Multi-pattern Matching with Wildcards

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Abstract—Multi-pattern matching with wildcards is to find all the occurrences of a set of patterns with wildcards in a text. This problem arises in various fields, such as computational biology and network security. But the problem is not extensively studied as the single pattern case and there is no efficient algorithm for this problem. In this paper, we present efficient algorithms based on the fast Fourier transform. Let $P = \{p^1, …, p^k\}$ be a set of patterns with wildcards where the total length of patterns is $|P|$, and a text $t$ of length $n$ over alphabet $a_1, …, a_\sigma$. We present three algorithms for this problem where patterns are matched simultaneously. The first algorithm finds the matches of a small set of patterns in the text in $O(n \log |P| + \sigma \log k)$ time where $\sigma$ is the total number of occurrences of $P$ in $t$. The words used in the algorithm are of size $\log k + \sum_{i=1}^{k} \lfloor \log |p^i| \rfloor$. The second algorithm is based on a prime number encoding. By this approach, if $m = n$, the algorithm runs in $O(n \log m)$ time by computing a single convolution.

Much research focuses on multi-string matching problems. The first algorithm to solve this problem in $O(n \log \sigma)$ time is presented by Aho and Corasick [1] which generalizes the Knuth-Morris-Pratt algorithm [15]. The Commentz-Walter [8] algorithm is a direct extension of the Boyer-Moore algorithm [4] which also combined the idea of AC algorithm. Several parallel multi-string matching algorithms are presented that are either precise [10] or approximate [27], [25], [24], [26]. The factor recognition approach based algorithms [9], [19], [2] use either suffix automata or factor oracles for precise or weak factor recognition. For short patterns, bit parallelism leads to algorithms that are efficient in practice, see [18].

However, the problem of matching a set of patterns with wildcards is not extensively studied as the single pattern case. To date, there is no efficient algorithm for this problem. But multi-pattern matching problem arises in many applications, such as intrusion detection systems [29], anti-virus systems [28] and computational biology [17]. A close but different problem: matching a set of patterns with variable length don’t cares was solved by Kucherov and Rusinowitch [16]. They proposed an algorithm that runs in time $O((n + L(P)) \log L(P))$, where $L(P)$ is the total length of keywords in every pattern of the pattern set $P$, and $|t|$ is the length of the input text. A faster solution was given by [22] which runs in $O((n + |P|) \log \kappa / \log \log \kappa)$ time, where $|P|$ is the total number of keywords in all the patterns in $P$, and $\kappa$ is the number of distinct keywords in all the patterns in $P$.

In this paper, we focus on the problem of matching a set of patterns with wildcards without preprocessing the text. We present three FFT based algorithms for...
this problem. The first one extends the Clifford and Clifford algorithm [5] to handle multi-pattern and runs in \(O(n \log |P| + occ \log k)\) time where \(|P|\) denotes the total length of all the patterns in \(P\). It can find the matches of a small set of patterns in the text by three convolutions. The words used in the algorithm are of size \(k\left(2\lfloor\log |\sigma|\rfloor + \sum_{i=1}^{k} \lfloor\log |p^i|\rfloor\right)\) bits. The second one uses the prime number encoding to encode both the pattern and the text. It runs in time \(O(n \log m + occ \log k)\) where \(m\) is the length of the longest pattern. The algorithm uses words with \(k\left(2\lfloor\log |\sigma|\rfloor + \log |m|\right)\) bits. The drawback of the two methods is that when \(|P|\) and \(k\) are large, the word will be too long to fit into a machine word of modern processors (normally 32 or 64 bits). To shorten the word length, we present an algorithm that uses words with \(\sum_{i=1}^{k} \lfloor\log |p^i|\rfloor\) bits. The algorithm finds the occurrences of patterns in the text in time \(O(n \log |P| \log |\sigma| + occ \log k)\) by computing the Hamming distances between the patterns and the text. The distances are computed by \(2\lfloor\log |\sigma|\rfloor\) convolutions. Moreover, we discuss the modular arithmetic based FFT and give all the necessary parameters for the FFT on the 64-bit architecture. The algorithms presented in this paper can be easily parallelized. On a \(q\)-processor PRAM model, the time complexity of the algorithms decreases by \(q\) times compared with that on a single processor.

The paper is organized as follows. Section II gives some basic notions. Section III presents the algorithms for multi-pattern matching with wildcards using Euclidean distance. In Section IV, we give the approach based on Hamming distance of bit vectors. Section V introduces the FFT based on modular arithmetic on 64-bit architectures. Some interesting issues are discussed in Section VI.

II. PRELIMINARIES

Let \(\Sigma\) be a finite alphabet and \('s'\) the wildcard symbol. Denote by \(|s|\) the length of a string \(s\). A text \(t = t[1] \ldots t[n]\) and a pattern \(p = p[1] \ldots p[m]\) are strings over \(\Sigma \cup \{\text{'}s\text{'}\}\). Given a pattern \(p\) and a text \(t\), which both may contain wildcards, we say that \(p\) occurs at location \(j\) in \(t\) if:

\[
p[i] = t[i + j - 1] \text{ or } p[i] = \text{'}s\text{'} \text{ or } t[i + j - 1] = \text{'}s\text{'},
\]

for \(1 \leq i \leq m\). \(1\)

We use \(P = \{p^1, \ldots, p^k\}\) to denote a set of patterns with wildcards. We use \(\text{"} \cdot \text{"} \) to denote the concatenation of two patterns, for example \(p^1 \cdot p^2\) is the concatenation of pattern \(p^1\) and \(p^2\). For an integer array \(x = x[1], x[2], \ldots, x[n]\), we use \(x[i_1, i_2]\) where \(1 \leq i_1 \leq i_2 \leq n\) to denote the array \(x[i_1], \ldots, x[i_2]\) and use \(2^\text{\textit{e}}\text{\textit{x}}\) to denote the integer array of length \(n\), such that

\[
(2^\text{\textit{e}}\text{\textit{x}})[i] = x[i]2^\text{\textit{e}}, \text{ for each } 1 \leq i \leq n.
\]

The following definition is a basic technique used in this paper.

**Convolution:** The convolution, or cross-correlation, of two vectors \(a, b\) is the vector \(a \oplus b\) such that \(\sum_{i=1}^{\lfloor|n|\rfloor} a[j]b[i+j-1] \mod |b|\), \(1 \leq i \leq |b|\). Note that this definition of convolution involves wrap-around (i.e., \(b\) is assumed to be a cyclic vector).

Our algorithms are based on FFTs. An important property of FFT is that in the RAM model, \(p \oplus t\) can be computed in \(O(n \log n)\) time. By a standard trick [12], the running time can be further reduced to \(O(n \log m)\). First, split the text into \(n/m\) pieces of length \(2m\). The starting positions of the pieces are in the set \(\{lm + 1 \mid 0 \leq l < n/m\}\). The convolution between the pattern and each piece of the text is computed using FFT in time \(O(m \log m)\) per piece. The overall time complexity is \(O((n/m) \log m) = O(n \log m)\).

III. EUCLIDEAN DISTANCE BASED MULTI-PATTERN MATCHING WITH WILDCARDS

In this section, we extend the wildcard matching algorithm of Clifford and Clifford [5] to multi-pattern. Generally speaking, the Clifford and Clifford algorithm first encodes each symbol by a unique positive number and replaces wildcards by 0’s. Then, for each location \(1 \leq i \leq n - m + 1\) in the text, the algorithm computes

\[
\sum_{j=1}^{m} p[j]t[i + j - 1] - t[i + j - 1]^2 = \sum_{j=1}^{m} p[j]t[i + j - 1] - 2p[j]t[i + j - 1]^2 + p[j]t[i + j - 1]^3\]

\(3\)

in \(O(n \log m)\) time using FFTs. Wherever there is an exact match this sum will be exactly 0.

The numbers computed by the algorithm are as large as \(4m(\sigma - 1)^4/27\). In [6], the authors modified the algorithm as follows. First by replacing non-wildcards by 1’s and wildcards by 0’s in the text and the pattern, we get \(p’\) and \(t’\) respectively. Then, for each location \(1 \leq i \leq n - m + 1\) in the text, the algorithm computes

\[
\sum_{j=1}^{m} p'[j]t'[i + j - 1] - t[i + j - 1]^2.
\]

\(4\)

The result can be viewed as the squire of the Euclidean distance between the pattern and the substring starting from a location \(i\) in \(t\). The maximal numbers used in the convolutions are reduced to \(m\sigma^2\).

A. Algorithm 1

In our approach, for a pattern \(p\) and each location \(1 \leq i \leq n\) in the text, we use the following wrap-around sum

\[
d(p, t)[i] = \sum_{j=1}^{\lfloor|p|\rfloor} p'[j]t'[l(i, j)](p[j] - t[l(i, j)])^2
\]

\(5\)

where \(l(i, j)\) denotes \((i + j - 1) \mod n\). We can compute (5) by the following formula which uses only three \((a \oplus b)[i] = \sum_{j=1}^{\lfloor|n|\rfloor} a[j]b[i+j-1] \mod |b|, 1 \leq i \leq |b|\). Note that this definition of convolution involves wrap-around (i.e., \(b\) is assumed to be a cyclic vector).
convolutions.

\[
p = p^1 \cdot p^2 \ldots p^k.
\]

Define \( \alpha_1 = 0 \), \( \alpha_j = \sum_{i=1}^{n} [\lg|p^i| + 2\lg\sigma] \) for \( 1 \leq j \leq k \) and \( \alpha_j = \sum_{i=1}^{n} [p^i] \), for \( 1 \leq j \leq k \).

We use \( I[1..l] \) to denote the array of length \( n \) where all the entries are 1’s. We construct \( I^P \) as follows

\[
I^P = (2^{\alpha_1 I[1..|p^1|]}) \cdot (2^{\alpha_2 I[1..|p^2|]}) \cdots (2^{\alpha_k I[1..|p^k|]}).
\]

Then we compute the following

\[
R[i] = \sum_{j=1}^{n} [\sum_{j=1}^{n} I^P[j]p[j]t[l(i,j)]\lg(p[j] - t[l(i,j)])]^2
\]

\[
= \sum_{j=1}^{n} I^P[j]p[j]t[l((i,j))]/2 - 2 \sum_{j=1}^{n} I^P[j]p[j]t[l(i,j)]
\]

\[
+ \sum_{j=1}^{n} I^P[j]p[j]t'[l(i,j)].
\]

(7)

\( R \) can be computed using three convolutions. By checking whether the bit vector from \( \alpha_j+1 \)th significant bit to \( \alpha_{j+1} \)th significant bit of the binary code of \( R[i] \), denoted by \( R[i][\alpha_j+1..\alpha_{j+1}] \), is all 0’s, we will know whether \( p^j \) occurs at position \( (i + \alpha_j) \) mod \( n \) in \( t \). That is to say, assume that \( t \) is a cyclic vector, then for each \( |P| \)-length factor of \( t \) starting from each position of \( t \), the result of the matching of each pattern is stored in a disjoint bit interval in a word. We give the algorithm in Figure 1.

According to (7), we can see that for the resulting array \( R \) computed by Algorithm 1,

\[
R[i] = \sum_{j=1}^{n} d(p^j,t)[i + \alpha_j]2^{\alpha_j}, \quad \text{for} \ 1 \leq i \leq n. \tag{8}
\]

For any \( p^j \in P \), according to (5), we have \( d(p^j,t)[i + \alpha_j] \leq |p^j|\sigma^2 \). So \( \alpha_{j+1} - \alpha_j \) (that equals \( \lg|p^j| + 2\lg\sigma \)) bits are enough to represent \( d(p^j,t)[i + \alpha_j] \).

As a result, the binary code of \( d(p^j,t)[i + \alpha_j] \) equals \( R[i][\alpha_j+1..\alpha_{j+1}] \). Thus we can get \( d(p^j,t)[i + \alpha_j] \) by computing \( (R[i] \mod 2^{\alpha_{j+1}})/2^{\alpha_j} \).

To verify the correctness of Algorithm 1, suppose that a pattern \( p^j \in P \) occurs in the text \( t \) starting from position \( x \). We have \( d(p^j,t)[x] = 0 \). If \( p^j \) does not occur in \( t \) starting from \( x \), we have \( d(p^j,t)[x] \neq 0 \). Let \( x = x_0 \). Thus \( R[i][\alpha_j+1..\alpha_{j+1}] = 0 \) indicates \( d(p^j,t)[i + \alpha_j] = 0 \), that is, \( p^j \) occurs at position \( (i + \alpha_j) \) mod \( n \) of \( t \).

The algorithm takes \( O(n \log|P|) \) time to compute \( R \) and uses \( O(nk) \) time to check \( R \) to find whether there is any occurrence of patterns. Each entry of \( R \) uses \( k[2\lg\sigma + \sum_{i=1}^{n} |\lg|p^i|] \) bits. The size of the words used

\[
\text{Algorithm 1}
\]

\begin{algorithm}
\begin{algorithmic}
\State Input: Text \( t \) and pattern set \( P = \{p^1, p^2, \ldots, p^k\} \).
\State 1. \( R \leftarrow \{0,0, \ldots, 0\} \).
\State 2. \( \alpha_1 \leftarrow 0, \alpha_1 \leftarrow 0 \).
\State 3. for \( j \leftarrow 2 \) to \( k \) do
\State 4. \( \alpha_j \leftarrow \alpha_{j-1} + |\lg|p^{j-1}| + 2\lg\sigma| \).
\State 5. \( \alpha_j \leftarrow \alpha_{j-1} + |\lg|p^{j-1}| \).
\State 6. for end do
\State 7. \( L \leftarrow \alpha_k + |p^k| \).
\State 8. \( p \leftarrow p^1 \cdot p^2 \ldots p^k \).
\State 9. \( I^P[i] = (2^{\alpha_1 I[1..|p^1|]}) \cdot (2^{\alpha_2 I[1..|p^2|]}) \cdots (2^{\alpha_k I[1..|p^k|]}) \).
\State 10. Compute \( (p^j)' \) where \( 1 \leq i \leq k \) and \( t' \) by replacing non-wildcards by 1’s and wildcards by 0’s in the \( t \) and \( p^j \).
\State 11. For \( 1 \leq i \leq n \), compute \( R[i] = \sum_{j=1}^{n} I^P[j]p[j]t[l(i + j - 1) \mod n] \cdot 2 \sum_{j=1}^{n} I^P[j]p[j]t[l(i + j - 1) \mod n + 1] \).
\State 12. for pos \( \leftarrow 1 \) to \( n \) do
\State 13. for \( j \leftarrow 1 \) to \( k \) do
\State 14. Output \( (p^j)' \) occurs at \( (pos + \alpha_j) \mod n \) in \( t' \) if \( R[pos][\alpha_j+1..\alpha_{j+1}] = 0 \).
\State 15. for end do
\State 16. for end do
\end{algorithmic}
\end{algorithm}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{algorithm1.png}
\caption{Algorithm 1.}
\end{figure}

in the FFTs are of the same size. If \( \sigma, |P| \) and \( k \) are small enough, each word can fit into a single machine word of modern processors that is typically 32 bits or 64 bits. For example, for DNA sequences where \( \sigma = 4 \), Algorithm 1 can process four patterns each with 16 symbols or three patterns each with 64 symbols. But when \( \sigma, |P| \) and \( k \) are not small the words used in FFTs have to be very long. For example, let \( \sigma = 256 \), even for two patterns, Algorithm 1 uses words exceeding 32 bits. The algorithm in Section IV tries to shorten the word size to cope with texts on larger alphabets and pattern sets that have larger number of patterns and longer length.

The time complexity of checking the matches of \( P \) in \( t \) can be further reduced. Let the length of the longest pattern in \( P \) be \( m \). Checking the matches of \( P \) in \( t \) can be done in time \( O(n \log(m\sigma + occ \log k)) \) where \( occ \) is the times of occurrences of patterns in \( t \). We first transform \( R \) to array \( g \) such that \( g[i] = 0 \) if \( l \notin \{\alpha_1 + 1, \alpha_2 + 1, \ldots, \alpha_k + 1\} \) and for \( 1 \leq j \leq k \),

\[
g[i][\alpha_{j+1}] = \begin{cases} 1 & \text{if } R[i][\alpha_{j+1}..\alpha_{j+1}] = 0; \\
0 & \text{if } R[i][\alpha_{j+1}..\alpha_{j+1}] \neq 0.
\end{cases}
\]

For \( p^j \) position \( \alpha_j + 1 \) is called the indication position (id position) of \( p^j \). The transformation is described as
follows. Let there be \(d\) different lengths of patterns. We order the lengths in an increasing order, use \(m_j\) where \(1 \leq j \leq d\) to denote the \(j\)th minimal length. First, set \(g[i] = 0\) for \(1 \leq i \leq n\). Then for each pattern \(p^i\) we compute the bit \(\bigwedge_{c=a_{i+1}}^{a_i+1} R[i]_c\) and set this bit to \(g[i]_{a_i+1}\). It follows that if \(R[i]_{a_{i+1} \ldots a_j+1} = 0\) then \(\bigwedge_{c=a_{i+1}}^{a_i+1} R[i]_c = g[i]_{a_j+1} = 1\). The computation is by bitwise shift right and bitwise and operations. In the transformation, We use \(d\) bit masks, say \(V\text{mask}[1], \ldots, V\text{mask}[d]\). The bit mask \(V\text{mask}[j]\) is a word where the bits on the indication locations for patterns whose length is \(m_j\) are set to 1 and other bits are set to 0s. The transformation is given in Figure 2. The time complexity of the algorithm is \(O(n \log (m^2))\).

Transform \(R\)

**Input:** An array \(R\) of length \(n\).

1. for \(i \leftarrow 1\) to \(n\) do
2. \(R[i] \leftarrow g[i]\)
3. end for
4. for \(j \leftarrow 1\) to \(d\) do
5. \(V\text{mask}[j] \leftarrow 0\)
6. for each pattern \(p^i\) such that \(|p^i| = m_j\) do
7. \(V\text{mask}[j]_{a_j+1} \leftarrow 1\)
8. end for
9. end for
10. for \(i \leftarrow 1\) to \(n\) do
11. \(j \leftarrow 1, x \leftarrow Rr \leftarrow R[i], RE \leftarrow 0\)
12. for \(s \leftarrow 1\) to \(\lceil \log(m^2) \rceil - 1\) do
13. \(Rr \leftarrow Rr \gg 1\)
14. \(x \leftarrow x \wedge Rr\)
15. if \(s = \lceil \log(m_j^2) \rceil - 1\) then
16. \(RE \leftarrow x \wedge V\text{mask}[j]\)
17. \(j \leftarrow j + 1\)
18. end if
19. end for
20. \(g[i] \leftarrow RE\)
21. end for

Fig. 2. Transform \(R\) to \(g\).

When \(g\) is available, we next check each entry of \(g\) to find matches. For an entry \(x = g[i]\), we use an implicit binary tree \(T\) to find the id positions on which the bit values 1. Each tree node corresponds to a subset of the indication positions. A node \(u = [n_1..n_2]\) consists of the indication positions of \(p_j\) where \(j \in [n_1..n_2]\). The root is the set of all id positions, denoted by \([1..k]\). Each leaf contains one id position. For node \(u\), the two children are \([n_1..(n_1+n_2)/2]\) and \([(n_1+n_2)/2..n_2]\). We compute a bit mask \(Mask(u)\) from \(u\) as follows: the bits of \(Mask(u)\) on positions in \([n_1..n_2]\) are set to 1s, other bits are set to 0s.

We start at the root of \(T\) and check \(x \wedge Mask([1..k])\). If it is 0, then there is no match for patterns \(p^1, p^2, \ldots, p^k\). If it is not 0, at least one pattern matches, we continue to search in the left subtree of the root by checking \(x \wedge Mask([1..k/2])\). If it is 0 then there is no match for patterns \(p^1, p^2, \ldots, p^{k/2}\), and we prune the left branch; otherwise, at least one pattern matches, so we continue to search the left subtree. After searching in the left subtree, we search in the right subtree. In this manner we traverse \(T\) depth-first, checking \(x \wedge Mask(u)\) for each visited node \(u\) and pruning the branch if it is 0 along the way. At last, all the occurrences will be found by visiting the leaves corresponding to the matched patterns.

The time complexity is \(O(n + oc\log k)\). So the total time complexity for finding the matches in \(R\) is \(O(n \log (m^2) + oc\log k)\). For \(|P| > m\) and \(|P| \geq \sigma\), the total time complexity of Algorithm 1 is \(O(n \log |P| + oc\log k)\).

B. Prime number encoding based algorithm

In this section, we introduce another strategy for matching a set of patterns with wildcards. Other than concatenating the patterns to a long one, this method aligns the patterns and generates a composed pattern with the length of the longest pattern. The algorithm is based on a prime number encoding of patterns. Let \(m\) be the length of the longest pattern in \(P\). We extend each pattern to a similar length \(m\) by padding ‘0’s to the end of a pattern. Denote the resulting pattern set by \(P'\). By padding \(m\) ‘0’s to the end of the input, any matching of \(P'\) in \(i\) is exactly a matching of \(P'\) in \(i\) for the same pattern on the same position.

We first pick up \(k\) distinct prime numbers \(\rho_1, \rho_2, \ldots, \rho_k\). Denote \(M = \rho_1 \cdot \rho_2 \cdots \rho_k\). We further require that \(\rho_i\) are larger than \(|p^i|^2\).

Now we consider the encoding method. For \(k\) non-negative integers \(x_1, x_2, \ldots, x_k\), where each \(x_i\) is not greater than \(\sigma\), define an integer \(X\) that is less than \(M\), such that for \(1 \leq i \leq k\)

\[
X \equiv x_i \pmod{\rho_i}. \tag{9}
\]

According to the Chinese Remainder Theorem (CRT, in short) [11], define \(c_i = M/\rho_i, (M/\rho_i)^{-1} \pmod{\rho_i}\). For \(c_i\), we have \(c_i \equiv 1 \pmod{\rho_i}\) and \(c_i \equiv 0 \pmod{\rho_i'}, i' \neq i\).

By the CRT, \(X = \sum_{i=1}^{k} c_i \cdot x_i\).

We construct a composed pattern \(\gamma\) of length \(m\) from \(P\), where

\[
\gamma[i] = \sum_{j=1}^{k} c_j \cdot p^j[i], \text{ for } 1 \leq i \leq m, \tag{10}
\]

and another composed pattern \(\gamma'\) of length \(m\), where

\[
\gamma'[i] = \sum_{j=1}^{k} c_j \cdot (p^j)'[i], \text{ for } 1 \leq i \leq m. \tag{11}
\]

The text is encoded as follows:

\[
\tau[i] = \sum_{j=1}^{k} c_j \cdot t[i], \text{ for } 1 \leq i \leq n. \tag{12}
\]
\[ \tau' = \sum_{j=1}^{k} c_j \cdot t'[i], \text{ for } 1 \leq i \leq n. \]  

(13)

Since \( \rho_j > |p^j|\sigma^2 \) and \( d(p^j, t'[i]) < |p^j|\sigma^2 \), for \( 1 \leq j \leq k \), we have

\[ \gamma \mod \rho_j = p^j, \tau \mod \rho_j = t. \]  

(14)

Then we can use Clifford and Clifford algorithm to match \( P^r \) in \( t \) as follows. We compute Equation (5) for \( \gamma \) and \( \tau \) and get array \( d(\gamma, \tau) \). For an arbitrary array \( A \), define \( A \mod p \) as the array of the same length of \( A \) where \( (A \mod p)[i] = A'[i] \mod p \). According to the CRT, for \( 1 \leq j \leq k \), we have

\[ d(\gamma, \tau)[i] \mod \rho_j = d(\gamma \mod \rho_j, \tau \mod \rho_j)[i] = d(p^j, t'[i]), \text{ for } 1 \leq i \leq n. \]  

(15)

Suppose that a pattern \( p^j \in P \) occurs in the text \( t \) starting from position \( i \). We have \( d(p^j, t'[i]) = 0 \). So, \( d(\gamma, \tau)[i] \mod \rho_j = 0 \). If \( p^j \) does not occur in \( t \) starting from \( x \), we have \( d(\gamma, \tau)[i] \mod \rho_j = 0 \). To find the matches of patterns, we have to check every entry of \( d(\gamma, \tau)[i] \mod \rho_j = 0 \). This straightforward method can find the occurrences in \( O(nk) \) time. However, by using the method of Linhart and Shamir [17], the time complexity can be reduced to \( O(n + \sigma \log k) \).

For \( x = d(\gamma, \tau)[i] \), we use an implicit binary tree \( T \) to find \( \rho_j \) such that \( x \mod \rho_j = 0 \). Each tree node contains one pattern. For a node \( u = [n_1, n_2] \), the two children are \([n_1, (n_2 + n_1)/2]\) and \([(n_2 + n_1)/2, n_2]\). For node \( u \), we compute an integer \( Modul(u) \) as follows:

\[ Modul(u) = \rho_{n_1 \cdot \rho_{n_1+1} \cdots \rho_{n_2}}. \]

We start at the root of \( T \) and check \( x \mod M \). If it is not 0, then there is no match for patterns. If it is 0, at least one pattern matches, we continue to search in the left subtree of the root by checking \( x \mod \rho_{n_1 \cdot \rho_{n_1+1} \cdots \rho_{n_2}} \). If it is not 0 then there is no match for patterns \( p^1, p^2, \ldots, p^k/2 \), and we prune the left branch; at least one of these patterns matches, so we continue to search the left subtree of the root. We traverse \( T \) in a depth-first manner, pruning some of the branches along the way. In the end, all the occurrences will be found by visiting the leaves corresponding to the matched patterns.

The algorithm takes \( O(n \log m) \) time to compute \( R \) and \( O(n + \sigma \log k) \) time to check \( R \) to find whether there is any occurrence of patterns. The total running time is \( O(n \log m + \sigma \log k) \).

Now we consider the size of words used in the algorithm. For the number \( \pi(x) \) of primes not exceeding \( x \), the following bound is well known:

\[ \forall x > 17 \quad \frac{x}{\ln x} < \pi(x) < 1.26 \cdot \frac{x}{\ln x}. \]  

(16)

According to the bound, we can verify that for \( m\sigma^2 > 17 \),

\[ \frac{2m^2 \sigma^2 + k^2}{2 \ln(2m^2 + k^2)} - \frac{1.26 \cdot m\sigma^2}{\ln(m\sigma^2)} > k. \]  

(17)

Thus, if we search for prime numbers between \( m\sigma^2 + 1 \) and \( 2m^2 + k^2 \), we will find at least \( k \) prime numbers. Using the sieve algorithm of Atkin et al. [3] it takes \( o(2m\sigma^2 + k^2) \) time to find these prime numbers. Each prime number is at most \( \lg(2m\sigma^2 + k^2) \) bits long. Therefore, \( \lg M \leq k \lg(2m\sigma^2 + k^2) \). Thus, each entry of \( R \) uses \( \lg(2m\sigma^2 + k^2) \) bits.

IV. HAMMING DISTANCE BASED APPROACH

A. Hamming distance of bit vectors with wildcards

Let \( B = b_1b_2 \ldots b_n \) be a binary pattern with wildcards. Denote \( \overline{b_1b_2 \ldots b_n} \) by \( B \) where if \( b_i = \ast \) then \( \overline{b_i} = \ast \). Given a binary pattern \( p \) and a bit string \( t \) (is assumed to be a cyclic vector), such that both \( p \) and \( t \) have wildcards. The matchings between the pattern and a factor of \( t \) of length \( m \) have seven cases: 01, 10, 00, 0*, and *. Alignments. The Hamming distance between the two strings is the sum of the number of nonmatchings, that is \#10 + \#01.

The Hamming distance between \( p \) and the factor starting from each position of \( t \) of length \( m \) can be computed by the following method: For a bit vector \( v \) with wildcards, denote by \( h^x(v) \) the bit vector where each wildcard of \( u \) is replaced by the bit \( x \). Then \( (h^0(\overline{p}) \oplus h^0(t)[i]) \) equals \#10 between \( p \) and the factor of \( t \) starting from \( i \), and \( (h^0(\overline{p}) \oplus h^0(t)[i]) \) equals \#10 between the same pair of strings. The Hamming distance between \( p \) and the factor of \( t \) starting from \( i \) can be computed by the following:

\[ H(p, t)|i = (h^0(\overline{p}) \oplus h^0(t)[i]) + (h^0(\overline{p}) \oplus h^0(t)[i]). \]  

(18)

This distance can be computed by two convolutions.

B. Multi-pattern matching by Hamming distance of bit vectors

We present an algorithm for multi-pattern matching with wildcards based on Hamming distance. In this approach, we derive \( \lg \sigma \) bit vectors from the input text and \( \lg \sigma \) patterns from the pattern and then perform \( \lg \sigma \) times of pattern matchings for \( \lg \sigma \) pairs of pattern and bit text. For \( 1 \leq s \leq \lg \sigma \), denote the \( s \)th bit of the integer encoding of a character \( c \in \Sigma \) by \( c_s \), for \( c = \ast, c_s = \ast \). For a string \( t \) over \( \Sigma \), denote the bit vector \( t[1]_{\ast}t[2]_{\ast} \ldots t[n]_{\ast} \) by \( t_s \).

To match a set of patterns, we first construct a series of patterns \( B_{(1)}, B_{(2)}, \ldots, B_{(|\lg \sigma|)} \) from \( P \), such that for \( 1 \leq s \leq \lg \sigma \),

\[ B_s = 2a_1p^1_s \cdot 2a_2p^2_s \cdots 2a_kp^k_s \]

where \( a_0 = 0, a_j = \sum_{i=1}^{j-1} \lg |p^i_s| \).
Fig. 3. An example of the run of Algorithm 3. Pattern \( p_2 \) is matched in \( t^2 \). The matching position is \( i + |p_1| + |p_2| \) and \( R^2[i] = \{ a_3 + 1 \ldots a_4 \} = 0 \).

Define

\[
B_{(s,0)} = 2^{2s}h^0(p_{[1]}^s) \cdot 2^{2s+1}h^0(p_{[2]}^s) \cdots 2^{2s+k}h^0(p_{[k]}^s),
\]

\[
\tilde{B}_{(s,0)} = 2^{2s}h^0(p_{[1]}^s) \cdot 2^{2s+1}h^0(p_{[2]}^s) \cdots 2^{2s+k}h^0(p_{[k]}^s). \quad (19)
\]

According to the computing of Hamming distance, for each \( B_{(s)} \) and \( P \), we compute

\[
R^s[i] = (\tilde{B}_{(s,0)} \oplus h^0(T[i]))[i] + (h^0(T[i]) \oplus B_{(s,0)})[i],
\]

for \( 1 \leq i \leq n \). \quad (20)

At last we compute the resulting array \( R \) such that

\[
R[i] = R^1[i] \lor R^2[i] \lor \ldots \lor R^{[\log_2 \sigma]}[i], \quad 1 \leq i \leq n.
\]

By checking whether \( R[i] \mod \sigma = 1 \) we will know whether \( p^j \) occurs at position \( (i + o_j) \mod n \) in \( t \).

According to the definition of \( B_{(s)} \), we can see that for a resulting array \( R^s \) computed by the algorithm,

\[
R^s[i] = \sum_{j=1}^{k} H(p_{[j]}^s, t[i])[i + o_j]2^{s_j}, \quad 1 \leq i \leq n. \quad (22)
\]

For \( p^j \in P \), we have \( H(p_{[j]}^s, t[i])[i + o_j] \leq |p^j| \).

So, \( a_{j+1} - a_j = \lfloor \log |p^j| \rfloor \) bits are enough to represent \( H(p_{[j]}^s, t[i])[i + o_j] \). Then we have

\[
H(p_{[j]}^s, t[i])[i + o_j] = R^s[i] \mod 2^{a_{j+1} - a_j}.
\]

Thus we can get \( H(p_{[j]}^s, t[i])[i + o_j] \) by computing \( (R^s[i] \mod 2^{a_{j+1}})/2^{a_j} \).

To see why this method works, suppose that a pattern \( p^j \in P \) occurs in \( t \) starting from position \( x \). We have

\[
H(p_{[j]}^s, t[i])[x] = 0 \quad \text{for} \quad 1 \leq s \leq \lfloor \log \sigma \rfloor.
\]

Otherwise \( H(p_{[j]}^s, t[i])[x] \neq 0 \) for at least one \( s \). Let \( x = i + o_j \).

Since \( R^s[i][i+1:a_{j+1}] = H(p_{[j]}^s, t[i])[i+o_j] \), we have that

\[
\sum_{s=1}^{\lfloor \log \sigma \rfloor} R^s[i][a_{j+1}:a_{j+1} + 1] = 0 \quad \text{indicates} \quad H(p_{[j]}^s, t[i])[i + o_j] = 0,
\]

for \( 1 \leq s \leq \lfloor \log \sigma \rfloor \). Thus \( R[i][a_{j+1}:a_{j+1} + 1] = 0 \) indicates that \( p^j \) occurs at position \( (i + o_j) \mod n \) of \( t \).

An example of the running of this algorithm is shown in Figure 3 where we only illustrate one bit vector.

The algorithm is given in Figure 4.

Algorithm 3 takes \( O(n \log |P| \log \sigma) \) time to compute \( R \) and uses \( O(nk) \) time to check whether there is any occurrence of patterns. By the method in Section III-A, the time for checking matches in \( R \) can be reduced to \( O(n \log m + o_c \log k) \). The total time complexity is therefore \( O(n \log |P| \log \sigma + o_c \log k) \). The words used in the algorithm are of size \( w = \sum_{i=1}^{k} \lfloor \log |p^i| \rfloor \).

If we use \( O(n \log m) \) time single pattern matching algorithm to match each pattern one by one, the total time is \( O(n \sum_{i=1}^{k} \lfloor \log |p^i| \rfloor + o_c \log k) = O(nw + o_c \log k) \). Thus if \( w > \log |P| \log \sigma \), Algorithm 3 takes less time for matching \( P \) against \( t \) than the time taken by single pattern matching algorithms. If \( w \leq \log |P| \log \sigma \), we can partition the pattern set into a set of subsets of \( P \) guaranteeing that \( w \leq \log |L_P| \log \sigma \) for each subset \( LP \). We then match all these subsets of \( P \) using our algorithm one by one. The overall running time is certainly less than the time for running a single pattern algorithm for all the patterns.

If the occurrences of patterns are rare, we can revise Algorithm 3 to have a better performance. Recall that to use FFT efficiently, the input text is split into \( n/w \) pieces of length \( 2/P \) and each piece is matched against \( P \) independently. Then in the running of Algorithm 3, the pattern set is matched against every piece. Denote the piece starting at position \( (l - 1)|P| + 1 \) in \( t \) where...
1 ≤ l ≤ n/P] by piece(t, l). The resulting arrays computed by Algorithm 3 taking piece(t, l) and P as input are denoted by $R^i(l)$, for each $1 ≤ s ≤ \lfloor \log_2 \sigma \rfloor$. We have $R^i(l)[i] = R^i[i + (l - 1)/P]$ where $1 ≤ i ≤ |P|$. Algorithm 3 is revised as follows. In computing $R^i$, for piece $piece(t[s], l)$, if $R^i(l)[i/m, a[i+1..a[i+1]] ≠ 0$ for all $1 ≤ i ≤ |P|$, then we have that no pattern occurs in piece $piece(t, l)$. In the followed computing, all the pieces $piece(t[s], l)$ where $s' > s$ are neglected. That is to say, $piece(t[s], l)$ will not be matched against $B(t[s')]$. The revised algorithm is correct for the situation that no pattern occurs in the neglected pieces. Since the occurrences of patterns are rare, the number of inspected pieces of bit vectors of the input by the revised algorithm is small. The cost is that in processing a bit vector of the input, the revised algorithm has to check the resulting array to determine which piece of the input can be neglected. In the original algorithm, the check is done only once when all the convolutions are finished.

V. FFT BASED ON INTEGER ARITHMETIC

In this section, we give an FFT implementation based on modular arithmetic other than complex numbers. In [21], Yap and Li present a fast integer multiplication algorithm based on an efficient FFT implementation built on the integer arithmetic. The approach aims at reducing the overhead in the implementation of FFT on machines in which the words are 32-bit or 64-bit. This FFT implementation fits the context of our multi-pattern matching algorithm. The basic idea of the approach is to perform FFT in the ring $\mathbb{Z}_M = \{0, 1, \ldots, M - 1\}$ of numbers modulo $M$. Here we choose $M$ as a specially chosen prime number. In a $w$-bit machine, $M$ is at most $w$-bits so that the component-wise product can be done in $O(1)$ machine operations. For an $n$-length vector, in the field $\mathbb{Z}_M$, we need primitive $n$-th roots of unity. According to [21], the modular $M$ is a prime number that can be expressed as

$$M = nd + 1. \quad (24)$$

We next pick up a primitive element of $\mathbb{Z}_M$, say $e$, and set

$$\omega = e^{d} \mod M. \quad (25)$$

Then we have that each of $\omega^n \mod M$ is distinct and $\neq 1$, for $i = 1, 2, 3, \ldots, n - 1$. We have $\omega^n \equiv e^{nd} \equiv e^{M-1} \equiv 1(\mod M)$ by Fermat’s little theorem. That is $\omega$ is an $n$-th primitive root of unity.

In practice, to proceed the recursion of FFT, we need $n$ to be a power of 2. For the case that the machine word is 32 bits, Yap and Li [21] choose $M_{32} = 2, 013, 265, 921$ as the modulo, a prime number with only 31 bits that can be expressed as $M_{32} = 2^{27}d + 1$ where $d = 15$ and $n = 2^{27}$. The number 31 can be proved to be a primitive element of $\mathbb{Z}_{M_{32}}$. The primitive $2^{27}$-th roots of unity can be chosen as $\omega = 311^{27} \mod M_{32} = 440, 564, 289$. Using $\omega$, we can implement the FFT algorithm on a 32-bit machine, where we compute everything mod $M_{32}$ and each component of the vectors is of the length 27 bits.

Based on Yap and Li’s approach, we present the parameters for the 64-bit architecture. We choose $M_{64} = 6, 269, 010, 681, 299, 730, 433$ as the modulo, a prime number with only 63 bits. $M_{64}$ has similar properties with $M_{32}$, which can be expressed as $M_{64} = 2^{32}d + 1$ where $d = 87$ and $x = 56$. Among the prime numbers that can be expressed as $2^{2d} + 1$, $M_{64}$ has the maximal $x$. The number 5 turns out to be the smallest primitive element of $\mathbb{Z}_{M_{64}}$. The primitive $2^{32}$-th roots of unity can be chosen as

$$\omega = 5^{87} \mod M_{64} = 4, 467, 632, 415, 761, 384, 939.$$

Using $\omega$, we can implement the FFT algorithm on a 64-bit machine, where we compute everything mod $M_{64}$ and each component of the vectors is of the length 56 bits.

VI. PARALLELIZED ALGORITHMS

The algorithms in this paper can be easily parallelized. For Algorithm 1, we design a parallel multi-pattern matching algorithm with no communication. According to the trick in Section II, we first split the text into $n/P$ pieces each of length $2/P$. The starting positions of these pieces are in the set $\{i | P + 1 | 0 ≤ i < n/P\}$. The convolution between the composed pattern and each piece of the text is computed using FFT in time $O(|P| \log |P|)$ per piece. We do not use the parallelized FFT, but use the sequential version of FFT conducted by processors. As each piece can be matched independently, no communication is needed. Since each piece is of length $2/P$ and completely overlaps with the adjacent pieces, any occurrence of a pattern will keep integrate in at least one piece. Therefore, the parallelized Algorithm 1 is correct. On a $q$-processor PRAM model, the overall time complexity of parallelized Algorithm 1 is $O(n \log |P| + \text{ooc log } k)/q$.

Algorithm 2 and Algorithm 3 can be parallelized in the same way as Algorithm 1. Readily, the time complexity of the parallelize Algorithm 2 and Algorithm 3 on a $q$-processor PRAM model are $O((n \log m + \text{ooc log } k)/q)$ and $O((n \log |P| \log \sigma + \text{ooc log } k)/q)$ accordingly.

VII. CONCLUSION AND FURTHER RESEARCH

We have presented three algorithms for multi-pattern matching with wildcards. The first one can find the matches of a small set of patterns in a text in $O(n \log |P| + \text{ooc log } k)$ time. The words used in the algorithm are of size $k[2 \lg \sigma + \sum_{i=1}^{k} \lg |p_i|]$ bits. The second algorithm finds the occurrences of patterns in time $n \log m + \text{ooc log } k$ based on a prime number encoding of the pattern set and the text, using the words of $k[\lg(2nα + k^2)]$ bits. The last algorithm is based on Hamming distance between bit vectors. It runs in $O(n \log |P| \log \sigma + \text{ooc log } k)$ time and uses the words with $\sum_{i=1}^{k} \lg |p_i|$ bits. If the number of wildcards in the patterns is very small, the problem can be solved by building a deterministic finite automaton (DFA) that detects all possible words that match the pattern. One advantage of our algorithm over finite automata based algorithms is that the patterns in our algorithm need not
be preprocessed but taken as the input. In automata based approaches, the pattern set is used to build automata that will be further optimized for a low memory usage or a better performance. Whenever built, it is difficult to add or remove patterns from the existing data structures for the automata. In our approach, as patterns are taken as the input, adding or deleting a pattern has very low costs, thus our approach has more flexibilities.

It remains to determine whether there exists an $O(n \log |P|)$ algorithm using words of a proper size. That is in the resulting array, each bit of an entry indicates whether a pattern occurs at the corresponding location of the text. The modifications on FFT itself would be necessary to reach this goal.

**ACKNOWLEDGMENTS**

This work was in part supported by Fundamental Research Funds for the Central Universities No.200903186, Education Department of Jilin Province No.599, NSF of Science & Technology Department of Jilin Province No.201101522, NSF grant OCI 0904179, Chinese NSF No.60703024 and Program for New Century Excellent Talents in University NCET-09-0428.

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