An Interior Point Penalty Method for Utility Maximization Problems in OFDMA Networks

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Abstract—This paper investigates the non-convexity of utility-based resource allocation problems in orthogonal frequency division multiple access (OFDMA) networks with heterogeneous traffic classes. Efficient transmission in OFDMA networks requires optimal resource allocation to users based on their current channel states. Also, utility-based resource allocation improves the network resource utilization and application level quality of service (QoS) provisioning. However, a major difficulty in using utility-based OFDMA resource allocation schemes is the non-convexity of corresponding optimization problem. In this paper, a continuous optimization technique is proposed to treat the non-convexity. The approach is based on a combination of penalty function methods and interior point methods. Numerical results demonstrate that the proposed approach solves the problem within limited time, and the solutions are close to near optimal solutions obtained by the search algorithm.

I. INTRODUCTION

Scarce spectrum and power of wireless networks in one hand, and diverse arrival traffic and bandwidth demand of heterogeneous services on the other hand, make the resource allocation problem very critical and challenging in the next generation wireless broadband networks. In practice, the procedure of allocating resource is modeled as an optimization problem where the objective function and constraints are determined based on services or users’ requirements and networks specifications.

A utility function, shortly utility, quantifies a user satisfaction of allocated rate [1]. For example, the utility of voice traffic is usually represented by a step function of rate which indicates that the user expects a threshold rate. As utilities represent the utilization of allocated rate by users numerically, they are used in utility based resource allocation schemes that maximizes networks resource utilization subject to the resource limits.

The problem of utility-based resource allocation was first addressed in [2] for wired networks and later found popular applications in fair allocation of resources, i.e., utility-fair resource allocation, both in wired and wireless networks [3]–[8]. Recently, the problem of utility-based OFDMA resource allocation, i.e., sub-carriers assignment to users and power allocation to sub-carriers, has attracted much research attention because the OFDMA technology is emerging in most wireless access technologies, such as, WiMAX, UWB, and WLANs. A major challenge in utility-based OFDMA resource allocation is the non-convexity of the optimization problem which usually is a mixed integer non-linear programming (MINLP) problem. The feasible region, i.e., the set of feasible solutions that satisfy all constraints, of the MINLP problem is non-convex due to exclusive sub-carrier assignment in OFDMA. In addition to the non-convexity of the feasible region, nonlinear or non-convex utility functions, when deployed in the problem, contribute to difficulty of solving the problem. For many concave utility functions, the MINLP problem can be solved by using some convex optimization techniques when the feasible region of the MINLP problem is approximated with a convex one [9]–[12]. However, approximating the feasible region reduces the accuracy of the solution, and the utility functions are not always concave in practice.

In this paper, we propose a new approach for the non-convexity of the OFDMA resource allocation problem. Unlike the existing approaches in the literature that use discrete optimization techniques to give an approximate solution, see [13] and the references therein, we propose a continuous optimization approach to solve the problem. Our proposed approach is based on a combination of penalty methods and interior point methods (PM/IPM) [14], [15]. We apply PM/IPM to a continuous model of the OFDMA resource allocation problem proposed in our earlier work [16]. The proposed approach is very comprehensive in the sense that users can have heterogeneous rate requirements and the objective function of the resource allocation scheme may be non-convex. We will compare the solutions obtained by PM/IPM with near optimal solutions obtained by an iterative search algorithm to evaluate the performance of the proposed approach in terms of convergence time and closeness of the solutions to near optimal solutions.

The remainder of the paper is organized as follows. System model for OFDM sub-carrier and power allocation problem is presented in section II. The PM/IPM algorithm is described in section III, and numerical results are presented in section IV. The paper is concluded in section V.

II. PROBLEM FORMULATION

The OFDMA resource allocation problem is presented in this section. The network platform, shown in Fig. 1, consists of...
a central controller, named base-station (BS), and several users located in one hop neighborhood from BS in a point to multi-point manner. Traffic arriving to BS is buffered in separate infinite queues dedicated to each user and then forwarded to users on the down-link path. BS assigns sub-carriers and allocates a fraction of the total BS power, \( P_{BS} \), to users on the down-link path. BS assigns sub-carriers and infinite queues dedicated to each user and then forwarded point manner. Traffic arriving to BS is buffered in separate queues located in one hop neighborhood from BS in a point to multi-point format, a central controller, named base-station (BS), and several users based on the network objective and resource constraints. The network parameters used in the optimization model are defined in Table I.

**Fig. 1. Network platform**

TABLE I

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M )</td>
<td>total number of users in the network</td>
</tr>
<tr>
<td>( K )</td>
<td>total number of sub-carriers in the network</td>
</tr>
<tr>
<td>( i )</td>
<td>user index belongs to ( \mathcal{M} := {1, 2, \ldots, M} )</td>
</tr>
<tr>
<td>( j )</td>
<td>sub-carrier index belongs to ( \mathcal{K} := {1, 2, \ldots, K} )</td>
</tr>
<tr>
<td>( \alpha_{ij} )</td>
<td>channel gain of user ( i ) on sub-carrier ( j )</td>
</tr>
<tr>
<td>( P_{ij} )</td>
<td>allocated power to user ( i ) on sub-carrier ( j )</td>
</tr>
<tr>
<td>( r_{ij} )</td>
<td>allocated rate to user ( i ) on sub-carrier ( j )</td>
</tr>
<tr>
<td>( P_{min} )</td>
<td>minimum service rate requirement of user ( i )</td>
</tr>
<tr>
<td>( P_{BS} )</td>
<td>BS total power budget</td>
</tr>
<tr>
<td>( U_i )</td>
<td>utility function of user ( i )</td>
</tr>
</tbody>
</table>

In our model, a solution of the resource allocation problem is denoted by a rate allocation vector \( r \):

\[
\mathbf{r} = [r_{11}, r_{12}, \ldots, r_{1K}, \ldots, r_{M1}, \ldots, r_{MK}]^T. \tag{1}
\]

The objective of the optimization problem is to maximize aggregate users’ utilities subject to the following constraints:

- Allocated rate to a user should be more than its minimum rate requirement;
- BS power is the upper bound for total allocated power to users over all sub-carriers;
- A sub-carrier can be allocated to only one user;

These constraints along with the objective function form the optimization problem \( P_1 \):

\[
P_1 : \quad \max_r \sum_{i=1}^{M} U_i(r) \tag{2}
\]

\[
\text{s.t} \quad \sum_{j=1}^{K} r_{ij} \geq R_{min}^i \forall i \in \mathcal{M}, \tag{3}
\]

\[
\sum_{i=1}^{M} \sum_{j=1}^{K} \frac{1}{\alpha_{ij}} (2^{\frac{r_{ij}}{P_{min}} K_{K}^i} - 1) \leq P_{BS} \forall i \in \mathcal{M}, \forall j \in \mathcal{K}, \tag{4}
\]

\[
r_{ij} \leq 0 \forall i \in \mathcal{M} \setminus \{i\} \forall j \in \mathcal{K}, \tag{5}
\]

\[
0 \leq r_{ij}, \forall i \in \mathcal{M}, \forall j \in \mathcal{K}. \tag{6}
\]

The feasible region of problem \( P_1 \) is closed and bounded. Thus, when the objective function is a continuous function of \( r \), Weierstrass Theorem [17] implies that problem \( P_1 \) has global optimal solution(s). Although Weierstrass Theorem guarantees that a global optimal solution exists, finding such a global solution for \( P_1 \) is hard as explained before.

### III. PENALTY FUNCTION AND INTERIOR POINT METHODS

We propose a solution based on a combination of penalty function and interior point methods for problem \( P_1 \). The success of interior point methods in solving a non-convex nonlinear problem strongly depends on how non-convexity of the problem is treated. We apply a penalty function method to deal with the non-convexity of \( P_1 \). In other words, using a penalty function method, we convert \( P_1 \) to a new problem with convex feasible region, and then, an interior point method is applied to solve the problem.

In problem \( P_1 \), all constraints except the set of constraints (5) are convex. We add this set of constraints to the objective function as a penalty term which is negative when one of the constraints is violated, and zero otherwise. After adding the penalty term to the objective function, the new objective becomes:

\[
\max_r u(r) = \sum_{i=1}^{M} U_i(r) - \frac{L}{2} \sum_{i=1}^{M} \sum_{j=1}^{K} r_{ij} r_{ij}, \tag{7}
\]

where positive constant \( L \) is the penalty parameter. The new objective function along with the constraints of \( P_1 \) form the following problem:

\[
P_2 : \quad \max_r u(r) \tag{8}
\]

\[
\text{s.t} \quad C(r) \geq 0, \tag{9}
\]

where \( C(r) \) is the vector of inequality constraints (3), (4) and (6), and it is represented as follows:

\[
C(r) = \left( \begin{array}{c}
\sum_{j=1}^{K} r_{1j} - P_{min}^1 \\
\vdots \\
\sum_{j=1}^{K} r_{Mj} - P_{min}^M \\
- \sum_{i=1}^{M} \sum_{j=1}^{K} \frac{1}{\alpha_{ij}} (2^{\frac{r_{ij}}{P_{min}} K_{K}^i} - 1) + P_{BS}
\end{array} \right). \tag{10}
\]
Instead of solving $P_1$, we solve $P_2$ whose feasible region is convex. However, an optimal solution of $P_2$ will not be an optimal solution of $P_1$, unless the (positive) penalty term is zero. The larger $L$ is, the more penalized the constraint violations of penalty term is. Indeed, for a large enough choice of $L$, global optimal solution(s) of $P_2$ is (are) optimal solution(s) of $P_1$ [14]. An appropriate value for $L$ can be found through a simple search method. Even though the objective function of $P_2$ is a non-concave nonlinear function, but its feasible region is convex. Convexity of the feasible region suggests to use interior point methods to solve $P_2$.

Before applying the interior point method, we first convert the inequality constraints in $C(r)$ to equality constraints by associating a positive slack variable to each constraint. Denote the $(2M+1)K$ vector of slack variables with $s$. Hence, $P_2$ is converted to the following minimization problem:

$$P_3 : \begin{align*}
    \min_t & \quad -u(r) \\
    \text{s.t} & \quad C(r) - s = 0, \\
    & \quad s \geq 0.
\end{align*}$$  \hspace{1cm} (11)

A necessary condition for a feasible solution of $P_3$ to be optimal is to satisfy the following conditions, called Karush-Kuhn-Tucker (KKT) conditions:

$$\begin{align*}
    \nabla u(r) - A^T(r)z &= 0, \quad (14) \\
    C(r) - s &= 0, \quad (15) \\
    Sz &= 0, \quad (16) \\
    s \geq 0, \quad z \geq 0. \quad (17)
\end{align*}$$

In the aforementioned KKT conditions, $S$ is a diagonal matrix with diagonal elements given by vector $s$, and vector $z$ contains $(2M+1)K$ Lagrange multipliers used in the definition of the Lagrangian function of $P_3$:

$$\mathcal{L}(r, s, z) = u(r) - z^T(C(r) - s). \quad (18)$$

The matrix $A$ in (14) is the Jacobian matrix of $C(r)$ represented by:

$$A = \begin{pmatrix}
    -K \ln(2) \frac{r_s}{r_{b,r}} & \Theta & 0 \\
    \vdots & \vdots & \vdots \\
    -K \ln(2) \frac{K r_{M,K}}{b_{s,M,K}} & \vdots & \vdots \\
    \end{pmatrix},$$  \hspace{1cm} (19)

where $I$ is an identity matrix of dimension $MK \times MK$, and $\Theta$ is the following $M \times MK$ matrix:

$$\Theta = \begin{pmatrix}
    1_{(1,K)} & 0_{(1,K)} & \cdots & 0_{(1,K)} \\
    0_{(1,K)} & 1_{(1,K)} & \cdots & 0_{(1,K)} \\
    \vdots & \vdots & \ddots & \vdots \\
    0_{(1,K)} & 0_{(1,K)} & \cdots & 1_{(1,K)}
\end{pmatrix},$$  \hspace{1cm} (20)

where $1_{(1,K)}$ and $0_{(1,K)}$ are $K$ vectors of $1$ and $0$, respectively.

To find an approximation for a local optimum of the nonlinear problem, interior point algorithms solve a series of perturbed KKT conditions in which only the right-hand-side in equation (16) is replaced by a vector $\mu e$:

$$\begin{align*}
    \nabla u(r) - A^T(r)z &= 0, \quad (21) \\
    C(r) - s &= 0, \quad (22) \\
    Sz &= \mu e, \quad (23) \\
    s \geq 0, \quad z \geq 0, \quad (24)
\end{align*}$$

with $e = [1, 1, \ldots, 1]^T$ and $\mu > 0$. Interior point methods start with an initial interior point in the feasible region that satisfies perturbed KKT conditions for some $\mu$ and proceeds to find another interior point that satisfies perturbed KKT conditions for a smaller value of $\mu$. As the algorithm proceeds, $\mu$ is decreased, and consequently the solution of the perturbed KKT conditions approaches the solution of the KKT conditions, in which $\mu = 0$. It is expected that after several iterations the solution will converge to a point that satisfies the KKT conditions of the problem [14].

In each iteration of the interior point method, the directions and lengths of movements from one point to another point are updated based on the first and second order gradients of objective function and constraints. At each iteration, movement directions for variables $r$, $s$, and $z$, i.e., $b = [b_r, b_s, b_z]^T$ is computed by solving the following linear system of equations:

$$\left( \begin{array}{ccc}
    \nabla^2 \mathcal{L} & 0 & -A^T(r) \\
    0 & S & 0 \\
    A(r) & -I & 0 \\
\end{array} \right) \left( \begin{array}{c}
    b_r \\
    b_s \\
    b_z \\
\end{array} \right) = \\
\left( \begin{array}{c}
    \nabla_r u(r) - A^T(r)z \\
    Sz - \mu e \\
    C(r) - s \\
\end{array} \right),$$  \hspace{1cm} (25)

where $Z$ denotes the diagonal matrix whose diagonal elements are given by vector $z$. Matrices $\nabla^2 \mathcal{L}$ and $\nabla_r u(r)$ depend on the chosen utility functions of the problem.

After obtaining movement directions, the length of movement in each direction, step length, denoted with $\alpha^\text{max}_s$ and $\alpha^\text{max}_z$, are specified as below:

$$\begin{align*}
    \alpha^\text{max}_s &= \max \{ \alpha \in (0,1] : s + \alpha b_s \geq (1 - \tau) s \}, \quad (26) \\
    \alpha^\text{max}_z &= \max \{ \alpha \in (0,1] : z + \alpha b_z \geq (1 - \tau) z \}, \quad (27)
\end{align*}$$

where $\tau \in (0,1)$. A Large value of $\tau$ close to one, e.g., $\tau = 0.995$, is usually chosen to avoid $s$ and $z$ approach zero too quickly. The new interior point, slack variables, and Lagrange multipliers, $(r^+, s^+, z^+)$, are determined with the information of movement directions and step lengths accordingly:

$$\begin{align*}
    r^+ &= r + \alpha^\text{max}_s b_r, \quad (28) \\
    s^+ &= s + \alpha^\text{max}_s b_s, \quad (29) \\
    z^+ &= z + \alpha^\text{max}_z b_z. \quad (30)
\end{align*}$$

For the next iteration, $\mu$ is updated to a smaller value, say $\mu^- = \mu^- \sigma < \mu$. There are several strategies to choose $\mu^-$. Among them we use a linear equation to update $\mu$:

$$\mu^- = \sigma \mu \quad \sigma \in (0,1).$$  \hspace{1cm} (31)
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Since $\sigma < 1$, $\mu$ approaches to zero over several iterations. However, choosing a very small $\sigma$ or a very large $\sigma$ will cause faster or slower convergence, respectively. Although fast convergence is always desired, it may cause some algorithm parameters, such as $s$ and $z$, approaching zero too quickly. This may reduce the performance of the algorithm, e.g., the offered solution may be infeasible or far from optimality.

The interior point algorithm is terminated when a stopping criterion is achieved. In this paper, the initial value of $\mu_0 = 1$ has been chosen, and when $\mu$ approaches to a very small value or the change in allocated rate vector, $r$, is negligible, the algorithm stops. Algorithm 1 presents a summary of the interior point algorithm used in our simulation.

Algorithm 1 The interior point algorithm for $P_3$

Input: $M, K, P_{BS}, B, \alpha, U_i, initial_x, s_0, \mu_0, r, \sigma$

Result: $r$

begin

Setting up and initialization:
Choose $initial_x$ and compute $s_0 > 0$.
Choose $\mu_0 > 0$ and compute $z_0 > 0$ accordingly.
Set parameters $r \in (0, 1)$ and $\sigma \in (0, 1)$.
Set $k = 0$.

while Exit_flag == 0 do

Solve (25) to obtain movement direction $h = (b_1, b_2, b_3)$.
Compute $\alpha_{max}^{\mu} \text{ and } \alpha_{max}^\sigma$ using (26) and (27).
Compute $(y^{k+1}, z^{k+1}, b^{k+1})$ using (28) to (30).
Set $p^{k+1} = p^k$ and $\sigma^{k+1} = \sigma^k + 1$.
Compute $Exit_flag$.
end

return $r$.

end

IV. NUMERICAL RESULTS

In this section, we evaluate PM/IPM solutions in terms of optimality and sensitivity to network parameters by comparing with a genetic algorithm (GA) ones. The iterative GA search algorithm is implemented in our earlier work [16].

We consider the network platform, shown in Fig. 1, for our simulation. The objective function is aggregate utility maximization. There are two sets of users with concave and convex utility functions expressed with equation (32) [18].

$$U_i(r) = \begin{cases} 
0 & \text{if } r \leq l_1, \\
\sin^k \frac{\pi}{2} \frac{r_{i1} - l_1}{l_2 - l_1} & l_1 < r \leq l_2, \\
1 & r > l_2.
\end{cases}$$  (32)

$r_i$ denotes allocated rate to user $i$, $l_1$ and $l_2$ are thresholds, and $k$ controls the shape of the utility function. The function is concave for $k < 1$ and convex for $k > 1$. $k = 0.7$ and $k = 2$ have been chosen for concave and convex utility functions, respectively. Other simulation parameters are listed in Table II.

A network of 4 users with the same convex utility functions, but diverse channel gain on sub-carriers, is considered. A small number of users is used because GA results are intractable for large number of users. A comparison between the convergence speed of GA and PM/IPM is shown in Fig. 2. The iterations of the algorithms stop when the improvement in rate allocation vector is less than $1e-13$. GA has a very slow convergence speed, although it starts from an initial allocation with better aggregate utilities than the ones of PM/IPM. In comparison, PM/IPM converges very fast while its maximum achievable aggregate utilities and convergence time depend on the value of $\sigma$. The smaller is $\sigma$, the faster is the algorithm, and the less accurate is the result. The data tips on the diagram show the time and aggregate utilities at the data tips with $x$ and $y$, respectively. It can be seen that, at 29.61 sec, PM/IPM with $\sigma = 0.95$ obtains the same aggregate utilities as the one of GA, i.e., 3.609, which is obtained in about 8916 sec. When $\sigma$ increases beyond 0.99, PM/IPM has no further improvement in achievable aggregate utilities or convergence speed.

A comparison between rate allocation of GA and PM/IPM, shown in Fig. 3, demonstrates the performance of PM/IPM in recognizing diverse channel status and capability of PM/IPM in allocating resources. It is assumed that users 1 and 3 as well as users 2 and 4 have the same channel status, besides average channel gain on sub-carriers is higher for users 1 and 3 than those of users 2 and 4. Therefore, the algorithms should allocate more resources to the users with better average channel quality to gain user diversity and maximize aggregate utilities. Fig. 3 illustrates users’ utilities versus rate derived by

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**TABLE II**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>maximum power budget of the BS</td>
<td>20 Watt</td>
</tr>
<tr>
<td>total bandwidth</td>
<td>2400 Hz</td>
</tr>
<tr>
<td>number of sub-carriers</td>
<td>4</td>
</tr>
<tr>
<td>minimum required rate of users</td>
<td>100 bit/symbol</td>
</tr>
<tr>
<td>with concave utility</td>
<td>1 bit/symbol</td>
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<tr>
<td>minimum required rate of users</td>
<td>300000</td>
</tr>
<tr>
<td>with convex utility</td>
<td>0.75</td>
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<tr>
<td>number of iterations</td>
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<td>crossover probability</td>
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<td>mutation probability</td>
<td></td>
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<td>initial population.</td>
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</table>
GA and PM/IPM. The data tips in Fig. 3 indicate that both GA and PM/IPM allocate more rate to users 1 and 3 than users 2 and 4. Also, PM/IPM allocates same equal rate to the users with the same average channel quality on sub-carriers.

![Utility allocation comparison of GA and PM/IPM](image)

Fig. 3. Utility allocation comparison of GA and PM/IPM

To obtain a deep insight into rate allocation and exclusive sub-carrier assignment by PM/IPM, the vectors of allocated rate to sub-carriers, $n = 1, \ldots, 24$, for users 1 to 4, $r_1$ to $r_4$, along with the corresponding channel gains of the users on the sub-carriers, $\alpha_1$ to $\alpha_4$, are shown in Table III. The gray rows of the table represent the assigned sub-carriers to the users, and the sub-carriers on white rows are unassigned. The result confirms the success of PM/IPM in exclusive sub-carrier assignment since no sub-carrier has been assigned to two users. In addition, a sub-carrier is assigned to a user that has the best channel gain on that sub-carrier, which results in a solution closer to the optimum. In numerical results given in Table III, all users achieve utility equal to one, so some sub-carriers are not needed to be assigned to any user.

### V. CONCLUSIONS

We use continuous optimization approaches to model and solve the problem of OFDMA resource (sub-carrier and power) allocation by combining a penalty function and an interior point method that can efficiently combat the non-convexity of the problem. The proposed approach is computationally-efficient and performs very well in achieving near optimal solutions.

### REFERENCES


**TABLE III**

<table>
<thead>
<tr>
<th>$n$</th>
<th>$r_1$</th>
<th>$r_2$</th>
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<th>$r_4$</th>
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