Fuzzy outranking approach: A knowledge-driven method for mineral prospectivity mapping

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ABSTRACT

This paper describes the application of a new multi-criteria decision making (MCDM) technique called fuzzy outranking to map prospectivity for porphyry Cu–Mo deposits. Various raster-based evidential layers involving geological, geophysical, and geochemical geo-data sets are integrated for mineral prospectivity mapping (MPM). In a case study, 13 layers of the Now Chun deposit located in the Kerman province of Iran are used to explore the region of interest. The outputs are validated using 21 boreholes drilled in this area. Comparison of the output prospectivity map with concentrations of Cu and Mo in the boreholes indicates that the fuzzy outranking MCDM is a useful tool for MPM. The proposed method shows a high performance for MPM thereby reducing the cost of exploratory drilling in the study area.

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1. Introduction

Mineral exploration aims to discover new mineral deposits in a region of interest. One of the main steps in mineral exploration is to distinguish prospective areas within the region of interest. Various thematic geo-data set (e.g., geological, geophysical and geochemical data set) are collected, analyzed and integrated for mineral prospectivity mapping (MPM) to delineate prospective areas. Thus, MPM is a multiple criteria decision-making (MCDM) task and produces a predictive model for outlining prospective areas. There are several approaches to MPM, which can be categorized into data-driven and knowledge-driven methods (Bonham-Carter, 1994; Pan and Harris, 2000; Carranza, 2008). In data-driven or empirical techniques, the known mineral deposits in a region of interest are used as ‘training points’ to establish spatial relationships between the known deposits and particular geological, geochemical and geophysical features (Carranza et al., 2008a). The relationships between evidential maps and the training points are quantified and used to establish the importance of each evidence map (Carranza and Hale, 2002c) and are finally integrated into a single mineral prospectivity map (Nykänen and Salmirinne, 2007). Examples of the empirical methods of MPM include weights of evidence (Bonham-Carter et al., 1989; Carranza and Hale, 2002d), logistic regression (Apterberg and Bonham-Carter, 1999; Carranza and Hale, 2001a), neural networks (Singer and Kouda, 1996; Porwal et al., 2003a, 2004; Abedi and Norouzi, 2012), evidential belief functions (Carranza and Hale, 2002a; Carranza et al., 2005, 2008b), Bayesian classifiers (Porwal et al., 2006; Abedi and Norouzi, 2012) support vector machines (Zuo and Carranza, 2011; Abedi et al., 2012b) and clustering methods (Abedi et al., 2012c). The other techniques, in which a geoscientist’s expert opinions are applied, are called knowledge-driven methods and include methods such as the use of Boolean logic (Bonham-Carter et al., 1989), index overlay (Bonham-Carter et al., 1989; Carranza et al., 1999), the Dempster–Shafer belief theory (Moon, 1990; Carranza et al., 2008b), fuzzy logic (Chung and Moon, 1990; An et al., 1991), wildcat mapping (Carranza and Hale, 2002e; Carranza, 2010), and outranking method (Abedi et al., 2012a).

If suitable data sets are available from previous mineral exploration projects, MPM can take the form of a classification process for outlining new prospective areas in a region of interest. There are two types of classification, supervised and unsupervised. Supervised classification is used to categorize every location as either prospective or non-prospective based on various evidential layers and a training data vector of known deposit locations and non-deposit locations (e.g., Carranza and Hale, 2002b). Unsupervised classification is based only on the statistical features of the evidential layers (Abedi et al., 2012c).

Among various knowledge-driven methods, the fuzzy logic method has been applied in many cases to represent the knowledge of different indicator layers in the MPM process (Bonham-Carter, 1994; Carranza and Hale, 2001b; Porwal et al., 2003b; Tangestani and Moore, 2003; Nykänen and Ojala, 2007; Nykänen et al., 2003).
2008a,b). Process understanding of a mineral system is the key to identification of targeting elements (Czarnota et al., 2010). Such understanding can be either conceptual or empirical (Porwal and Kreuzer, 2010). Fuzzy logic methods have been used to represent conceptual process understanding of mineral systems (D’Ercole et al., 2000; Groves et al., 2000; Knox-Robinson, 2000). However, hybrids methods, such as fuzzy weights-of-evidence modeling (Cheng and Agterberg, 1999), data-driven fuzzy modeling (Luo and Dimitrakopoulos, 2003; Porwal et al., 2003b; Zuo et al., 2009) and neuro-fuzzy modeling (Porwal et al., 2004) have been proposed to optimize utilization of both conceptual knowledge of mineral systems and empirical spatial associations between mineral deposits and evidential features in geocomputational modeling of exploration targets (Carranza, 2011).

Selecting the best area for exploratory drillings by considering many alternatives as a MCDM procedure is developed in our study, in which the results of application of a new proposed approach that is called fuzzy outranking are considered. This method is applied to real data pertaining to the Now Chon copper deposit located in Kerman, central part of Iran. The Now Chon deposit is also located among some operational copper mines such as Sar Cheshmeh, Saridun and Darrehzar. The Now Chon area is chosen in the reconnaissance stage based on analyzing the Advanced Spaceborne Thermal Emission and Reflection Radiometer (ASTER) remote sensor data. After mapping typical alterations of porphyry type deposits including argillic, phyllic and propylitic, the study area is selected for further ground-based exploration analysis (i.e., detail consideration of surface exploratory geo-data sets involving geological, geophysical, and geochemical data layers). The main motivation of this study is to determine whether additional drilling is needed in the study area. A comparison with 21 boreholes drilled in the Now Chon prospect is also provided.

2. Methodology

Problems in MCDM are encountered under various situations where a number of alternatives need to be chosen based on a set of criteria or attributes. Comparing the alternatives is the key to MCDM in such cases. However, in case of conflicting alternatives, decision making must also consider imprecise or ambiguous data, which is normal in this type of decision problems. The theory of fuzzy set is ideally suited for handling ambiguity encountered in solving MCDM problems (Aouam et al., 2003).

A fuzzy MCDM problem consists of two phases. The first phase requires the finding of suitable fuzzy utility functions (fuzzy ratings) for every alternative. The second phase requires the application of fuzzy ranking methods. A new methodology that combines the concepts of fuzzy outranking and fuzzy attributes to provide a more flexible way for comparing alternatives has been applied for MCDM problems (Aouam et al., 2003). This approach is described in the following section and is developed for MPM.

In general, a MCDM problem can be formulated as follows. Given a set of alternatives \( A = \{ a, b, c, d, \ldots \} \), evaluated by \( n \) criteria: \( g_1, g_2, \ldots, g_n \). The problem consists of choosing the best alternative in the set \( A \) based on the comparison of criterion vector \( g(k) \), \( k \in A \). For example, the criterion vector formed based on an alternative \( a \) in \( A \) can be represented by \( g(a) = [g_1(a), g_2(a), \ldots, g_n(a)] \). It is defined a fuzzy outranking function in \( A \times A \) as a function \( f: A \times A \rightarrow R \) in which the difference values \( f(a, b) \) indicate the degree of outranking associated with a pair of alternatives \( (a, b) \). This function reflects the credibility of a preference for \( a \) over \( b \). Here,

\[
f(a, b) = f[g(a) − g(b)], \quad \forall (a, b) \in A \times A
\]

where \( g(a) \) and \( g(b) \) are normalized vectors of performances of alternatives \( a \) and \( b \), respectively. All attributes are in a comparable scale (Aouam et al., 2003).

The fuzzy outranking function is based on two parts: the concordance and the discordance. The concordance part is based on the comparison of the fuzzy concordance numbers \( C \), which are an aggregation of fuzzy ranks \( d_j \), \( j = 1, 2, \ldots, n \) called the partial outranking fuzzy numbers for considering the performance of the alternatives for each criterion \( j \). The discordance part is based on \( n \) fuzzy sets \( D_j, j = 1, 2, \ldots, n \) called the discordance fuzzy numbers for considering the importance of the criteria that disagree with the concordance part. There are three phases in the approach.

- First, fuzzy sets or fuzzy numbers \( d_j, D_j \), and \( C \) are defined to evaluate the differences between the performances of the various alternatives.
- Second, a fuzzy ranking method is used to compare these fuzzy and to define the concordance and discordance functions \( dC \) and \( dD \), respectively.
- Finally, both \( dC \) and \( dD \) are combined to obtain the explicit form for the outranking function \( f \) and an overall outranking intensity \( I \) is calculated as an index to extract the decision making rule.

All fuzzy numbers considered in this study are defined by triangular fuzzy membership functions because they are easy to manipulate and provide sufficient accuracy for modeling fuzziness (Aouam et al., 2003).

2.1. Fuzzy numbers for evaluating the criteria

This section consists of three steps,

Step 1: Partial outranking fuzzy numbers.

The triangular functions of fuzzy numbers \( d_j, j = 1, 2, \ldots, n \) are defined so that \( d_j(a, b) \) represents the average of concordance with the outranking of alternative \( b \) by alternative \( a \) regarding the \( j \)th criterion. The notion of the maximum non-significant threshold \( s_j \) is used for each criterion \( j \). Thus, if \( g(b) − g(a) > s_j \), we can say with “certainty” that for the criterion \( j \), alternative \( b \) is better than alternative \( a \). However, if \( 0 < g(b) − g(a) > s_j \), alternative \( a \) is still considered “at least as good” alternative \( b \) for the criterion \( j \), but the certainty of this statement decreases as the difference increases (Siskos et al., 1984). Instead of using crisp numbers, \( d_j(a, b) \) is defined using a triangular fuzzy functions of numbers reflect degree of concordance with the criterion that “\( a \) outranks \( b \).” However, the performances of the alternatives can be either crisp or fuzzy. Performance, as defined for any real number \( x \), is the positive real number \( x^+ \) as

\[
x^+ = \max(0; x)
\]

where ‘max’ is the function that returns the maximum of values. For any pair of alternatives \((a, b) \in A \times A \) with \( a \neq b \), and for any criterion \( j \), performance of alternatives is defined the partial outranking fuzzy numbers as follows:

- if attribute \( j \) is crisp:

\[
\langle (g(a) - g(b))^+, (g(a) - g(b))^+, (g(a) - g(b))^+ + s_j \rangle
\]

- if attribute \( j \) is fuzzy:

\[
\langle (g(a) - g(b))^+, (g(a) - g(b))^+, (g(a) - g(b))^+ + s_j \rangle
\]
where \( g_l(a)^k \), \( g_m(a)^M \), and \( g_r(a)^R \) are the left, middle, and right values, respectively, of the triangular fuzzy membership function of \( g_i(a) \) (Aouam et al., 2003).

Step 2: Fuzzy concordance number.

The aim of this step is to aggregate the partial outranking fuzzy numbers corresponding to each concordant criterion into the fuzzy numbers \( C(a, b) \) that measures the concordance degree with \( a \) outranks \( b \). These numbers are called the fuzzy concordance numbers, which accumulate the favorable differences to “\( a \) outranks \( b \)” by consideration of all criteria. Let the weight \( p_j \) represents the assigned weight for each criterion \( j \). For any pair of alternatives \((a, b) \in A \times A\) with \( a \neq b \), the fuzzy concordance numbers are defined as

\[
C(a, b) = \sum_{j=1}^{n} p_j^f \times d_j(a, b)
\]

where

\[
p_j^f = \begin{cases} p_j & \text{if } g_j(a) - g_j(b) \geq 0 \\ 0 & \text{otherwise} \end{cases}
\]

where \( p_j^f \) is a concordance weight for each criterion \( j \). The partial outranking fuzzy numbers \( d_j(a, b) \), Eqs. (3) and (4), are defined as triangular function fuzzy numbers. Since multiplication of real numbers (the weights) with triangular function fuzzy numbers results in a fuzzy number, the summation operation to obtain \( C(a, b) \) still results in a triangular function fuzzy number (Aouam et al., 2003).

Step 3: Fuzzy discordance number

The fuzzy discordance numbers are introduced to evaluate the importance of discordant criteria. A criterion \( j \) becomes discordant, while evaluating the superiority of \( a \) over \( b \), when \( g_j(b) - g_j(a) \) becomes significant, i.e., \( g_j(b) - g_j(a) > s_j \).

The criterion \( j \) with a large divergence plays the role of “dictator” when two alternatives are compared. Considering the notion of veto \( v_j \) as a threshold, beyond which (i.e., \( g_j(b) - g_j(a) > v_j \)) \( a \) cannot outrank \( b \) in any case as suggested by the term “veto”, this situation would correspond to: \( d(a, b) = 0 \) (Roy, 1977; Siskos et al., 1984).

In some cases, which often happen in practice, for a particular pair of alternative \((a, b) \) it may happen that \( g_j(b) - g_j(a) > v_j \) \( d(a, b) = 0 \) and \( g_j(a) - g_j(b) > v_j \) \( d(a, b) = 0 \), \( j \neq l \). This situation implies that the situation “\( a \) cannot outrank \( b \) in any case” and “\( b \) cannot outrank \( a \) in any case” would coexist. This leads to the situation that \( a \) and \( b \) are incomparable. In fact, the derivation of the outranking structure is very sensitive to the value of the veto threshold. To resolve this issue, veto thresholds \( \{v_j\} \) are considered as a fuzzy numbers for each criterion \( j \).

For any pair of alternatives \((a, b) \in A \times A \) with \( a \neq b \), and for any criterion \( j \), the fuzzy discordance numbers are defined as follows:

- if attribute \( j \) is crisp:
  \[
  D_j(a, b) = (g_j(b) - g_j(a), (g_j(b) - g_j(a)) + s_j)
  \]

- if attribute \( j \) is fuzzy:
  \[
  D_j(a, b) = (g_j(b)^L - g_j(a)^L + (g_j(b)^M - g_j(a)^M)^+ + (g_j(b)^R - g_j(a)^R)^+ + s_j),
  \]

In order to evaluate the degree of discordance with the superiority of \( a \) over \( b \), for each criteria \( j \), the fuzzy discordance numbers \( D_j(a, b) \) are considered in fuzzy discordance functions (Aouam et al., 2003).

2.2. Ranking fuzzy numbers

The fuzzy concordance and discordance numbers are compared to construct the fuzzy concordance and discordance functions. However, since fuzzy numbers are generally partial order, ranking or comparison of fuzzy numbers is complicated. Various approaches have been proposed for comparing fuzzy numbers. The concept of fuzzy distance is used for this purpose (Chang and Lee, 1994). Since the concordance and discordance numbers are defined as triangular function fuzzy numbers, the difference (Diff) between two fuzzy numbers \( A_1 \) and \( A_2 \) is defined as

\[
\text{Diff}(A_1, A_2) = \text{OM}(A_1) - \text{OM}(A_2)
\]

\[
\text{OM}(A_j) = \frac{4A_j^L - A_j^L + A_j^R}{4}
\]

where \( A_j^L, A_j^M \) and \( A_j^R \) are the middle, left, and right values, respectively, of the fuzzy numbers \( A_j \). The fuzzy numbers \( A_j \) in Eq. (10) corresponds to a crisp value \( \text{OM}(A_j) \). This difference is used to compare the fuzzy concordance and discordance relations.

2.3. Fuzzy functions for comparing the criteria

This section consists of three steps

Step 1: Fuzzy concordance function.

The fuzzy concordance function \( dC : A \times A \rightarrow R \), can be defined as

\[
dC(a, b) = \text{OM}(C(a, b)) - \text{OM}(C(b, a))
\]

or

\[
dC(a, b) = \text{OM}(C(a, b)) - \text{OM}(C(b, a))
\]

If \( dC(a, b) > 0 \), then there is concordance with the outranking of alternative \( b \) by alternative \( a \). The degree of this concordance is \( dC(a, b) \).

Step 2: Fuzzy discordance function.

The fuzzy discordance function \( dD : A \times A \rightarrow [0, \infty) \), can be defined as

\[
dD(a, b) = \max\{dD_j(a, b), \quad j \in \{1, 2, \ldots, n\}\)
\]

where

\[
dD_j(a, b) = \begin{cases} \frac{\text{OM}(D_j(a, b))}{\text{OM}(v_j)} & \text{if } g_j(b) - g_j(a) > 0 \\ 0 & \text{otherwise} \end{cases}
\]

Step 3: Fuzzy outranking function.

To obtain an explicit form for the fuzzy outranking function \( f : A \times A \rightarrow R \), we have to combine the fuzzy concordance and discordance functions. The participation of a decision maker (i.e., an expert) is important because this operation involves his/her opinion concerning the importance given to the discordance part while reducing the concordance part with the outranking of an alternative by another one. Thus, the following formula is used to define the fuzzy outranking relation that compares two alternatives \( a \) and \( b \):

\[
f(a, b) = \begin{cases} dC \times [1 - (\beta \times dD)] & \text{if } dC > 0 \\ \frac{dC}{dC} & \text{otherwise} \end{cases}
\]

where \( \beta \) is a factor between 0 and 1 reflecting the importance given by the decision maker to the influence of the discordance (this value may also be greater than 1). The discordance weight \( \beta \) is so designed.
that, when it increases, more importance is given to the discordance part. We set $\beta$ value equal to 0.5 by trial-and-error in this study.

2.4. Overall outranking intensity index

To extract the ranking of the alternatives using the fuzzy outranking structure, a measure of some kind of overall outranking is needed. Given the outranking function $d(a, b)$, the outranking intensity of $b$ by $a$ is defined as

$$d_f(a, b) = f(a, b) - f(b, a)$$

Thus, the overall outranking index or the measure of the overall outranking intensity $l(a)$ for the alternative of $a$ over all the other alternatives in $A$ is

$$l(a) = \sum_{b \in A} d_f(a, b), \quad b \neq a$$

Higher values of $l(a)$ imply higher capability of selection (Aouam et al., 2003).

3. Geological setting

The study area is located within the Urumieh-Dokhtar (Sahand-Bazman) magmatic arc of the Central Iran zone, where extensive Tertiary to Plio-Quaternary extrusive and intrusive units are exposed along a NW-SE trend (Fig. 1). In general, many studies suggest a subduction-related magmatic model for the Urumieh-Dokhtar magmatic arc (Berberian and Berberian, 1981; Ahmad and Posht Kuh, 1993; Hassanzadeh, 1993; Moradian, 1997; Omrani et al., 2008). The subduction zone resulted from the closure of the Neo-Tethyan ocean between Arabia-Turkish and Eurasia (Takin, 1972; Ricou et al., 1977; Dercourt et al., 1986, 1993; Sengor et al., 1988; Agard et al., 2005; Omrani et al., 2008).

The Urumieh-Dokhtar magmatic arc generally contains two major mineralization regions, the Chahar Gonbad region (Kerman province) to south and the Sun gun region to the north-west. Dominant mineralization is porphyry type associated with Eocene–Pliocene–Quaternary granitoids, plutonic bodies and volcanic rocks. Each of the two major mineralization regions has porphyry copper deposits such as the Sar Cheshmeh and Darrehzar copper mines and prospects such as the Now Chun.

3.1. Analysis of remote sensing data

The area has a semi-arid climate, a mountainous topography and vegetation cover is poor. Thus, it has great potential for exploration porphyry copper deposits using remote sensing data. Generalized geological and mineral occurrence map (1:100,000 scale) of the Now Chun deposit located in Kerman province (Iran), (1) recent alluvium, younger gravel fan and calcareous terraces (Quaternary), (2) neogene sediments, mostly arenites with pebbles and boulders of volcanic and intrusive rocks, (3) dacitic rocks (Quaternary), (4) oligocene–miocene granodiorite, quartz diorite, diorite porphyries and monzonite dykes, (5) trachyandesites, trachybasalts, basaltic andesites and pyroclastics (Eocene), (6) fault, (7) working mine and copper deposit, and (8) village. (For interpretation of the references to color in the artwork, the reader is referred to the web version of the article.) Reproduced from Ranjar and Honarmand (2004).

Fig. 2. Generalized 1:100,000 scale geological and mineral occurrence map of the Now Chun deposit located in Kerman province (Iran), (1) recent alluvium, younger gravel fan and calcareous terraces (Quaternary), (2) neogene sediments, mostly arenites with pebbles and boulders of volcanic and intrusive rocks, (3) dacitic rocks (Quaternary), (4) oligocene–miocene granodiorite, quartz diorite, diorite porphyries and monzonite dykes, (5) trachyandesites, trachybasalts, basaltic andesites and pyroclastics (Eocene), (6) fault, (7) working mine and copper deposit, and (8) village. (For interpretation of the references to color in the artwork, the reader is referred to the web version of the article.) Reproduced from Ranjar and Honarmand (2004).

Fig. 1. Generalized tectono-sedimentary zones of Iranian plateau (Hezarkhani, 2009).
sensor is a suitable tool for mapping hydrothermal alteration zones associated with porphyry–Cu deposits. Hydrothermal alteration zones associated with porphyry–Cu deposit such as phyllic, argillic, and propylitic mineral assemblage can be discriminated from one another by virtue of their spectral absorption features, which are detectable by ASTER data through the shortwave length infrared (SWIR) spectral bands. Six recording bands (bands 4–9) of SWIR out of 14 ASTER bands exist from 1.6 to 2.43 μm at a spatial resolution of 30 m (Fig. 3) (Beiranvand Pour and Hashim, 2011).

The broad phyllic zone is characterized by illite/muscovite (sericite) that indicates an intense Al–OH absorption feature centered at 2.20 μm coinciding with ASTER band 6. The narrower argillic zone including kaolinite and alunite displays a secondary Al–OH absorption feature at 2.17 μm that corresponds with ASTER band 5. The mineral assemblages of the outer propylitic zone are epidote, chlorite and calcite that exhibit absorption features situated in the 2.35 μm, which coincide with ASTER band 8 (Fig. 3) (Mars and Rowan, 2006; Beiranvand Pour and Hashim, 2011, 2012).

Relative Absorption Band Depth (RBD) is a useful three-point ratio formulation for displaying Al–OH, Fe, Mg–OH, and CO₃ absorption intensities. Al(OH)–bearing minerals such as kaolinite, alunite, muscovite and illite show major absorption in bands 5–7 (2.14–2.28 lm). Fe, Mg(OH)–bearing minerals such as chlorite, as well as carbonates such as calcite and dolomite have distinctive absorption in bands 8 and 9 (2.29–2.43 lm) of ASTER data (Fig. 3) (Hunt and Ashley, 1979; Mars and Rowan, 2006). For each absorption feature, the numerator is the sum of the bands representing the shoulders and the denominator is the band located nearest the absorption feature minimum (Crowley et al., 1989; Beiranvand Pour and Hashim, 2011). Three RBD ratios have adopted in this study namely RBD₅ (mineral features: alunite/kaolinite/pyrophyllite), RBD₆ (mineral features: sericite/muscovite/illite/smectite), and RBD₈ (mineral features: carbonate/chlorite/epidote) are used to delineate argillic, phyllic and propylitic hydrothermal alteration zones (van der Meer et al., 2012). The RBD ratios have been derived based on Crowley et al. (1989) as follows:

$$\text{RBD}_5 = \frac{\text{Band 4} + \text{Band 6}}{\text{Band 5}}$$

$$\text{RBD}_6 = \frac{\text{Band 5} + \text{Band 7}}{\text{Band 6}}$$

$$\text{RBD}_8 = \frac{\text{Band 7} + \text{Band 9}}{\text{Band 8}}$$

Absorption Band Depths (RBDs) consists of RBD₅, RBD₆, and RBD₈ images have used in this study to delineate argillic, phyllic and propylitic mineral assemblages using ASTER SWIR bands. In this regard, alteration mineral assemblages are demonstrated with different colors. Phyllic alteration zones have green color and rarely argillic alteration is shown by red color. Propylitic zone as blue color that surrounded outside of these hydrothermal alteration zones is wider (Fig. 4).

Based on alteration zones in Fig. 4, a new prospect area, i.e., the Now Chun, is chosen for ground-based measurement of geological, geochemical and geophysical data layers.
3.2. Geological background of the Now Chun deposit

The study area includes the Now Chun Cu-Mo prospect. Its exploration was initiated in the 1970s by Yugoslavian geologists. The detailed prospecting work was carried out solely in the recent years. In general, the main lithological units exposed in the area comprise volcanic–subvolcanic complex and intrusive bodies. Volcanic rocks cover most parts of the study area and consist of Eocene andesitic, dacitic to rhyodacitic lavas and its associated breccias tuffs. Intrusive bodies consist of granite to diorite porphyry distributed in the south to southwest of the area (Elyasi, 2009).

Most of the geological units in the study area have been altered hydrothermally and the most intensive alteration occurred in subvolcanic rhyodacitic bodies. The general trend of altered rhyodacitic unit and hydrothermal dissemination is NE-SW. Because there is comprehensive correlation between hydrothermal zones and faults, we assume that the alteration zones have probably been controlled by major faults and fractures. However, the Cu-bearing mineralization zones are mainly associated with azurite and malachite stockwork with minor chalcopyrite as inclusions within quartz. Some Cu-Mo mineralized outcrops are also shown on the detailed geological map (Fig. 5).

4. Mineral prospectivity mapping

4.1. Criteria for MPM

Magnetic and electrical surveys are two common geophysical methods for prospecting porphyry deposits. Magnetic methods are used worldwide in the exploration and characterization of porphyry deposits. The primary control of the bulk magnetic properties of the host rock and magnetic intrusions is the partitioning of iron between oxides and silicates (Clark, 1999). However, sulfide minerals associated with hydrothermal alteration also provide localized geophysical targets (John et al., 2010). Simple models for porphyry deposits involve contrasting zones of alteration centered on the deposit. Magnetic anomalies, at least in principle, reflect the location of these zones (Thoman et al., 2000): weak local magnetic highs occur over the potassic zone; low magnetic intensity occurs over the sericitic zones; and magnetic intensities increase gradually over the propylitic zone.

The resistivity (RS) method is one of the oldest techniques used in geophysical exploration. The RS is a measure of the ability of an electrical charge to form currents that move through a geological section. Minerals and rocks associated with hydrothermal alteration often have anomalous electrical properties. Thus, geophysical methods that detect and model such properties are mainstays in exploring for and characterizing porphyry deposits. Like the distribution of magnetic minerals, electrical properties reflect the type and degree of hydrothermal alteration. Hydrothermal minerals relevant to geophysical exploration include pyrite, chalcopyrite, chalcocite, biotite, and sericite. As with magnetic anomalies, we would expect to see the intensity and type of alteration reflected in RS anomalies, with the lowest RS associated with sericitic alterations that develop in the zones with the greatest extent of fracturing and fluid flow (Thoman et al., 2000; John et al., 2010).

The dispersed nature of sulfide minerals in porphyry systems is particularly suitable for induced polarization (IP) methods (Sinclair, 2007). The IP method was originally developed for the exploration of porphyry deposits (Brant, 1966) and is still commonly used. Induced polarization is a complex phenomenon. In the simplest terms, IP anomalies reflect the ability of a mineral, rock, or lithology to act as an electrical capacitor. In porphyry deposits, the strongest IP responses correlate with quartz–sericite–pyrite alterations (Thoman et al., 2000; John et al., 2010). Typically, the zone of potassic alteration in the core of the deposit is depleted in total sulfide minerals; the surrounding zone of sericitic alteration has high sulfide content, including pyrite; and the distal zone of propylitic alteration has low amounts of pyrite. Thus, the sericitic zone of alteration is an important IP target (John et al., 2010). As illustrated, various geophysical layers of information can be derived from magnetic and electrical surveys for the MPM process.

4.2. Data treatments for MPM

In this study, thirteen layers of information are used for MPM. Table 1 describes the criteria, the evidence layers and the reasons for using them in this study.

Table 1

<table>
<thead>
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<th>Criteria</th>
<th>Evidence layer</th>
<th>Reason of using</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geology</td>
<td>Alteration zones, Fault areas, Mineralization indicators, Host rock</td>
<td>Suitable area for ore–metal occurrence</td>
</tr>
<tr>
<td>Geochonmy</td>
<td>Copper anomaly (Cu), Molybdenum anomaly (Mo), Additive map (Cu + Mo)</td>
<td>Ore–metal enrichment</td>
</tr>
<tr>
<td>Geophysics</td>
<td>Induced polarization (IP), “chargeability map”, Resistivity map (RS), Metal factor (MF)</td>
<td>Reflect the type and degree of hydrothermal alteration</td>
</tr>
<tr>
<td>Electric</td>
<td>Residual magnetism 10 m upward continued (UP10), Reduced to pole of magnetic data (RTP), Analytic signal of magnetic data (AS)</td>
<td>Reflect the location of alteration zones</td>
</tr>
</tbody>
</table>

Fig. 5. Detailed 1:5000 scale geological and mineral occurrence map of the Now Chun deposit located in Kerman. (For interpretation of the references to color in the artwork, the reader is referred to the web version of the article.) Reproduced from Elyasi (2009).
According to the geology of the study area, rhyodacite is the host rock of the porphyry deposit. Four 25-m-interval buffers are considered around the rhyodacite units to represent the presence of mineralization beneath the adjacent rocks. Then, the classes in the host rock buffer map are given fuzzy scores by geologists. The fuzzy scores for each layer class were obtained from a group of geoscience experts in the National Iranian Copper Industries Company (NICIC), who considered the data and compared them with their knowledge about each layer as evidence of porphyry Cu–Mo deposit occurrence (Elyasi, 2009). Other types of rocks (granite and diorite porphyrite) have no role in the mineralization; thus, their scores were considered minimally significant (Fig. 6a). Because no hydrothermal alterations are indicated in the geological map (Fig. 5), hydrothermal alterations in the study area based on guides to ore were delineated and subsequently classified (Fig. 6b). Rocks in areas adjacent to NE-SW faults are crushed and altered. Therefore, such areas are given maximum scores in comparison with no-fault zones (Fig. 6c). Four 25-m-interval buffers were considered to represent the parts of the study area adjacent to observed mineralized outcrops (Fig. 6d).

Soil geochemical data of Cu and Mo were interpolated by an inverse distance weighted method to map their anomalies (Fig. 7a and b). After transformation of each of the Cu and Mo dataset into standard normalized distributions (Zscore), an additive map as a summation of the Cu and Mo concentrations was created (Fig. 7c).

A ground-based magnetic survey was performed in the area, in which the distances between profiles and stations are 50 m and 20 m, respectively. It was conducted by a group of geoscience experts in the NICIC. The geomagnetic field is 46,000 nT (inclination = 46°, declination = 2.5°). Analytical upward continuation was used because it is suspected that porphyry deposit exists at depth. This method calculates the magnetic field farther from the source; consequently, it results in a better map of deeper deposits and reduces the effects of shallow structures with high magnetic frequency. Residual magnetic data that were upward-continued to 10 m (UP10) are shown in Fig. 8a.

The general filter operation applied to the magnetic data is reduced to the pole (RTP), which is a technique that converts magnetic anomalies to a symmetrical pattern that would have been observed with vertical magnetization. The RTP technique eliminates the dipolar nature of magnetic anomalies and converts their asymmetric shape to a symmetric shape (Ansari and Alamdar, 2009). The RTP map of the Now Chun deposit is shown in Fig. 8b.

Many filters are available to enhance magnetic field data, including downward continuation, horizontal and vertical derivatives, and other forms of high-pass filters. One of these techniques is the analytic signal method. The basic concepts of the analytic signal method for magnetic data were extensively discussed by Nabighian (1972, 1974, 1984). Fig. 8c shows the analytic signal image of the magnetic data.

The most common geophysical methods for the exploration of sulfide deposits are electrical techniques. In this study, RS map, IP map “chargeability map” and a metal factor map (ratio of chargeability to resistivity) are used. A rectangular array with 1600 m of space as the current electrode was used in the area, with distances between profiles and stations of 100 m and 20 m, respectively. These maps are shown in Fig. 8d–f.

5. Applications to real data

To use the fuzzy outranking MCDM method, 13 layers were considered as key criteria. Cell values of raster data sets associated with these criteria were extracted and stored in 13 separate columns.
Thus, the occurrence of the mineral deposit of interest. Thus, the expert geoscientists divided each layer into four classes, namely non-anomalous, possibly anomalous, probably anomalous, and anomalous. Assuming a normal or log-normal distribution of data, any numerical data layer can be reclassified by its standard deviation (σ) and mean (μ) into μ ± σ, μ ± 2σ, and μ ± 3σ classes. Values less than (μ + σ) classified as non-anomalous and values more than (μ + 3σ) are considered anomalous. The values of μ ± 2σ and μ ± 3σ are thresholds to define possibly and probably anomalous areas. By considering the statistic of data, the maximum non-significant threshold sj is chosen a crisp value equal to μ ± 2σ in this study. The veto threshold vj is also chosen as a fuzzy triangular function number equal to (μ ± σ, μ ± 2σ, μ ± 3σ) by trial-and-error method. Considering Eq. (10), the veto value corresponds to a crisp value equal to μ ± 3σ. The point is that the domain of each alternative is between [xmin, xmax] so based on Eq. (1) this interval (xmax − xmin) must be considered for choosing suitable non-significant and veto thresholds. Selection of these thresholds is based on experts’ opinion. The program would maximize the effect of all layers except the RS layer that can be minimized in most porphyry prospect. Thus, it is suggested to use a transformation for RS data in which high values of resistive zones correspond to low values of transformed data. Therefore, we use a normalized function in the form of (xmax − μ)/(xmax − xmin) for RS data. However, all data layers must be maximized to generate final MPM.

It is a common practice in MPM to determine the weights of attributes (i.e., feature data including geological, geophysical and geochemical layers) using knowledge-driven methods, such as the Delphi method. Therefore, the Delphi method was applied to determine the weights of each of the 13 layers by asking questions to a group of experts at the NICIC. The Delphi method is based on interviewing a group of experts using a series of questionnaires. Judgments derived from several experts are generally more accurate than results from only one expert. However, group processes often lead to sub-optimal judgments. Between 5 and 20 experts with appropriate expertise in a particular field of knowledge must be employed to acquire a reasonable answer. Final weights of evidence layers for MPM can be obtained by this method (Rowe and Wright, 2001). Normalized weights of the evidential layers are shown in Table 2 (Abedi et al., 2012a).

The normalized values of the final MPM generated using the fuzzy outranking method are shown in Fig. 9 in which high values correspond to zones with high potential for further study and additional drillings. A large area under prospect has been excluded for detail studies.

### Table 2
Weights extracted using the Delphi method.

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alteration zone</td>
<td>0.1138</td>
</tr>
<tr>
<td>Fault area</td>
<td>0.0506</td>
</tr>
<tr>
<td>Mineralization indicators</td>
<td>0.1013</td>
</tr>
<tr>
<td>Host rock</td>
<td>0.1013</td>
</tr>
<tr>
<td>Copper anomaly (Cu)</td>
<td>0.0868</td>
</tr>
<tr>
<td>Molybdenum anomaly (Mo)</td>
<td>0.0767</td>
</tr>
<tr>
<td>Additive map (Cu + Mo)</td>
<td>0.1635</td>
</tr>
<tr>
<td>Induce polarization (IP) &quot;chargeability map&quot;</td>
<td>0.06885</td>
</tr>
<tr>
<td>Resistivity map (RS)</td>
<td>0.02295</td>
</tr>
<tr>
<td>Metal factor (MF)</td>
<td>0.0918</td>
</tr>
<tr>
<td>Residual magnetic that 10 m upwarded (UP10)</td>
<td>0.0306</td>
</tr>
<tr>
<td>Reduced to pole of magnetic data (RTP)</td>
<td>0.04896</td>
</tr>
<tr>
<td>Analytic signal of magnetic data (AS)</td>
<td>0.04284</td>
</tr>
</tbody>
</table>
6. Discussion

Based on 21-drilled boreholes in the high potential zones of the Now Chun prospect, the 3D distributions of Cu and Mo concentrations (Fig. 10) show that high potential zones of the final MPM are satisfactory matching with the known Cu—Mo mineralization. Considering economical values of Cu—Mo for mine operation, Fig. 11a and c show economical boundaries of ore occurrence for mine operation. The fuzzy outputs for these boundaries are also indicated in Fig. 11b and d showing good match with ore occurrence in Fig. 11a and c. A depth level equal to 150 m was chosen to evaluate the fuzzy results with Cu—Mo concentration distribution.

The fuzzy outranking MCDM method can be a useful and easily understood method for MPM process and interested readers are recommended to work on it for further development. The main aspects of developing this method can be related to the concordance and discordance functions and as well as considering optimum values of non-significant and veto thresholds. However,
• Different decision criteria with diverse dimensions and scales can be taken into account.
• All collected information in the decision matrix can be fully and efficiently considered when making the final decision.
• Input data can be crisp or fuzzy numbers.
• Threshold values can be easily determined based on simple statistics of each criterion.
• The concordance and discordance functions can vary based on decision makers’ knowledge.

7. Conclusion

In this study, a new knowledge-driven method called fuzzy outranking was proposed to produce a prospectivity map in the area comprising the Now Chun copper deposit of Iran. A comparative evaluation of the results was carried out using Cu and Mo concentrations in 21 boreholes in the study area. The results of this study indicate that the fuzzy outranking method can be for prioritizing parts of the prospect area. The main motive for implementing the fuzzy outranking method was that it could be easily applied in MPM. The final result of the fuzzy outranking technique as a multi-criteria decision making method for prioritizing parts of the Now Chun prospect showed an adequate match between mapped potential zones and distributions of Cu and Mo concentrations in known economical zones of the prospect. The proposed method can be applied as a reliable knowledge-based tool to prepare prospectivity maps in other application areas.

Acknowledgements

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interested readers are referred to Aouam et al. (2003) for additional details of fuzzy MCDM techniques and also for a practical example applied for a small database. The main characteristics of the fuzzy outranking MCDM method are as follows:

• It could be applied readily in the context of MPM.

Fig. 10. 3D representation of ore body: (a) Cu concentration cutoff of 0.25%; and (b) Mo concentration cutoff of 300 ppm. (For interpretation of the references to color in the artwork, the reader is referred to the web version of the article.)

Fig. 11. Validation of the results: (a) Cu concentration distribution using economical cutoff of 0.15% at depth 150 m; (b) fuzzy outrank MPM for Cu boundary; (c) Mo concentration distribution using economical cutoff of 300 ppm at depth 150 m; and (d) fuzzy outrank MPM for Mo boundary. (For interpretation of the references to color in the artwork, the reader is referred to the web version of the article.)
Meer and an anonymous referee for their comments, which helped us improve our paper.

References


