Simultaneous versus Sequential Group-Buying Mechanisms

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This paper studies the design of group-buying mechanisms in a two-period game where cohorts of consumers arrive at a deal and make signup decisions sequentially. The group-buying deal succeeds whenever the total number of committed participants exceeds a pre-specified threshold. A firm can adopt either a sequential mechanism where the firm discloses to second-period arrivals the number of sign-ups accumulated in the first period, or a simultaneous mechanism where the firm does not post the number of first-period sign-ups and hence each cohort of consumers faces uncertainty about another cohort’s size and willingness-to-pay when making signup decisions. Our analysis shows that, compared to the simultaneous mechanism, the sequential mechanism leads to higher expected signup rates for both cohorts of consumers and hence higher deal success rates. This result holds when first-period arrivals can postpone their signup decisions strategically. Under the sequential mechanism, the deal success rate is higher when consumers discount future values less, when the deals are more attractive, and/or when the deals are applied to luxury purchases (versus necessity purchases). We find that, when the firm can manage the sequence of arrivals, it should inform the smaller cohort of consumers first, and then the larger cohort, given that each cohort has information on the stochastic characterizations of the other. We also extend our analysis to explore the conditions under which a group-buying mechanism leads to a higher expected profit than a posted-price mechanism.

Key words: group buying; assurance contract; mechanism design; Bayesian equilibrium; rational expectations equilibrium

1. Introduction

A group-buying mechanism is a scheme designed to help coordinate a group of interested buyers so that they can reach their common purchase goals. In a typical group-buying mechanism, no transaction will take place unless the total number of committed purchases exceeds a specified threshold within a certain time period. Various forms of group-buying mechanisms have been observed on a wide range of occasions and for a long time. One version is Street Performer Protocol (SPP), an agreement between an artist and a group of potential users. The artist does not start the creative work until the potential funders have pledged a required amount of support. Renowned
musicians like Beethoven and Mozart used such arrangements to ensure that enough tickets were sold for their concerts. The recent emergence of social media has popularized these funding concepts and generated similar concepts like threshold pledge systems and crowd funding. Websites like Kickstarter.com allow artists, museums, and entrepreneurs to post project proposals and seek funding from interested donors. If the total amount pledged for a project exceeds the amount requested within a given time period, the project is on, and in return the donors will receive some product or benefit created by the project once it is completed. On the other hand, if the amount pledged is less than what was requested, the project is off and the donors’ pledges will be refunded.

In another version of the group-buying mechanism, a number of buyers pool their purchases, often through the facilitation of a third party, to obtain a quantity discount from sellers. For example, many resellers form purchase consortiums to obtain more favorable wholesale prices from manufacturers. Online group-buying websites first appeared in the late 1990s, as part of the wave of innovative online market-based mechanisms. To illustrate how these group-buying sites worked, consider the sale of a digital camera with the retail price of $800 available at retail stores like BestBuy. A group-buying website might offer a deal price of $600 if a minimum of 200 people committed to purchase within three days. Usually the consumers had to make the purchase commitment through escrow payment systems. Most of the representative group-buying websites that became popular in late 1990s, including Mercata, Mobshop, and Letsbuyit, either ceased operating or changed their business models a few years later (Kauffman and Wang 2002).

Interestingly, despite the failure of these pioneering group-buying sites, a decade later another generation of social buying websites like Groupon and LivingSocial emerged. Led by the market leader, Groupon, these newcomers typically offer “a deal a day” tailored to each local market (Wortham 2009). The market enthusiasm for online group buying peaked when Groupon declined a $6 billion offer from Google (Weiss 2010).

This paper investigates the optimal design of group-buying mechanisms. Previous research on group-buying mechanisms used static models to study the private provision of public goods (Palfrey and Rosenthal 1984, Bagnoli and Lipman 1989, Tabarrok 1998). A static model assumes that all the participants make decisions simultaneously without knowing the decisions of others. This static nature might be consistent with many situations in which public goods are provided. However, the most prevalent forms of group-buying mechanisms, typified by such sites as Groupon and Kickstarter, employ a dynamic process by updating the progress towards the group goals, in the form of cumulative number of sign ups or cumulative amount of donations. We take the perspective of third-party group-buying platforms like Groupon and Kickstarter and investigate the impact
of these alternative information management mechanisms on deal success rate. There are two important premises in our research. First, group-buying deals do not always succeed. For example, between the launch of Kickstarter.com in April 2009 and March 2011, a total of 20,371 projects were proposed. Among them, 7,496 projects attracted enough funds, leading to a success rate of 43 percent (blog.kickstarter.com). Zhang and Liu (2011), who studied requests for microloans listed in Prosper.com, reported a success rate of 12.3 percent among 49,693 listings. Since firms like Kickstarter.com and Prosper.com earn revenues from successful listings only, it is important for them to know how to improve the success rates. Second, a consumer’s decision to sign up depends on his or her belief in the deal success rate. Despite the guarantee of a full refund when a deal is not on, a consumer may not want to commit to a deal with a low success rate, for reasons that include both opportunity cost and psychological loss. In the case of Kickstarter.com, instead of waiting 90 days and seeing the project fail, a consumer can invest the funds on other projects in Kickstarter.com or other sites. Psychological loss can occur if a consumer has committed to a project that eventually fails. The underlying reason for the psychological loss can be similar to what drives the bidding frenzy discussed in the auction bidding literature (Ariely and Simonson 2003). The goal of this research is to examine how different information management mechanisms may affect consumer beliefs and the expected success rate.

To investigate the influence of information management mechanism on deal success rate, we develop a two-period model where two cohorts of consumers arrive at the deal sequentially. The two-period model is a stylized capture of the fact that earlier arrivals are faced with more uncertainty in the deal success rate than later arrivals. The firm being studied chooses between a “sequential mechanism” where the firm posts the number of sign-ups at the end of first period, and a “simultaneous mechanism” where the firm does not post the first-period outcome. Assuming that the firm has adopted the group-buying mechanism where the deal characteristics and minimum-requirement threshold are determined exogenously, our study focuses on comparing success rates under different information disclosure structures. The firm’s decision does not appear straightforward because uncertainty about the number of consumer arrivals and their individual sign-up probabilities. Looking forward, it can be beneficial to post the number of sign-ups if one has a large cohort of consumers with high individual sign-up probabilities in the first period, but it can be detrimental if the first cohort of consumers turns out to be small and have low individual sign-up probabilities. Zhang and Liu (2011) shows the strong dependence of a listing’s funding outcome on the first-day momentum. Surprisingly, our analysis shows that the deal success rate is always higher under the sequential mechanism. To see the reason for this result requires a backward approach,
starting with the second period and then moving back to the first period. A sequential mechanism increases the sign-up rates of the second cohort of consumers by reducing the uncertainty facing them. These increased sign-up rates of the second cohort enhance the confidence of the first cohort of consumers, thereby increasing the sign-up rates of the first cohort. This result underscores the importance of modeling and investigating the dynamics of sign-up behavior under group-buying mechanisms. The result also offers a potential explanation for why firms like Groupon and Kickstarter display the updated number of sign-ups along with the minimum number required to unlock the deals.

Our further analysis shows that this result is robust in a number of model extensions. First, we allow consumers arriving in the first period to postpone their sign-up decisions strategically till the end of the game. Under the sequential mechanism, we find that the expected success rate for a group-buying deal is always higher with strategic consumers. Those consumers with relatively low valuations for the deal will buy if and only if they can wait till the end of game and observe enough accumulated sign-ups. Thus, a sequential mechanism leads to a higher expected success rate than a simultaneous mechanism, with or without strategic consumers. Second, we extend our model to one with more than one period and we find that, for any group-buying deals requiring all consumers to participate, the sequential mechanism always leads to a higher expected success rate than the simultaneous mechanism. Since all of the consumers have to commit to purchases in order for the deal to be on, the backward perspective described earlier applies here, too. And since the later arrivals will sign up only if they know that all the earlier arrivals have signed up, the sequential mechanism leads to a higher success rate.

To further understand the sequential mechanism, we extend the model in three directions. In the first extension, we allow the firm to decide the sequence of arrivals for two cohorts of consumers. Controlling the arrival sequence becomes feasible when the firm can contact and inform different cohorts of consumers separately. Such control becomes more valuable when the firm can predict the sizes and preferences of each cohort according to their past responses. Interestingly, our analysis finds it is optimal for the firm to inform the cohort with a smaller number of consumers first, and then the larger cohort of consumers, given that the consumers in each cohort know the stochastic characterizations of the other. The reason for this result can again be seen from the backward approach described earlier. When the firm is uncertain about the size of each cohort and hence is uncertain about the number of sign-ups, it is better to inform the potentially larger cohort of consumers later because the confidence of consumers arriving earlier can be boosted by a “stronger” cohort behind them.
In the second extension, our analysis shows that under the sequential mechanism, the deal success rate tends to be higher when the deal is more attractive, e.g., price discount is larger and/or consumers discount the future benefit less. First, deals providing greater benefits give consumers more incentives to participate in group buying. This may explain a shift from the necessity goods often sold by first-generation group-buying sites to luxury services frequently sold by the second-generation group-buying sites. The markets for necessity goods such as digital cameras are very competitive because the established retailers are selling the same or similar products. Since the volume sold through group-buying sites is usually much smaller than the volume sold through large retail stores, it is difficult for group-buying sites to offer much better deals than retail giants like BestBuy and Wal-Mart. In contrast, luxury services like spa and leisure activities are more differentiated and the discounts offered by group-buying sites tend to be more valuable to consumers.

Second, larger time discounting can arise from longer horizons. The first-generation group-buying sites simultaneously offered a large number of products, as eBay does. These deals were often extended over weeks in order to gain enough sign-ups. However, as the sequentially arriving consumers became suspicious that a deal would not unlock and hence were reluctant to sign on, the success rate would still be low in spite of the long duration. In contrast, firms like Groupon offer deals expiring within 24 hours, thereby increasing the deal success rate substantially. These findings may explain the stark contrast between the fate of the first-generation group-buying sites and the enormous success enjoyed by their counterparts in the current generation.

In the third extension, given the focus of this paper on comparing the sequential and simultaneous mechanisms for group-buying deals, we treat the firm’s decision on the adoption of group-buying deals as exogenous. To explore the adoption of a group-buying selling mechanism versus a posted-price mechanism, we analyze a simpler model with two potential consumers who arrive sequentially. The firm may benefit from economy of scale by selling to both consumers; however, if it uses a group-buying mechanism, it may risk not selling to any consumer when the deal is not on. We find that the optimal price is always lower with the group-buying mechanism than that with the posted-price mechanism. This is because with the group-buying mechanism, a consumer requires a sufficiently low price as well as a sufficiently high success rate if he or she is going to sign up to the deal. Thus, attracting consumers to a group-buying deal necessarily requires a lower price. Overall the group-buying mechanism results in a higher profit if and only if the economy of scale is substantial and the cost of selling to a single consumer is high enough.

Our paper is related to the small but growing theoretical literature on group-buying mechanisms in two ways. First, like Bagnoli and Lipman (1989) and Tabarrok (1998), we study assurance
contracts in the provision of discrete public goods. Group-buying mechanisms belong to assurance contracts because consumers are reimbursed when a deal is not on. The output is discrete public goods because the outcome is discrete (either on or off) and the same outcome applies to the entire cohort of consumers. Our paper contributes to this literature by examining a dynamic model and, more specifically, analyzing how the sequential order of sign-up decisions can affect the success rate. Second, our paper is related to research that compares the relative performance of the group-buying mechanism to the posted-price mechanism. In the presence of demand uncertainty, the group-buying mechanism is shown to outperform posted pricing under demand heterogeneity, economies of scale (Anand and Aron 2003), and risk-seeking sellers (Chen et al. 2007). Chen et al. (2010) compares the uniform group-buying price, where buyers pay the same unit price obtained by the cohort, with non-uniform-price group-buying mechanisms. Jing and Xie (2007) explores another strategic function of group buying, that of facilitating consumer social interaction, and shows that group buying dominates other social-interaction schemes under certain conditions. Two mechanisms, price discrimination and advertising, by which online group buying can potentially benefit affiliated vendors, are carefully examined by Edelman et al. (2010). Unlike these papers, we assume the firm has adopted the group-buying mechanism and we focus on the deal success rate under various sign-up mechanisms.

2. Model Setup and Preliminary Analysis

In this section we develop a two-period model to study the firm’s design of a group-buying mechanism and subsequent consumer responses. We will base our description of the model on group-buying websites like Groupon; nevertheless, the results also apply to settings like Kickstarter. We consider a market where a firm (either a producer or a broker) uses the group-buying format to promote a product or service to consumers. At the beginning of the first period, the firm posts its group-buying deal, which is characterized by three elements: group-buying price $w$, minimum number of buyers $N$ required, and a time horizon of two periods for signing up. In practice, firms may use variations of such formats depending on the specific products or services being promoted. For instance, Kickstarter sets the total amount of dollar commitment as the threshold. When firms such as Groupon use group-buying mechanisms to promote price discounts, the firm also posts a regular price $p$ that consumers would otherwise have to pay in the market. We assume that the threshold number $N$ and the group-buying price $w$ are exogenous. The threshold may be the minimum number at which the goods or service provider can enjoy economies of scale and offer the discount. Naturally the group-buying price should be at a discount to the regular price, i.e.,
$w < p$. Or, as in this case, a firm may replace the deal price with the size of the discount in its announcement. For example, a group-buying deal may offer a 50 percent discount for a lunch buffet at a popular Japanese Sushi restaurant, regularly priced at $20 per person, on condition that the number of buyers surpasses 150 within one day. This is equivalent to a deal price of $10.

We consider a two-period model as an efficient way to capture the sequential nature of consumer arrivals and the interdependence between purchase decisions of early and late arrivals. In practice the length of the time horizon can vary from one day to several months. For example, most deals offered by Groupon.com expire within 24 hours. However, project proposals posted in Kickstarter typically last for several weeks. In our model, if the total number of committed purchases at the end of the second period either reaches or exceeds the minimum requirement $N$, then the group-buying deal is on and every consumer who has signed up receives the product or service at price $w$. Otherwise, the deal is off and no transaction takes place. Since the firm gives refunds, our group-buying mechanism is a type of assurance contract (Tabarrok 1998). We define the success rate of a deal as the likelihood that the deal will be on at the end of second period. Consumers evaluate the success rate of a deal at the beginning of the first or second period, depending on their times of arrival.

There are two cohorts of consumers, indexed by $i = 1, 2$: one arrives in the first period, and the other in the second period. For simplicity, we assume all consumers within the same cohort make purchase decisions simultaneously. Each consumer makes only one decision upon arrival. Consumers do not wait strategically; that is, if a consumer arriving in the first period decides not to sign on to the deal, that consumer will not return to the deal later. Before the game starts, the firm is uncertain about the size of each cohort (number of consumers), but holds a belief about the distribution for the size of each cohort. The firm knows that consumers within each cohort may have different valuations for the firm’s product or service. Although the firm does not know the individual consumer’s valuations even after their arrival at the group-buying deal, it knows the distribution of product or service valuation among consumers within each cohort. We use a set of random variables to represent these distributions with respect to cohort sizes and consumer valuations. Specifically, we denote the number of consumers arriving in period $i$ by a discrete random variable $M_i$ and assume the product valuations of consumers arriving in period $i$ are independently and identically distributed random variables. The individual product valuation for cohort $i$ is denoted by $V_i$, which is drawn from a given cumulative distribution function $F_i(\cdot)$. These two cohorts of consumers can differ in both size and product valuations; in other words, $M_1$ can be different from $M_2$ and $V_1$ can be different from $V_2$. 
The distributions for the two cohorts’ sizes and valuations of consumers in each cohort are public information, known to the firm as well as to all consumers. We assume that consumers know exactly how much they themselves value the product (denoted by \( v \) for each individual’s realization) and that their valuations are not influenced by other people’s purchase decisions. We further assume that when one cohort of consumers arrive at the deal and need to make sign-up decisions, these consumers find out the size of the cohort and its members’ valuations. Such information becomes available to all consumers within the cohort, but not to the other cohort. Again, the information does not alter any consumer’s valuation for the product. However, the communication within each cohort resolves any information asymmetry within the cohort so that all individuals in a cohort share the same belief about the success rate of group buying, which depends on the size and valuations that may still remain unresolved for the other cohort. In summary, in the assumed information structure, uncertainty for consumers arises from imperfect information regarding the size and valuation of another cohort. We assume certainty on consumer’s self-valuation for the deal, therefore eliminating the need to learn from others and the possibility of herding behavior or information cascade.

Each consumer demands and may purchase up to one unit of the product or service. (This assumption can be easily relaxed to accommodate quantity decisions as in Kickstarter.) When a consumer arrives at a group-buying deal, he or she needs to decide whether or not to sign on to the deal. To sign on to the deal is a commitment to purchase if the deal is on. A consumer decides to sign on to the group-buying deal if and only if he or she expects a discounted utility from the deal at least as high as that of not signing on to the deal. In making the decision, consumers take into consideration their own valuations for the deal as well as their expectations about the success rate of the group-buying deal. We assume that consumers are rational in the sense that their purchase decisions depend on their expectations of the success rate at the end of the second period. We further assume that consumers form rational expectations (RE) and focus on the pure strategy solutions of the game. The use of the rational expectations hypotheses is standard in the economics and management literature (Frydman 1982, Jerath et al. 2010).

The firm considers two alternative group-buying mechanisms: a sequential or a simultaneous mechanism. The distinction between these two mechanisms lies in the information revealed to the cohort of consumers who arrive in the second period; more specifically, the second cohort of consumers can see the cumulative number of sign-ups in the first period under a sequential mechanism, but not under a simultaneous mechanism. When a firm adopts the simultaneous mechanism, it does not post the current number of consumers who have already committed, and consumers do
not know if the transaction will be on or not until the end of second period. Under this mechanism, although the second cohort of consumers arrive and make decisions later than the first cohort, two cohorts of consumers have the same information when making decisions. The firm’s decision not to disclose the outcome of the first period makes the process equivalent to one of simultaneous decisions with time discounting that corresponds to their actual arrival time. This distinction between simultaneous and sequential mechanisms is similar in nature to that between sealed and sequential auctions.

We summarize all notations in Table 1.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
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<tbody>
<tr>
<td>$w$</td>
<td>Group-buying price</td>
</tr>
<tr>
<td>$p$</td>
<td>Regular price</td>
</tr>
<tr>
<td>$N$</td>
<td>Minimum number of buyers required</td>
</tr>
<tr>
<td>$M_i$</td>
<td>Number of consumers for cohort $i$</td>
</tr>
<tr>
<td>$m_i$</td>
<td>Realized number of consumers for cohort $i$</td>
</tr>
<tr>
<td>$V_i$</td>
<td>Individual product valuation for cohort $i$</td>
</tr>
<tr>
<td>$F_i(\cdot)$</td>
<td>Cumulative distribution function of the individual product valuation for cohort $i$</td>
</tr>
<tr>
<td>$v_{ij}$</td>
<td>Realized valuation of consumer $j$ in cohort $i$</td>
</tr>
<tr>
<td>$q$</td>
<td>Success rate</td>
</tr>
<tr>
<td>$H(q)$</td>
<td>Likelihood of an individual consumer signing on a group-buying deal given his or her success rate $q$</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>Time discounting factor in period $i$</td>
</tr>
<tr>
<td>$a$</td>
<td>Surplus of the non-purchase option for luxury goods</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>Realized type of cohort $i$</td>
</tr>
<tr>
<td>$s_i(\theta_i)$</td>
<td>Decision rule that cohort $i$ uses to decide the number of sign-ups under simultaneous mechanism given the realized type $\theta_i$</td>
</tr>
<tr>
<td>$q^*$</td>
<td>Equilibrium success rate under simultaneous mechanism</td>
</tr>
<tr>
<td>$S_i(\theta_i)$</td>
<td>Decision rule that cohort $i$ takes to decide the number of sign-ups under sequential mechanism given the realized type $\theta_i$</td>
</tr>
<tr>
<td>$Q^*_i$</td>
<td>Equilibrium success rate under sequential mechanism when cohort $i$ moves first</td>
</tr>
</tbody>
</table>

The success of a group-buying mechanism depends on consumers’ sign-up decisions. It is thus essential for us to first understand the individual consumer’s decision process and the determinants for their signup decisions. Next we provide a preliminary analysis of consumer decisions to glean some insights that can help us understand the equilibrium results to be presented in the following sections.
2.1. Preliminary Analysis of Consumer Decisions

We denote by $H(q)$ the likelihood of an individual consumer signing on to a group-buying deal where $q$ is the success rate the consumer expects. In this notation, we emphasize that the likelihood of signing up depends on the expected success rate, and suppress its dependence on the characteristics of the deal and the individual consumer. (To minimize the notation complexity, here we omit the subscripts for consumer identity and arrival time.) The success rate for a group-buying deal depends, not only on the number of consumer arrivals in each period, but also on their product valuations. For our analysis, we assume two very general properties for the signing-up probability function $H(q)$.

**Assumption 1 (Signing Up Likelihood).** For $q \in [0, 1]$, we assume

(i) $H(q = 0) = 0$;

(ii) $H(q)$ is non-decreasing in $q$.

Assumption 1(i) states that if a consumer expects with certainty that the deal will be off, then he or she expects zero benefits from signing on to the deal and will definitely not sign up. Assumption 1(ii) indicates that the higher the likelihood that a consumer expects the group-buying deal to be on, the higher the probability that the consumer will sign up.

To gain granularity on how exactly consumers’ purchase decisions may depend on the success rate, we define and examine two distinct types of products or services that fit into our framework: necessity goods and luxury goods. We define necessity goods as those products considered essential for a certain consumption purpose or necessary to daily life. More specifically, if a consumer does not buy from the group-buying deal, he or she will buy the product elsewhere, e.g., from a local store. In contrast, we define luxury goods as those products and services that provide hedonic experiences. Examples of such luxury goods might be a dinner at an expensive restaurant, an hour of pampering at a fancy spa, or a donation to a music project proposed by a remote museum. Note that, first, we do not follow the standard income-elasticity approach for defining necessity versus luxury goods. However, our definitions seem consistent with the standard approach. Second, we merely define these two categories for purpose of illustration and our results are by no means limited to these special cases. For instance, we can define the expected individual donation amount function $H(q)$ as a function of the project’s success rate $q$, for donors at Kickstarter according to the distribution of potential donation amount and the characteristics of the proposed project. As long as Assumption 1 is satisfied for $H(q)$ which can be a likelihood function in the Groupon’s case or a general function in the Kickstarter’s case, all results in the subsequent sections apply except for those specifically related to the Groupon context.
Example 1 (Necessity Goods). Consumers sign up for group-buying deals of necessity goods to obtain the desired product at a bargain price, but take the risk that the deal may not be successful and they will have to go elsewhere to buy the same product at a regular price and with a delay. For example, a consumer shopping around for a camcorder before a family vacation may want to sign up for a group-buying bargain if time permits, making a trade-off between the buying from a local store with the ability to enjoy it right away versus waiting for the bargain to be realized at a later time with a possibility that the deal may not be successful. In such a case, when a consumer arrives at a group-buying deal, he or she will sign up if and only if the present value of the expected surplus resulting from signing on to the group-buying deal is no less than the present value of buying the same product immediately through another channel. The expected surplus from signing up to a group-buying deal has two components: if the deal turns out to be successful, the consumer enjoys the product at the bargain price \( w \); otherwise, the consumer purchases the product elsewhere at a later time after the deal has expired. The time discounting factor used in calculating the present value, denoted by \( \rho_t \), depends on the specific time \( t \) of arrival. For a consumer with a success rate \( q \) and an individual product valuation \( v \), the mathematical form of this trade-off statement is

\[
[q(v - w) + (1 - q)(v - p)] \rho_t \geq (v - p).
\]

From the above condition we can derive the range of consumers’ valuation under which they will sign up for the group-buying deal as follows:

\[
w \leq v \leq p + \frac{\rho_t}{1 - \rho_t} (p - w)q.
\]

If a consumer has a product valuation higher than \( p + \frac{\rho_t}{1 - \rho_t} (p - w)q \), then the consumer will resort to immediate access to the product through the alternative channel instead of going after the group-buying deal. Let consumers arriving at time \( t \) have their valuations drawn from a continuous cumulative distribution function \( F \), we can then derive the probability of signing up to the group-buying deal as follows.

\[
H(q) = \begin{cases} 
0 & \text{if } q = 0, \\
F \left( p + \frac{\rho_t}{1 - \rho_t} (p - w)q \right) - F(w) & \text{if } 0 < q \leq 1,
\end{cases}
\]

which is non-decreasing in \( q \). As a standard feature of rational expectation equilibrium, consumer belief plays an important role in signup decisions. When a consumer believes that the deal will be off \( (q = 0) \), it is certain that he or she will not sign on.
From the seller’s perspective, $H(q)$ is the signing-up likelihood for an individual consumer. The market for necessity goods tends to be very competitive because retailers may have already enjoyed economies of scale. As a result, the benefit of group-buying deals over purchasing from a retailer as measured by price discount $p - w$ is likely to be limited, and therefore the individual signing-up likelihood is low. Later we will show that the expected low individual signing-up likelihood may spread the reduce other consumers’ confidence in the deal’s success, eventually leading to a low success rate of group buying for necessity goods.

**Example 2 (Luxury Goods).** Consumers may budget some of their discretionary income for purchases of luxury goods, with reservation values for the expected benefits of such purchases. More specifically, we assume that consumers will buy luxury goods only if the present value of the expected surplus from signing on to a group-buying deal exceeds a predetermined threshold $a \geq 0$, which is the surplus of the non-purchase option. In practice, the luxury goods sold through group-buying sites are usually the “try-it-for-the-first-time” products or services. We assume the same value of outside option for all consumers within a cohort. (This assumption can be easily relaxed by incorporating the heterogeneous outside option values into the distribution of consumers’ valuation.) The expected surplus of signing up has two components: if the deal turns out to be successful, the consumer enjoys the product at price $w$; otherwise, the consumer takes the no-purchase option at a later time after the deal turns off. Consumers’ purchase decisions upon arrival at time period $t$ for this type of products can be characterized as

$$[q(v - w) + (1 - q)a] \rho_t \geq a.$$

In the case of buying luxury goods through group-buying discounts, the regular price of this type of product or service is usually higher than the valuation of consumers. In other words, for the target consumers, we have $v - p < a$, which indicates that the consumers would otherwise not purchase the luxury goods at the regular price. We can then derive the range of consumers’ valuation of signing up to the group-buying deal as:

$$\frac{a[1 - (1 - q)\rho_t]}{q \cdot \rho_t} + w \leq v < p + a.$$

Given the distribution of valuations $F$ for consumers arriving at time period $t$, we can specify the sign-up probability for luxury goods as

$$H(q) = \begin{cases} 0 & \text{if } q = 0, \\ F(p + a) - F\left(\frac{a[1 - (1 - q)\rho_t]}{q \cdot \rho_t} + w\right) & \text{if } 0 < q \leq 1, \end{cases}$$
which is non-decreasing in $q$. Again, if a consumer believes with certainty that the deal will not be on ($q = 0$), he or she will not sign on. Note that $a[1 - (1 - q)\rho_1] \geq 0$ is the loss of surplus from signing up to a deal that is off at the end. The larger such a loss, the less likely it is that a consumer will sign on to the deal.

3. Model Analysis and Results

In this section we analyze ex ante success rates of group-buying deals offered by the firm. We start with equilibrium analysis under the simultaneous mechanism where the firm does not post the number of sign-ups after the first period. We then move to an analysis of the sequential mechanism where at the beginning of the second period the firm posts the number of sign-ups from the first period. Finally, we compare the ex ante success rates under these two mechanisms and provide comparative statics.

3.1. Equilibrium Analysis under Simultaneous Mechanism

When the firm adopts the simultaneous mechanism, neither cohort of consumers are informed of sign-up decisions of another cohort. As a result, consumers in each cohort base their sign-up decisions on their beliefs on the size and valuations of another cohort. According to the model setup, individual consumers within each cohort share the same belief about the deal’s success rate by resolving any information asymmetry within the cohort. The game is a Bayesian game in the Harsanyi sense (Harsanyi 1968) where types are defined by cohort sizes and valuations. Specifically, the type of a cohort $i$ is defined by its realized size $m_i$ and realized valuation vector $\vec{v}_i(m_i) = (v_{i1},\ldots,v_{im_i})$ of the $m_i$ members, which is private information known to the cohort itself but not to the other cohort. For notational simplicity, we denote by $\theta_i = (m_i, \vec{v}_i(m_i))$ the realized type of cohort $i$. Similarly, we denote by $\Theta_i = (M_i, \vec{V}_i(M_i))$ the corresponding random variable of cohort $i$’s types, whose distribution is public information. For any cohort $i$, $s_i(\theta_i)$ denotes the decision rule that cohort $i$ takes to decide the number of sign-ups given the realized type of cohort $i$ being $\theta_i$. Given cohort $i$’s sign up decision $s_i(\theta_i)$ and the other cohort $-i$’s sign up decision $s_{-i}(\theta_{-i})$, the total surplus of cohort $i$ is denoted as $\pi_i(s_i(\theta_i), s_{-i}(\theta_{-i}))$.

Definition 1 (Bayesian Equilibrium). The Bayesian equilibrium $\{s^*_i(\theta_i)\}$ to the simultaneous game is defined by the best-response strategy played by each cohort $i$

\[
s^*_i(\theta_i) \in \arg\max_{s_i(\theta_i)} E_{\Theta_{-i}} \{ \pi_i(s_i(\theta_i), s^*_{-i}(\Theta_{-i})) \}.
\]

The existence of Bayesian equilibria follows standard arguments. When making signup decisions under the simultaneous mechanism, members in one cohort face the same uncertainty in the deal’s
success rate as those in the other cohort. The solution scheme for this type of game is a threshold strategy: given a realized cohort size $m_i$, there exists the same valuation range for every consumer in cohort $i$ such that any consumer signs up if and only if its valuation falls into the range. Consumers within the same cohort follow the same strategy because they hold the common belief about the deal success rate.

At the beginning of the game, the ex ante belief of the firm on the success rate under a Bayesian equilibrium $\{s_i^*(\theta_i)\}$ can be characterized by

$$q^* = P \left( \sum_i s_i^*(\Theta_i) \geq N \right),$$

which is the success rate at equilibrium. The success rate is of particular interest to group-buying sites because the revenue of group-buying sites depends on there being no fewer subscribers than the minimum requirement. From this point on, we focus solely on the success rate from the seller’s point of view when comparing different mechanisms. Following the preceding discussion, we can characterize the success rate under the simultaneous mechanism as follows.

**Lemma 1 (Success Rate of Simultaneous Mechanism).** The success rate $q^*$ at equilibrium can be characterized by

$$P \left( \sum_{k=1}^{M_1} X_{1k}(q) + \sum_{k=1}^{M_2} X_{2k}(q) \geq N \right) = q,$$

where $X_{ik}(q)$, $i = 1, 2$, all $k$, are independent, $X_{1k}(q)$, all $k$, are identically distributed Bernoulli random variables with success probability $H_1(q)$, and $X_{2k}(q)$, all $k$, are identically distributed Bernoulli random variables with success probability $H_2(q)$.

Equation (1) characterizes equilibria in the sense that any Bayesian equilibrium results in an equilibrium belief $q^*$ which satisfies the equation, and any $q^*$ that satisfies the equation corresponds to a Bayesian equilibrium where each cohort behaves as if the success rate is $q^*$. The existence of such an success rate $q^*$, and equivalently, the existence of a Bayesian equilibrium in pure strategies, is a direct consequence of Tarski’s Fixed Point Theorem, since the left-hand-side of Equation (1) is non-decreasing in $q \in [0,1]$.

However, there may exist multiple equilibria: to minor perturbation in belief, some may be robust and others may be sensitive. To show how the computation of equilibria works, let us consider a concrete example of a deal selling luxury goods. Assume that consumers’ valuations are homogeneous, and uniformly distributed between $[\underline{v}, \overline{v}]$. For simplification, we set the time
discounting factor $\rho = 0.5$. Recall how consumers make decisions when facing luxury goods, the signup probability for each individual consumer is given by

$$H(q) = \frac{1}{\overline{v} - \underline{v}} \max \left[ \min (\overline{v}, p + a) - \max \left( \underline{v}, \frac{a(1 + q)}{q} + w \right), 0 \right].$$

Let us specify the upper bound and lower bound of consumers’ valuations to be $\underline{v} = $1 and $\overline{v} = $30, respectively. Similarly, we assume that the size of cohort $i$ follows a uniform distribution with a discrete support from 1 to 200. Additionally, we choose the value of parameters as $p = $30, $w = $16 and $N = 50$. These values are typical in the current practice of online group buying and the discount level of this deal is equivalent to 47% off. We also set the surplus of the non-purchase option to $a = $2. The equilibria of this example are illustrated in Figure 1. The solid line in the figure is a plot of the left-hand-side of Equation (1), which maps the belief $q$ in the success rate to the expected outcome $\hat{q}$ of the success rate if consumers make signup decisions based on the belief $q$. Based on Lemma 1, equilibria are characterized by the crossing points of the mapping with the 45 degree line. At the equilibrium belief, consumers optimally make signup decisions that leads to the same success rate on expectation as originally believed.

It is useful to work through the details of why values of $q$ other than 0, $q'$ and $q''$ do not constitute equilibria. Suppose that the shared expectation of the deal’s success rate is $\tilde{q}$ that is not one of the three equilibrium values.

- If $\tilde{q}$ is between 0 and $q'$, then there is a “downward pressure” on the success rate: consumers expect the deal will succeed with probability $\tilde{q}$, but then the outcome underperforms this expectation. In this case, consumers with valuation $a/\tilde{q} + w$ will value the non-purchase option more than the deal and hence will wish they had not signed up at the very beginning. This would push the demand downward and consequently decrease the success rate.

- If $\tilde{q}$ is between $q'$ and $q''$, then there is an “upward pressure” on the success rate: consumers expect the deal will succeed with probability $\tilde{q}$, but then the outcome outperforms this expectation. Consequently, consumers with valuations close to but less than $a/\tilde{q} + w$ have not signed up for the deal but will wish they had. This would drive the success rate upward.

- If $\tilde{q}$ is above $q''$, then there is again a downward pressure, which is similar to the first case.

In summary, there exists a downward pressure for those beliefs that lead to an outcome that is lower than the original belief, and there exists an upward pressure for those beliefs that lead to an outcome that is higher than the original belief.

Different properties of the non-equilibrium belief values lead to distinct properties of the equilibria. Consider the vicinity of the equilibrium point $q^* = q'$. If consumers expect the success rate to be
slightly higher than \( q' \), say \( q' + \epsilon \), the realized success rate will be higher than consumers’ expectation since the solid line is above the 45 degree line in the interval \( (q', q' + \epsilon] \). The higher-than-success rate will adversely boost consumers’ confidence, thus yielding an even higher success rate. That is, with a positive perturbation of deal success rate from \( q' \) to \( q' + \epsilon \), the “upward pressure” will drive the success rate away from \( q' \) toward the higher equilibrium \( q^* = q'' \). Similarly, if consumers expect the success rate to be slightly lower than \( q' \), then the “downward pressure” will drive the success rate away from \( q' \) in the other direction, down toward the equilibrium \( q^* = 0 \). In summary, if the success rate slightly deviates from \( q' \), the outcome will spiral up or spiral down to a significant extent. However, the other two equilibria, \( q^* = 0 \) and \( q^* = q'' \) show a strong stability property. We observe from Figure 1 that 0 is an equilibrium, which is generally true due to Assumption 1(i). It is self-fulfilling that the deal will fail if every individual is convinced that the deal will not take off. However, the equilibrium of \( q^* = 0 \) is trivial and is not of our particular interest.

Following the preceding discussions, we can categorize equilibria into two sets depending on their sensitivity to minor perturbation in belief. In fact, \( q' \) is a tipping point in the sense that if the group-buying site can convince consumers the success rate exceeds \( q' \), then they may drive the success rate up to the even higher stable equilibrium at \( q'' \).

**Definition 2 (Stable Equilibrium and Tipping Point).** A non-zero equilibrium belief \( q^* \) is a stable equilibrium (tipping point) if there exists \( \epsilon' > 0 \) such that for all \( \epsilon \in (0, \epsilon') \), when all...
consumers make decisions with the belief $q^* - \epsilon$, the deal success rate will be higher (lower) than $q^* - \epsilon$; when all consumers make decisions with the belief $q^* + \epsilon$, the deal success rate will be lower (higher) than $q^* + \epsilon$.

3.2. Equilibrium Analysis under Sequential Mechanism

Under the sequential mechanism, at the beginning of the second period the firm posts the number of consumers who signed up in the first period. Since two cohorts of consumers make decisions sequentially, the first cohort needs to take into consideration the second cohort’s decisions to form an expectation of the success rate. The sequential game we analyze follows the concept of rational expectations (RE) equilibrium. Specifically, the realized deal’s success rate resulting from the collective actions of both cohorts should be consistent with the success rate that is originally believed by individuals in each cohort.

Definition 3 (RE Equilibrium). For any realization $\theta_i = (m_i, \vec{v}_i(m_i))$ of size and valuations for cohort $i$ who moves first, an RE equilibrium $Q^*_i(\theta_i)$ to the sequential game satisfies: (i) Cohort $i$ plays an optimal strategy $S^*_i(\theta_i)$ given belief $\hat{Q}_i(\theta_i)$ about the success rate; (ii) Given the number of sign-ups $S^*_i(\theta_i)$ from cohort $i$ and any realization $\theta_{-i} = (m_{-i}, \vec{v}_{-i}(m_{-i}))$ of size and valuations, cohort $-i$ plays a best-response strategy $S^*_{-i}(\theta_{-i}; S^*_i(\theta_i))$; (iii) The belief is consistent with the equilibrium outcome: $\hat{Q}_i(\theta_i) = Q^*_i(\theta_i) = P(S^*_i(\theta_i) + S^*_{-i}(\theta_{-i}; S^*_i(\theta_i)) \geq N)$.

Without loss of generality, let us assume that cohort 1 arrives in the first period and cohort 2 arrives in the second period. The sequence of move is fixed for now, and comprehensive discussion about endogenous sequencing is presented in Section 4.4. We solve the equilibrium behavior by using backward induction. Note that when consumers in the second cohort make decisions, there is no more uncertainty about the future. Given the number of sign-ups from the first cohort revealed as $n_1$, the optimal strategy for any individuals in the second cohort can be characterized by a valuation range that depends on its realized size: to sign up if and only if (a) one’s valuation falls into the range for signing up with the success rate being 1 and (b) the total number of consumers in cohort 2 satisfying condition (a) is no less than $N - n_1$. In other words, it is optimal for consumers in the second cohort to do the following: assume in advance that the deal will definitely be successful, and then count and check if the potential subscribers are no fewer than $N - n_1$; if this is the case, the potential subscribers fulfill the expectation of a successful deal by signing up; otherwise, nobody in the second cohort signs up and the deal is off. After solving the best response from the second period, we move backwards to the first period when the first cohort makes signup decisions. For each realization $\theta_1$ of size and valuations of the first cohort, there exists a self-enforced belief $Q^*_1(\theta_1)$ about the success rate, given the best response from the second cohort.
At the beginning of the first period, the firm expects the success rate at a RE equilibrium $Q^*_1(\theta_1)$ to be $Q^*_1 = E_{\theta_1}[Q^*_1(\Theta_1)]$. We can characterize the success rate at the equilibrium strategy by the following result, which is informative for the purpose of comparison.

**Lemma 2 (Success Rate of Sequential Mechanism).** Suppose cohort $i$ arrives in the first period and cohort $-i$ arrives in the second period. From the seller’s perspective, the success rate $Q^*_i$ at equilibrium can be characterized by

$$P\left(\sum_{k=1}^{M_i} X_{ik}(q) + \sum_{k=1}^{M_{-i}} X_{-ik}(1) \geq N\right) = q,$$

(2)

where $X_{ik}(q), X_{-ik}(1), \forall k$, are independent, $X_{ik}(q), \forall k$, are identically distributed Bernoulli random variables with success probability $H_i(q)$, and $X_{-ik}(1), \forall k$, are identically distributed Bernoulli random variables with success probability $H_{-i}(1)$.

Again, by Assumption 1(ii), the existence of such a success rate $Q^*_i$ at a RE equilibrium is a direct consequence of Tarski’s Fixed Point Theorem. It is also noteworthy that multiple equilibria may arise and that categorization specified in Definition 2 applies to these equilibria as well. Unlike the simultaneous mechanism, 0 is an equilibrium belief under the sequential mechanism if and only if the second cohort is short of potential subscribers and it is the only cohort to sign up for the group-buying deal, namely, $P(\sum_{k=1}^{M_{-i}} X_{-ik}(1) < N) = 1$.

### 3.3. Equilibrium Mechanism: Simultaneous or Sequential?

Given the potential presence of multiple equilibria, how can we compare the success rate of simultaneous and sequential mechanisms? In this paper, we use the approach proposed in Jackson and Yariv (2007) because it allows us to compare the set of equilibria regardless of multiplicity. We formalize our criteria for “better” mechanism by the following definition.

**Definition 4 (Higher Success Rates).** One scenario generates higher success rates than another if, for any stable equilibrium of the latter, there exists a higher stable equilibrium of the former, and for any tipping point of the latter there is a lower tipping point of the former or no lower tipping points of the former at all.

Consider two scenarios under the simultaneous mechanism whose equilibria are as depicted in Figure 2(a). The scenario corresponding to the solid line is the same as the example introduced in Section 3.1. The scenario corresponding to the dashed line differs from the benchmark example only in the group-buying price. That is a lower group-buying price $w' = $10 in the new scenario compared with $w = $16 in the original example. We observe from the figure that the scenario with a higher group-buying price has a higher tipping point $A$ and a lower stable equilibrium $B$ than...
the tipping point $A'$ and the stable equilibrium $B'$ in the other scenario with a more attractive group-buying price. The reason why the latter is better is twofold: (1) As the tipping point is lower, it is more likely that consumers’ expectation will go beyond it and then the self-enforced upward pressure will drive up belief; (2) When the success rate is eventually stabilized at equilibrium, the success rate is always reached at a higher level.

In Figure 2(b), we compare the benchmark example with another scenario in which the seller applies the sequential mechanism and all other parameters remain the same. The distinct feature of this scenario is that it has a higher stable equilibrium $B'$, but no tipping points. Applying the same reasonings as above, we conclude that this scenario of the sequential mechanism is more prone to a deal success than the benchmark scenario of the simultaneous mechanism that has a lower stable equilibrium $B$ and one tipping point $A$. We will show that this result holds in general when comparing simultaneous mechanisms with sequential mechanisms.

Figure 2  Comparison between various scenarios with a benchmark scenario as depicted in Figure 1.

(a) Existence of a lower tipping point and a higher stable equilibrium.  
(b) No tipping points but a higher stable equilibrium.

Note. The solid lines in subfigures (a) and (b) are the benchmark scenario when the seller applies the simultaneous mechanism, $V_i$ and $M_i$, $i = 1, 2$, follow uniform distributions with supports $[1, 30]$ and from 1 to 200, and $p = 30, w = 16, N = 50, \rho = 0.5, a = 2$. The dashed line in subfigure (a) differs from the benchmark only in that $w' = 10$. The dashed line in subfigure (b) differs from the benchmark only in that the seller applies the sequential mechanism.

To facilitate mechanism comparison, we present the following lemma.
Lemma 3 (Scenario Comparison). Suppose $X_1(q)$ and $X_2(q)$ are random variables parameterized by $q \in [0,1]$. If $X_1(q) \geq_{st} X_2(q)$ for all $q \in [0,1]$, then $P(X_1(q) \geq N) = q$, $q \in [0,1]$ yields higher success rates than $P(X_2(q) \geq N) = q$, $q \in [0,1]$.

The notation $\leq_{st}$ stands for the usual stochastic order (Shaked and Shanthikumar 2007). Let $X$ and $Y$ be two random variables such that $P(X > x) \leq P(Y > x)$ for all $x$, then $X \leq_{st} Y$. The proof of Lemma 3 is straightforward, and thus it is omitted. The basic idea is that if a function, say $f(q)$ always yields a higher value than another function $\tilde{f}(q)$ for all $q$, for any left-to-right cross point with 45 degree line of the latter, there always exists a higher one of the former, and for any right-to-left cross point of the latter, there exists either none or a lower one of the former.

Lemma 3 sets the stage for the comparison of simultaneous and sequential mechanisms. In all of the following comparisons between mechanisms (simultaneous or sequential) or sequences of move under the sequential mechanism, we assume the timing of each cohort’s decision making is about the same, no matter under which mechanism or which sequence to follow. Hence the individual consumer’s signing-up likelihood function $H_i(q)$ in each cohort given the same success rate $q$ is the same regardless of any mechanism or move sequence. This is a reasonable assumption when the time discounting does not play a big role as in today’s daily-deals practice.

We start by considering the following stochastic order:

$$
\sum_{k=1}^{M_i} X_{ik}(q) \leq_{st} \sum_{k=1}^{M_i} X_{ik}(1), q \in [0,1], \quad i = 1, 2.
$$

Because of the non-decreasing property of $H(q)$, it is easy to verify that the stochastic orders above hold. Due to Lemma 3, we immediately obtain the following proposition by comparing the left-hand-side of Equation (1) and (2).

Proposition 1 (Mechanism Comparison for Two Cohorts). Given all others the same, the sequential mechanism always yields a higher success rate than the simultaneous mechanism.

The driving force behind this result is that in the simultaneous mechanism, each cohort faces uncertainty about the other cohort when making decisions. However, in the sequential mechanism, the second cohort decides after the uncertainty about the first cohort has been resolved; moreover, in our setting, the information asymmetry within the second cohort can be eliminated by communication. In anticipation that the second cohort will sign up to the deal without discounting belief in the success rate, the first cohort’s confidence about the future success rate is consequently boosted. This result may explain why the first wave of group-buying websites, which often used simultaneous mechanisms, failed while the second wave has been more profitable.
3.4. Comparative Statics for Sequential Mechanism

In keeping with the prevalence of adoption of sequential mechanisms in the current group-buying sites, we have shown that the sequential mechanism always yields higher success rates than does the simultaneous mechanism. But how does the success rate vary across different types of products? Which products are more suitable for the group-buying selling format? We now conduct comparative statics analysis on the success rate under the sequential mechanism with the moving sequence exogenously determined.

Consider signing up probability functions $H_i(q)$ and $\tilde{H}_i(q)$, which are sustained by two distinct group-buying deals. Suppose that $\tilde{H}_i(q) \geq H_i(q)$ for all $q \in [0,1]$ and $i = 1, 2$, then we have

$$\sum_{k=1}^{M_i} X_{ik}(q) \leq \sum_{k=1}^{M_i} \tilde{X}_{ik}(q), \quad q \in [0,1], \quad i = 1, 2.$$  

Recall that if cohort $i$ moves first, the success rate $Q_i^*$ at equilibrium given the signing up probability function $H$ can be characterized by

$$P \left( \sum_{k=1}^{M_i} X_{ik}(q) + \sum_{k=1}^{M_{-i}} X_{-ik}(1) \geq N \right) = q,$$

where $X_{ik}(q)$, $X_{-ik}(1)$, all $k$, are independent, $X_{ik}(q)$, all $k$, are identically distributed Bernoulli random variables with success probability $H_i(q)$, and $X_{-ik}(1)$, all $k$, are identically distributed Bernoulli random variables with success probability $H_{-i}(1)$. Similarly, the success rate $\tilde{Q}_i^*$ at equilibrium with $\tilde{H}$ can be characterized by

$$P \left( \sum_{k=1}^{M_i} \tilde{X}_{ik}(q) + \sum_{k=1}^{M_{-i}} \tilde{X}_{-ik}(1) \geq N \right) = q,$$

where $\tilde{X}_{ik}(q)$, $\tilde{X}_{-ik}(1)$, all $k$, are independent, $\tilde{X}_{ik}(q)$, all $k$, are identically distributed Bernoulli random variables with success probability $\tilde{H}_i(q)$, and $\tilde{X}_{-ik}(1)$, all $k$, are identically distributed Bernoulli random variables with success probability $\tilde{H}_{-i}(1)$.

Given the discussions above, the following proposition is an immediate result of Lemma 3.

**Proposition 2 (Comparative Statics).** Consider $\tilde{H}$ and $H$ corresponding to two group-buying deals. If $\tilde{H}_i(q) \geq H_i(q)$ for all $q \in [0,1]$ and $i = 1, 2$, then in the sequential mechanism the deal $\tilde{H}$ yields higher success rates than the deal $H$.

Proposition 2 simply states that when it is more likely for each individual consumer to sign up the group-buying deal, the collective number of subscribers as a whole after the sequential signing up will be more likely to exceed the threshold. In other words, higher individual-level signup
probability aggregates sequentially to a larger number of total sign-ups. The result in Proposition 2 implies that one can compare the success rates of two group-buying deals by simply comparing the individual signup probability. To derive more specific insights, we resort to two special product categories – the necessity and luxury goods with their sign-up likelihood function $H(q)$ specified as in Examples 1 and 2. We summarize the results in the following corollaries, which are proved in appendix.

**Corollary 1 (Time Discounting).** Consider either necessity goods or luxury goods. A deal $\tilde{H}$ yields higher success rates than another deal $H$ in the sequential mechanism, if, ceteris paribus, the deal $\tilde{H}$ has higher discount rates (less discounting) than those of the deal $H$ for each cohort, i.e., $\tilde{\rho}_i \geq \rho_i$, $i = 1, 2$.

Corollary 1 reveals that the success rate is higher if the products are of less immediate need. Because of the distinctiveness of each type of goods, luxury goods are arguably more suitable for group buying since consumers are more tolerant of waiting for the goods not considered to be necessities. On the other hand, shorter deal duration can also help boost the success rate since consumers’ potential payoff is less discounted by waiting. This may partially explain why we observe lower success rates among the earlier wave of group-buying deals that usually lasted for a week or 10 days. Meantime, today’s successful firms like Groupon usually offer deals that expire within 24 hours. This view is echoed in a speech by Groupon’s CEO Andrew Mason in Kelsey Marketplaces Conference 2010.

“I thought about why collective buying sites had failed in the past... Mercata was from the dot-com era. The more people bought, the lower the price would go. The trouble was it took a week to get enough people to drive the price down. They might buy a camera, but they’d have to wait a week.”

**Corollary 2 (Price Discount).** Consider either necessity goods or luxury goods. A deal $\tilde{H}$ yields higher success rates than another deal $H$ in the sequential mechanism, if, ceteris paribus, the consumer valuations for two deals are identically uniformly distributed and the deal $\tilde{H}$ has a higher regular price and a lower group-buying price than those of the deal $H$, i.e., $\tilde{p} \geq p$ and $\tilde{w} \leq w$.

Corollary 2 suggests that as a group-buying deal becomes more attractive with a lower discounted price or a deeper discount, consumers are more willing to take the risk of signing up to the deal in anticipation of getting the product for a substantially lower price. Recall that, for the necessity
goods, \( H_i(q) \) is in the form of \( H_i(q) = F_i \left( p + \frac{\rho_i}{1 - \rho_i} (p - w)q \right) - F_i(w) \) for \( q \in (0, 1] \). The length of the valuation interval where the consumers sign up for group buying is

\[
\left[ p + \frac{\rho_i}{1 - \rho_i} (p - w)q \right] - w = (p - w) \frac{1 - (1 - q)\rho_i}{1 - \rho_i}.
\]

For the luxury goods, \( H_i(q) \) is in the form of \( H_i(q) = F_i(p + a_i) - F_i \left( \frac{a_i[1 - (1 - q)\rho_i]}{q \cdot \rho_i} + w \right) \) for \( q \in (0, 1] \). The length of the valuation interval where the consumers sign up for group buying is

\[
(p + a_i) - \left( \frac{a_i[1 - (1 - q)\rho_i]}{q \cdot \rho_i} + w \right) = (p - w) - \frac{a_i(1 - \rho_i)}{q \cdot \rho_i}.
\]

The number of potential consumers captured by group buying depends directly on the discount level for both types of goods. In reality, the discount \( p - w \) can be small for necessity goods because the regular price is already very competitive. It is hard for group-buying websites to offer a lower price than what is already available at Wal-Mart or BestBuy, since these companies have already had buying power and enjoyed high level of economies of scale.

However, the typical discount level offered by current group-buying firms, like Groupon and Livingsocial, is 50% off regular prices. The reasons why the vendors are willing to offer such a high discount are economies of scale and advertising: (1) these vendors usually provide luxury services, like spa and teeth whitening, where the fixed cost is high but the variable cost for each additional consumer is relatively low; (2) they are willing to sacrifice outright profit in the hope of attracting new consumers who will return regularly. With such substantial discounts, the number of consumers captured by group buying can be fairly large. Combining the results from the above two corollaries, we conclude that necessity goods are less suitable than luxury goods for selling under the group-buying format.

**Corollary 3 (Non-Purchase Option).** Consider the luxury goods. A deal \( \tilde{H} \) yields higher success rates than another deal \( H \) in the sequential mechanism, if, ceteris paribus, the consumer valuations for two deals are identically uniformly distributed and the environment \( \tilde{H} \) has higher sum of the regular price and non-purchase value and lower non-purchase value than those of the deal \( H \) for each cohort, i.e., \( \tilde{p} + \tilde{a}_i \geq p + a_i \) and \( \tilde{a}_i \leq a_i \), \( i = 1, 2 \).

It is intuitive that with a lower non-purchase value, the deal is more likely to succeed. However, this is not the entire story. The success of deals for luxury goods also depends on the regular prices of the goods since a lower regular price may entice consumers towards a regular purchase. Consequently, higher regular prices along with lower non-purchase values are more likely to contribute to a successful group-buying deal for luxury goods.
4. Model Extensions

In this section, we present several extensions to our basic model.

4.1. Strategic Consumers

Under the sequential mechanism, early arrivals who did not sign up in the first period may return to the deal later when there is less or no uncertainty in the deal’s success. Consumers whose valuations are in the intermediate range may be willing to buy the goods at the group-buying price if the deal is on but may not sign up to the deal upon arrival when the future of the deal is still ambiguous. Rather than turning to the alternative channel for an immediate purchase, they may return later and make decisions again on the basis of the updated cumulative number of sign-ups. We incorporate consumers’ strategic waiting into our basic model to examine the comparison between the sequential and simultaneous mechanism. To do this, we add a “last-minute” period at the end of the time horizon. There are still two cohorts of consumers, indexed by \( i = 1, 2 \): one arriving in the first period, and the other in the second period. As in the basic model, upon arrival an individual consumer in the two cohorts will sign on to the deal if and only if he or she expects a discounted utility from the deal at least as high as that from not signing on to the deal. In this extension, we allow those consumers who do not sign on in the first period to come back in the last minute and make decisions again according to the updated cumulative number of sign-ups from the two periods. Consumers in the second cohort will know that there may be some returners from the first period in the last minute, but the timing does not allow those who do not sign up in the second period to make a different decisions on the basis of the updated last-minute signups.

Unlike the basic model, in which we can characterize the success rate at the equilibrium strategy by a single equation, we need to use more complicated dynamic programming to characterize the success rate in this case, belief the belief held by the second cohort is a consequence of both the expected number of sign-ups in the last minute and the cumulative number of sign-ups in the first period. Nevertheless, we can make a simple sample path comparison and still obtain the dominance of the sequential mechanism over the simultaneous mechanism even with the existence of strategic waiting.

Note that the existence of strategic consumers does not affect the success rates under simultaneous mechanism. Based on Proposition 1, it suffices to show that the sequential mechanism always yields higher success rates with strategic consumers than with no strategic consumers. To establish that one scenario yields higher success rates than another, it suffices to show that given the same ex ante belief in success rates for the firm, which is also the belief held by the first cohort upon arrival, the realized success rates in the former scenario are always higher than those in the latter.
Suppose the belief held by the first cohort upon arrival is the same in both scenarios – the sequential mechanism with and without strategic consumers. If an individual’s valuation falls into the parameter range so that the individual should sign up with the success rate of one, then we define the individual sign-up condition without uncertainty be satisfied. The number of sign-ups from the first cohort upon arrival should be the same in both scenarios. Now let us consider two types of sample paths (in terms of number of sign-ups over time) after the first cohort makes initial decisions. (1) In the second period, the total number of consumers satisfying the individual sign-up condition without uncertainty is more than or equal to the minimum number of signups required for the deal to be on. If this is the case, the deal will succeed in both scenarios. (2) In the second period, the total number of consumers satisfying the individual sign-up condition without uncertainty is less than the minimum required number. In this case, it is certain that the deal will fail in the scenario with no strategic consumers. However, with the existence of strategic consumers from the first cohort, some sample paths in which the number of sign-ups from those strategically waiting consumers in the last minute is enough to fill the gap between the threshold and the cumulative number of sign-ups in the first two periods, will lead to the success of the deal, while others still will not. Consequently, the success rates under the sequential mechanism with strategic consumers are always higher than those in the scenario with no strategic consumers, given that the belief held by the first cohort is identical. We summarize the result in the proposition below.

**Proposition 3 (Mechanism Comparison in the Existence of Strategic Consumers).** *Other things being equal, the sequential mechanism always yields higher success rates than the simultaneous mechanism with the existence of strategic consumers.*

**4.2. A Multi-Period Model**

So far we have used a two-period structure to model the fact that earlier arrivals may face more uncertainty in the deal’s success rate than later arrivals. To fully capture the dynamics of uncertainty, we extend our basic model to a multi-period model. For ease of exposition, we assume that exactly one consumer will arrive during each of \( m \) periods. Let \( V_i \) denote the valuation of the \((m - i)^{th}\) arrival, which is drawn from a given cumulative distribution function \( F_i(\cdot) \). Given the deal’s characteristics and the discount factor of each arrival, the likelihood of the \((m - i)^{th}\) arrival to sign up to the deal, as a function of the belief \( q \) on the deal’s success, is denoted by \( H_i(q) \). The valuation of each consumer is private information, but sign-up likelihood functions are public information.
**Sequential Mechanism.** Under the sequential mechanism, the firm reveals the cumulative number of sign-ups at the end of each period. Consumers arrive and make the sign-up decision sequentially on the basis of the cumulative number of sign-ups in all previous arrivals. We derive the ex ante success rate by dynamic programming as follows. Let \( Q_i(n) \) denote the ex ante success probability for the firm at the RE equilibrium when \( n \) more sign-ups needed to reach the preset threshold \( N \), and the number of consumers yet to arrive is \( i \). We use \( v_i \) to denote the realization of the \((m-i)\)\(^{th}\) arrival’s willingness-to-pay. Then, the success rate given the arrival’s valuation realized as \( v_i \) can be written recursively as

\[
Q_i(n; v_i) = \begin{cases} 
Q_{i-1}(n-1) & \text{if } v_i \in I_i(Q_{i-1}(n-1)), \\
Q_{i-1}(n) & \text{if } v_i \notin I_i(Q_{i-1}(n-1)),
\end{cases}
\]

where \( I_i(q) \) is a range of valuations for the \((m-i)\)\(^{th}\) arrival such that with his or her valuation falling into the range, the consumer is better off signing up to the deal with the belief in the deal’s success rate being \( q \). The recursive equation says if the \((m-i)\)\(^{th}\) arrival’s willingness-to-pay \( v_i \) is within the range of valuations such that the consumer is better off signing up to the deal with belief being \( Q_{i-1}(n-1) \), he or she indeed wants to sign up acting on this belief, and this belief is self-sustained since it needs only \( n-1 \) more sign-ups with \( i-1 \) periods to go. If his valuation \( v_i \) is not within the range \( I_i(Q_{i-1}(n-1)) \), \( v_i \) certainly falls outside the range of valuations \( I_i(Q_{i-1}(n)) \) because intuitively \( Q_{i-1}(n) \leq Q_{i-1}(n-1) \) and thus \( I_i(Q_{i-1}(n)) \) should be a subset of \( I_i(Q_{i-1}(n-1)) \); hence based on the belief \( Q_{i-1}(n) \), it is better off for the consumer not to sign up and this belief is indeed self-sustained because it still needs \( n \) more sign-ups with \( i-1 \) periods to go.

Taking expectation over \( v_i \), we have the ex ante success rate right before the \((m-i)\)\(^{th}\) arrival if the consumer follows RE equilibrium afterwards, as follows

\[
Q_i(n) = H_i(Q_{i-1}(n-1))Q_{i-1}(n-1) + [1 - H_i(Q_{i-1}(n-1))]Q_{i-1}(n). \tag{3}
\]

The boundary condition is \( Q_0(n) = 0 \) for all \( n > 0 \) (at the very end if the group buying slots have not been filled up) and \( Q_i(0) = 1 \) for all \( i \geq 0 \) (there are \( i \) customers yet to come and the slots have already been filled up). For the firm, the ex-ante success rate of a group-buying deal with threshold \( N \) before the consumers arrive is \( Q_m(N) \), which can be computed recursively by Equation (3).

**Simultaneous mechanism.** Under the simultaneous mechanism, the firm does not post the number of cumulative sign-ups with the result that consumers arrive and make decisions without knowing how other consumers behave. As under the basic model, we can characterize the ex ante success rate \( q_m(N) \) under the simultaneous mechanism as follows:

\[
P\left(\sum_{i=1}^{m} X_i \geq N\right) = q_m(N), \tag{4}
\]
where \( X_i \) is a Bernoulli random variable with parameter \( H_i(q_m(N)) \). Any consumer \( i \) (the \((m-i)\)th arrival) acts upon the belief of \( q_m(N) \) and signs up with probability \( H_i(q_m(N)) \). This belief is self-sustained at Bayesian equilibrium ensured by Equation (4).

**Mechanism Comparison.** Now we proceed to compare the ex ante success rates \( Q_m(N) \) and \( q_m(N) \) under the sequential and simultaneous mechanisms. For tractability, we consider a special case when the threshold is equal to the market size, i.e., \( m = N \). In this case, Equation (3) reduces to

\[
Q_i(i) = H_i(q_{i-1}(i-1))Q_{i-1}(i-1), \quad i = 1, 2, \ldots, N.
\]  

(5)

and Equation (4) reduces to

\[
q_N(N) = \prod_{i=1}^{N} H_i(q_N(N)),
\]

(6)

Unlike in the two-period model, where each cohort is composed of multiple individual consumers, the ex ante success rate under the sequential mechanism at RE equilibrium, \( Q_N(N) \) in this multi-period model is unique. This is given recursively by Equation (5). To conduct a fair comparison, we let \( q_N(N) \) be the largest root to Equation (6). We can prove by induction that the dominance of success rates under the sequential mechanism over the simultaneous mechanism is further confirmed in this multi-period model.

**Proposition 4 (Mechanism Comparison for Multi-Period Model).** Given all others being equal, the success rate under the sequential mechanism is always higher than or equal to the success rate under the simultaneous mechanism, i.e., \( Q_N(N) \geq q_N(N) \).

4.3. **Optimization of \( w \) and \( N \): Group Buying or Posted Price**

So far, our models assumed that the group-buying price \( w \) and the minimum threshold \( N \) are exogenous. However, in certain situations, the firm may have the power to choose these parameters. Selling through group-buying can be particularly beneficial when the firm faces cost structures with economies of scale. To simplify the analysis and also to compare group buying to the listed-price mechanism under economies of scale in Anand and Aron (2003), we assume that the firm is selling to two individual consumers. We consider the firm endogenizes the group-buying price and the minimum threshold requirement under the sequential mechanism because we want to compare the best group-buying mechanism with the posted-price mechanism. When \( N = 1 \), the group-buying mechanism is reduced to a posted-price mechanism. Each consumer buys at most one unit of the good, and makes purchase decisions according to the rule specified in the example of necessity goods. We denote the cost of supplying \( i \) units to be \( c_i \), \( i = 1, 2 \) after observing the demand. We
assume there are economies of scale at supply side, i.e., $0 \leq c_2 - c_1 \leq c_1$. This is the scenario of “production postponement” in Anand and Aron (2003).

Consumers’ valuations are assumed to be uniformly distributed with supports $[\underline{v}, \overline{v}]$. The distribution is common knowledge, but consumer’s own valuation is private. To ease the analysis, we assume the range of consumers’ valuations are sufficiently large that with positive probability, some consumers are not willing to pay for the good even at the group-buying price, and some consumers are better off buying the good from an alternative source and enjoying it right away. That is $\underline{v} \leq w$ and $\overline{v} \geq p + \frac{\rho}{1-\rho}(p-w)$. We have a constraint that the endogenous group-buying price needs to be no more than the exogenous regular price, i.e., $w \leq p$. Given the assumptions above, individual consumer’s sign-up probability is given by

$$H(q) = \frac{1}{\overline{v} - \underline{v}} \left[ p + \frac{\rho}{1-\rho}(p-w)q - w \right], \quad q \in [0,1)$$

and $H(1) = \alpha(p-w)$, where $\alpha = \frac{1}{\underline{v} - \overline{v}} \cdot \frac{1}{1-\rho}$, and the time discount factor is assumed to be identical between the two consumers.

The firm needs to choose between $N = 1$ and $N = 2$ and decide the optimal group-buying price $w^* \leq p$. When $N = 1$, the firm’s expected profit is

$$\pi_1(w) = 2H(1)(1-H(1))(w-c_1) + H(1)^2(2w-c_2)$$

$$= 2\alpha(p-w)[1-\alpha(p-w)](w-c_1) + \alpha^2(p-w)^2(2w-c_2).$$

In this case, consumers do not need to factor the behavior of other consumers into their purchase decisions. Consequently, a consumer’s decision is solely contingent on his or her own valuation. Thus, with probability $2H(1)(1-H(1))$, only one of the two consumers will sign up for the deal, and with probability $H(1)^2$, both of them will sign up.

When $N = 2$, we know from Lemma 2 that the success rate of the deal, $Q^*$, is the solution of equation $H(q)H(1) = q$. Plugging the expression of $H(q)$ into the equation, the success rate at equilibrium is obtained by

$$Q^* = \frac{\alpha^2(p-w)^2(1-\rho)}{1-\alpha^2\rho(p-w)^2}$$

Consequently, the expected profit when $N = 2$ is given by

$$\pi_2(w) = Q^*(2w-c_2) = \frac{\alpha^2(p-w)^2(1-\rho)}{1-\alpha^2\rho(p-w)^2}(2w-c_2).$$

Let $w_i^* := \arg\max_{w \leq p}\{\pi_i(w)\}$, for $i = 1, 2$. We compare the expected profit $\pi_i(w_i^*)$, $i = 1, 2$, through comprehensive numerical analyses, seek to compare group buying with posted price, and identify
the impact of cost structures on the optimal threshold and group-buying price. We consider the case when consumers’ valuations are uniformly distributed within the interval $[0, 20]$. The characteristics of the deal are specified as $p = 10$ and $\rho = 0.5$. Both costs $c_1$ and $c_2$ vary from 5 to 10 with $0 \leq c_2 - c_1 \leq c_1$.

Figure 3  $w^*_1$, $w^*_2$, and $w^*_1 - w^*_2$ as a function of $c_1$ and $c_2$.

![Figure 3](image)

(a) $w^*_1$.  
(b) $w^*_2$.  
(c) $w^*_1 - w^*_2$.

Note. The plots in subfigures (a), (b) and (c) are generated by the following parameters: consumers’ valuations are uniformly distributed with supports $[0, 20]$, and $p = 10$ and $\rho = 0.5$. The seller chooses $N = 1$ in subfigure (a) and $N = 2$ in subfigure (b). The plot in subfigure (c) illustrates the difference between $w^*_1$ and $w^*_2$.

The plots of the optimal group-buying price as a function of $c_1$ and $c_2$ are shown in Figure 3. When $N = 1$, ceteris paribus, the optimal price $w^*_1$ is increasing in $c_1$ and $c_2$. Intuitively, the firm increases its group-buying price to balance profit margin per unit as costs increase. Notice that $w^*_2$ is independent of $c_1$ because the deal is on if and only if both consumers sign up to the deal when $N = 2$. It turns out that $w^*_1$ is always greater than $w^*_2$ under all examined cost structures. When $N = 1$, the firm is able to make a profit even if only one consumer has high product valuation; however, both consumers need to possess sufficiently high valuations for the deal to kick in when $N = 2$. In order to facilitate the deal’s success when $N = 2$, it is to the firm’s benefit to reduce the group-buying price to entice consumers at the cost of a relatively lower profit margin per unit than with a posted price. In return the firm has downside protection, since the lower wholesale price is valid when both consumers sign up.

Figure 4 depicts the expected profit as a function of $c_1$ and $c_2$. When costs increase, $c_1$ or $c_2$ or both, the absolute profitability decreases in both cases when $N = 1$ and $N = 2$. The difference
Figure 4 \(\pi_1(w_1^*), \pi_2(w_2^*), \text{ and } \pi_1(w_1^*) - \pi_2(w_2^*)\) as a function of \(c_1\) and \(c_2\).

Note. The curves in subfigures (a), (b) and (c) are generated by the following parameters: consumers’ valuations are uniformly distributed with supports \([0, 20]\), and \(p = 10\) and \(\rho = 0.5\). The seller chooses \(N = 1\) in subfigure (a) and \(N = 2\) in subfigure (b). The plot in subfigure (c) illustrates the difference between \(\pi_1(w_1^*)\) and \(\pi_2(w_2^*)\).

in the expected profits decreases when the degree of scale economies increases (fix \(c_1\), decrease \(c_2\); or fix \(c_2\), increase \(c_1\)). In the case when the degree of scale economies is significant and \(c_1\) is relatively high, setting \(N = 2\) with a more appealing \(w_2^*\) is more profitable because it avoids the unprofitable scenario when only one consumer signs up to the deal. In other cases, posted price (\(N = 1\)) may yield higher profits than the group-buying mechanism (\(N = 2\)). This is a stark contrast to a result obtained in Anand and Aron (2003) that under production postponement, the profits from group buying dominate those under posted pricing. This is because the group-buying mechanism considered in Anand and Aron (2003) is essentially a quantity-discount schedule that consists of price points corresponding to different quantities demanded. The schedule includes the posted price mechanism as a special case.

The prevalent online group-buying mechanism considered in this paper is different in that the deal can fail - no purchase transaction is undertaken if the quantities demanded are lower than the minimum threshold. The difference between group buying and posted price is also subtler. With the posted-price mechanism, the firm captures every possible demand with willingness-to-pay no less than the posted price but may face a risk of ending up with few purchases so that a relatively high marginal cost applies; given such a risk, the firm tends to set a less appealing posted price ex ante. With the group-buying mechanism, the firm sells only when enough consumers commit to purchase so that a lower marginal cost applies and hence ex ante, being able to post a more appealing group
buying price. However, the firm suffers the loss of potential consumers who discount their utility in making sign-up decisions when facing the uncertainty of the deal’s success.

4.4. Endogenous Sequencing

So far we have analyzed the two-period game, where the sequence of arrivals of two different cohorts is exogenously determined. However in the existence of heterogeneity of two cohorts, in terms of size and valuations, varying the sequence of their arrivals may yield different success rates. One question naturally arises: what is the sequence of arrivals that yields the highest success rate? The answer may shed light on how group-buying firms can manipulate the sequence of arrivals of different cohorts to maximize the success rate. To investigate this problem, we add a stage 0 at the beginning of our current two-period game. In the stage 0, each cohort decides whether it wants to be the leader and move first, or to be the follower and move later. If both cohort decide to be leaders or followers, we assume the simultaneous mechanism applies. This is because if the sales horizon is a finite time period, then the desire to be the leader (follower) will force cohorts to make decisions right after (before) the sales horizon starts (ends). We assume at the stage 0, the uncertainty of size and valuations in each cohort has not been resolved. Once the sequence is decided in the stage 0, uncertainty starts to be resolved within each cohort and the game plays out according to the determined sequence.

The success of a group-buying deal is a win-win outcome for all players, both consumers and the firm. Specifically, there is a one-to-one correspondence between consumers’ expected surplus and the success rate. Hence we can use the success rate as a proxy for the payoff and compare success rates in the sense of Definition 4.

Table 2 The Payoff Matrix.

<table>
<thead>
<tr>
<th>Cohort 1 as leader</th>
<th>Cohort 2 as leader</th>
<th>Cohort 2 as follower</th>
</tr>
</thead>
<tbody>
<tr>
<td>(q∗, q∗)</td>
<td>(q∗, q∗)</td>
<td>(Q∗, Q∗)</td>
</tr>
<tr>
<td>(Q∗, Q∗)</td>
<td>(Q∗, Q∗)</td>
<td></td>
</tr>
<tr>
<td>(Q∗, Q∗)</td>
<td>(Q∗, Q∗)</td>
<td></td>
</tr>
</tbody>
</table>

At the stage 0, the firm is facing the payoff matrix summarized in Table 2. As an immediate result of Proposition 1, we can identify the equilibria of the stage 0 game.

Proposition 5 (The Two-Stage Game). In the stage 0 of the two-stage game, there exist two equilibria (Q∗, Q∗) and (Q∗, Q∗), both of which correspond to sequential mechanisms.
Which one of the two sequential equilibria is better? Let us compare $Q^*_1$ and $Q^*_2$ and investigate under what conditions one of the equilibria Pareto-dominates the other. When we allow for heterogeneity in cohort size $M_i$ and cohort members’ valuation $V_i$, we show that the following result, which is proved in the appendix, provides a sufficient condition such that the equilibrium where cohort $-i$ moves first Pareto-dominates the other equilibrium where cohort $i$ moves first.

**Proposition 6 (Heterogenous Cohort Size and Valuation).** If $M_i \geq_{st} M_{-i}$ and

\[
H_i(1) = H_{-i}(1),
\]
\[
H_i(q) \leq H_{-i}(q), \quad \text{for all } q \in [0, 1),
\]

then the equilibrium where cohort $-i$ acts as the leader Pareto-dominates the other equilibrium where cohort $i$ acts as the leader.

The condition $M_i \geq_{st} M_{-i}$ indicates that cohort $i$ is the larger cohort in the probabilistic sense. The proposition says that the cohort that has a potentially smaller size is preferred by all to move first, given that all the other characteristics of consumers in both cohorts are identical. This result is intriguing as the first reaction may be to have the stronger cohort that has more potential buyers move first to ensure that the weaker cohort is the follower. Our result predicts that the stronger cohort in the sense of being potentially larger is preferred by both cohorts as the information free-riders in the sequential coordination game. It is better to have the cohort that has more potential consumers to enjoy the luxury of deciding later without uncertainty discounting. Such an arrangement can eventually benefit both cohorts by boosting the confidence of the early cohort because there is a “stronger” cohort behind.

The condition $H_i(1) = H_{-i}(1)$ says that if the deal is sure to succeed, consumers in both cohorts have the same likelihood of signing up. One sufficient condition for $H_i(q) \leq H_{-i}(q), \ q \in [0, 1)$ is that given everything else being identical, consumers in cohort $-i$ are less risk-averse to the possibility that the deal may be off. The proposition says that the cohort that is less risk-averse is preferred by all to move first, given that all the other characters such as the cohort size distribution are identical between two cohorts. In other words, if the cohort size is distributed identically, the weaker cohort in the sense of being more risk-averse is preferred by both cohorts as the information free-riders in the sequential coordination game. This is because if one of two cohorts has the chance to decide later when uncertainty regarding the first period is resolved, it is better to have the cohort that is more risk-averse to enjoy such a luxury in order to minimize the chances of walking away from the deal due to risk-aversion to success uncertainty.
In reality, consumers may not be able to coordinate spontaneously. However, it is possible for firms to endogenize the sequence of arrivals by selecting one cohort of consumers and informing them first. Group-buying firms can predict the purchase likelihood and cohort size of their members for any given product by tracking their purchase histories. Consequently, the firm can decide which cohort of consumers to inform first.

5. Conclusion

This paper studies optimal group-buying mechanisms in a two-period game where cohorts of consumers arrive at the deal sequentially. Specifically, we examine the success rate of a group-buying deal under two alternative mechanisms: a sequential mechanism that reveals to late arrivals the cumulative number of signups, and a simultaneous mechanism that does not provide late arrivals with any updates with the result that all consumers face the uncertainty about the other cohort when making sign-up decisions. Our analysis shows that, all other things being the same, a sequential mechanism dominates a simultaneous mechanism. Interestingly, posting the cumulative number of sign-ups in the first period can reduce uncertainty and thus increase the expected purchases among the second cohort of consumers. The increased second-period sign-ups can in turn improve the confidence among the first cohort of consumers and lead to a higher expected number of sign-ups in the first period, thus further increasing the deal success rate. This backward perspective, starting from the second period and going back to the first period, is crucial for understanding the intuition behind our result. This result is consistent with the observed dominance of sequential mechanisms in practice. Moreover, we find this result robust in a number of model extensions: specifically, when consumers can postpone their sign-up decisions strategically, or when a group-buying deal requires the entire set of consumers to participate in a multiperiod model. Our analysis also provides guidance on how a group-buying firm may properly arrange the sequence of consumer arrivals to increase its success rate and the expected number of sign-ups. Having the smaller cohort come first and the larger cohort come later, or having the less risk-averse cohort come first and the more risk-averse cohort come later, can increase the deal’s success rate.

Our analysis offers some useful insights into what types of product or market are more suitable for the group-buying format. We find that luxury services are more suitable than necessity goods because deals on luxury services tend to provide greater benefits. We also find that a longer sign-up horizon and hence greater discounting could hurt the success rate. The importance of having a short horizon is reflected by the surging popularity of 24-hour deals by Groupon and Livingsocial. Finally, when compared to the posted-price mechanism, the group-buying mechanism is a better
selling format if and only if there are substantial benefits associated with selling to more consumers, i.e., if there are economies of scale.

We can visualize a number of directions for extending this line of research in the future. One direction for research would be to incorporate social learning under the sequential mechanism. When consumers are unsure of the value of deals, they may infer the value from the number of early sign-ups. This issue can be particularly important when people face deals and projects in unfamiliar domains. Many art projects posted in Kickstarter involve novel ideas, and many investment projects seeking financing are from areas remote to the potential donors. When people find it difficult to judge the value of deals, they may rely on other people’s decisions (Zhang and Liu 2011). Another direction for research would be to consider the competition between deals. More and more online group-buying sites are being launched, all competing for the same set of consumers. Even on the same group-buying site, multiple deals can be offered. For example, Kickstarter posts a number of project proposals simultaneously. Since consumers with limited budgets have to judge carefully the relative value of these projects and choose one to invest in, the design of the group-buying mechanism becomes more critical to ensure a high success rate. The third direction for research is to control the process of sign-ups dynamically. The success rates of deals can be crucial for those sites selling physical products, because they usually have low success rates. To keep up satisfactory success rates, one way to jump-start the sign-up process is to dole out free slots to consumers in the beginning of the process according to the number of sign-ups realized and the remaining time-to-go. Last but not least, future research should conduct empirical analysis to confirm the external validity of theoretical results. Online group-buying sites may experiment with different mechanism formats to search for the best designs.

Appendix. Proofs.

Proof of Corollary 1. By Proposition 2 and Assumption 1(i), it is sufficient to show that \( \tilde{H}_i(q) \geq H_i(q) \), \( i = 1,2 \) for all \( q \in (0,1] \).

Recall that, for the necessity goods, \( H_i(q) \) is in the form of \( H_i(q) = \int_i \left( p + \frac{\rho_i}{1-\rho_i} (p-w)q \right) - F_i(w) \) for \( q \in (0,1] \). If \( \tilde{\rho}_i \geq \rho_i \), ceteris paribus, then \( \tilde{H}_i(q) \geq H_i(q) \) for \( q \in (0,1] \), since the function \( \int_i \left( p + \frac{\rho_i}{1-\rho_i} (p-w)q \right) \) is increasing in \( \rho_i \).

For the luxury goods, \( H_i(q) \) is in the form of \( H_i(q) = \int_i (p + a_i) - F_i \left( \frac{a_i[-(1-q)\rho_i]}{q \rho_i} + w \right) \) for \( q \in (0,1] \). If \( \tilde{\rho}_i \geq \rho_i \), ceteris paribus, then \( \tilde{H}_i(q) \geq H_i(q) \) for \( q \in (0,1] \), since the function \( -F \left( \frac{a_i[-(1-q)\rho_i]}{q \rho_i} + w \right) \) is increasing in \( \rho_i \). \( \square \)
Proof of Corollary 2. By Proposition 2 and Assumption 1(i), it is sufficient to show that $\tilde{H}_i(q) \geq H_i(q)$, $i = 1, 2$ for all $q \in (0, 1]$.

Recall that, for the necessity goods, $H_i(q)$ is in the form of $H_i(q) = F_i\left(p + \frac{\rho}{1 - \rho_i}(p - w)q\right) - F_i(w)$ for $q \in (0, 1]$. If $\tilde{p} \geq p$ and $\tilde{w} \leq w$, then $\left[\tilde{w}, \tilde{p} + \frac{\rho}{1 - \rho_i}(\tilde{p} - \tilde{w})q\right] \supseteq \left[w, p + \frac{\rho}{1 - \rho_i}(p - w)q\right]$. Moreover, if $\tilde{V}_i = \tilde{V}_i U[c_i, c_i + \delta_i]$, ceteris paribus, then $\tilde{H}(q) = \left[\tilde{w}, p + \frac{\rho}{1 - \rho_i}(\tilde{p} - \tilde{w})q\right] \cap [c_i, c_i + \delta_i] / \delta_i \geq \left[w, p + \frac{\rho}{1 - \rho_i}(p - w)q\right] \cap [c_i, c_i + \delta_i] / \delta_i = H(q)$ for $q \in (0, 1]$. □

Proof of Corollary 3. By Proposition 2 and Assumption 1(i), it is sufficient to show that $\tilde{H}_i(q) \geq H_i(q)$, $i = 1, 2$ for all $q \in (0, 1]$. Recall that, for the luxury goods, $H_i(q)$ is in the form of $H_i(q) = F_i(p + a_i) - F_i\left(\frac{a_i[1 - (1 - q)\rho_i]}{q_{\rho_i}} + w\right)$ for $q \in (0, 1]$. If $\tilde{p} \geq p + a_i$ and $\tilde{a}_i \leq a_i$, then $\left[\tilde{a}_i[1 - (1 - q)\rho_i] + w, \tilde{p} + \tilde{a}_i\right] \supseteq \left[a_i[1 - (1 - q)\rho_i] + w, p + a_i\right]$. Moreover, if $\tilde{V}_i = \tilde{V}_i U[c_i, c_i + \delta_i]$, ceteris paribus, then $\tilde{H}(q) = \left[\tilde{a}_i[1 - (1 - q)\rho_i] + w, \tilde{p} + \tilde{a}_i\right] \cap [c_i, c_i + \delta_i] / \delta_i \geq \left[a_i[1 - (1 - q)\rho_i] + w, p + a_i\right] \cap [c_i, c_i + \delta_i] / \delta_i = H(q)$ for $q \in (0, 1]$. □

Proof of Proposition 6. We first present the following lemma.

Lemma 4. If $M_i \geq M_{-i}$ and for all $q \in [0, 1]$,

\[
H_i(1)\left[1 - H_i(q)\right] \geq H_{-i}(1)\left[1 - H_{-i}(q)\right], \tag{7}
\]

\[
\left[1 - H_i(1)\right]H_i(q) \leq \left[1 - H_{-i}(1)\right]H_{-i}(q), \tag{8}
\]

then in the two-stage game, the equilibrium where cohort $-i$ acts as the leader Pareto-dominates the other equilibrium where cohort $i$ acts as the leader.

Proof of Lemma 4. Recall $X_{ik}(q)$ is a Bernoulli random variable with success probability $H_i(q)$ and $X_{ik}(1)$ is a Bernoulli random variable with success probability $H_i(1)$. Hence,

\[
X_{ik}(1) - X_{ik}(q) = \begin{cases} 
1, & \text{with prob. } H_i(1)[1 - H_i(q)], \\
-1, & \text{with prob. } [1 - H_i(1)]H_i(q), \\
0, & \text{otherwise.}
\end{cases}
\]

Given inequality (7), we know that $X_{ik}(1) - X_{ik}(q)$ has a larger mass in value of 1 than $X_{-ik}(1) - X_{-ik}(q)$. By inequality (8) alone, $X_{ik}(1) - X_{ik}(q)$ has a smaller mass in value of $-1$ than $X_{-ik}(1) - X_{-ik}(q)$; equivalently,
$X_{ik}(1) - X_{ik}(q)$ has a larger mass in values of 0 and 1 combined than $X_{-ik}(1) - X_{-ik}(q)$. Hence, under conditions (7) and (8), we have $X_{ik}(1) - X_{ik}(q) \geq_{st} X_{-ik}(1) - X_{-ik}(q)$. Furthermore, if $M_i \geq_{st} M_{-i}$, by Shaked and Shanthikumar (2007) Theorem 1.A.4., we have
\[
\sum_{k=1}^{M_i} [X_{ik}(1) - X_{ik}(q)] \geq_{st} \sum_{k=1}^{M_{-i}} [X_{-ik}(1) - X_{-ik}(q)],
\]
which is equivalent to
\[
\sum_{k=1}^{M_i} X_{ik}(q) + \sum_{k=1}^{M_{-i}} X_{-ik}(1) \leq_{st} \sum_{k=1}^{M_{-i}} X_{ik}(1) + \sum_{k=1}^{M_i} X_{-ik}(q).
\]
Note that the left-hand-side (right-hand-side) of the above inequality corresponds to the random variable evaluated in Equation (2) when cohort $i$ (cohort $-i$) takes the lead. By Lemma 3, $Q^*_i \leq Q^*_{-i}$ and the desired result follows. \(\square\)

It is easy to verify that Proposition 6 provides a sufficient condition for inequalities (7)-(8). Thus, the announced result is an immediate result of Lemma 4. \(\square\)

Proof of Proposition 4. To facilitate the comparison, we present the following lemma.

**Lemma 5.** Under the simultaneous mechanism, the success rate of a larger market is always lower than or equal to that of a smaller market, i.e., $q_N(N) \geq q_{N'}(N')$, $\forall N \leq N'$.

**Proof of Lemma 5.** The success rate under the simultaneous mechanism is characterized by Equation (6), $x = \Pi_{i=1}^{N} H_i(x)$. For any $0 \leq x \leq 1$, we have $0 \leq H_i(x) \leq 1, \forall i = 1, 2, \ldots, N$. Note that $q_N(N)$ is the largest root to the equation $x = \Pi_{i=1}^{N} H_i(x)$ and $q_N(N')$ is the largest root to the equation $x = \Pi_{i=1}^{N'} H_i(x)$. Since $\Pi_{i=1}^{N} H_i(x) \geq \Pi_{i=1}^{N'} H_i(x)$, the function $\Pi_{i=1}^{N} H_i(x)$ will cross the 45 degree line at higher positions, hence $q_N(N) \geq q_N(N'), \forall N \leq N'. \square$

Given Lemma 5, we prove by induction on $N$ that $Q_N(N) \geq q_N(N)$. If $N = 1$, it is obvious $Q_1(1) = q_1(1)$. Suppose we have $Q_N(N) \geq q_N(N)$ for some $N \geq 1$, we want to show that $Q_{N+1}(N + 1) \geq q_{N+1}(N + 1)$.

We know that $Q_N(N + 1) = 0$. Hence by Equation (5),
\[
Q_{N+1}(N + 1) = H_{N+1}(Q_N(N))Q_N(N) \\
\geq H_{N+1}(q_N(N))q_N(N) \\
= H_{N+1}(q_N(N)) \cdot \Pi_{i=1}^{N} H_i(q_N(N)) \\
\geq \Pi_{i=1}^{N+1} H_i(q_{N+1}(N + 1)) \\
= q_{N+1}(N + 1),
\]

where the first inequality is due to the inductive assumption, the second inequality is due to Lemma 5, the first and second equalities are due to Equation (6). □

References


