-stationary

[8]We are thankful to an anonymous referee for the suggestion to consider another valuation ratio as well.

[9]Full results are available from the authors upon request.

which means that the no-bubbles condition is not fulfilled for a sub-sample of the data hinting at a rational bubble. Both approaches to test for rational bubbles have their limitations. In this paper, we consider a generalization of a test for changing persistence under long memory proposed by Sibbertsen and Kruse (2009). This testing procedure has the advantage that it permits both, long memory and changing persistence. In addition to the structural break test, we apply a unit root test against long memory proposed by Demetrescu et al. (2008). Our findings can be summarized as follows: the log dividend-price ratio exhibits significant long memory until the early 1990s and has a unit root afterwards. Therefore, the results are consistent with the existence of a rational bubble in the S&P500 stock market.

Moreover, our result of a changing memory parameter of the dividend-price ratio has an important implication for the literature on return predictability. While standard forecasting models for (expected) stock returns use the dividend-price ratio as a predictor (see for instance Stambaugh 1999, Lewellen 2004), they assume that both, the return series and the forecasting variable, are stationary. The latter is not the case when a bubble occurs, which implies that the forecasting regression is unbalanced. Thus, the shift from a stationary dividend-price ratio to a unit root process in 1991 is likely to have caused the well-documented failure of conventional return prediction models since the 1990s (Lettau and Van Nieuwerburgh 2008).

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