Detection of anomalous bids in procurement auctions

Pier Luigi Conti\textsuperscript{1} and Maurizio Naldi\textsuperscript{2}

\textsuperscript{1} Università di Roma “La Sapienza”, Dip. di Statistica, Probabilità e Statistiche Applicate, Piazzale Aldo Moro, Rome, Italy
\textsuperscript{2} Università di Roma “Tor Vergata”, Dip. di Informatica, Sistemi e Produzione, Via del Politecnico 1, Rome, Italy

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Extended abstract

Procurement of goods, services, or public works, is often accomplished by reverse auctions, where suppliers provide their competitive biddings to a buyer. Since the assignment is typically awarded to the supplier providing the lowest bid, each tenderer is spurred to provide the lowest possible bid, taking into account the expected level of competition and its expected rate of return. In some cases the tenderer presents an anomalous bid, whose value has been set by a line of reasoning different from that of regular competitors. Such cases are represented, e.g., by bridging situations, or by predatory bidding, or cover pricing. In particular, abnormally low bids are a cause of deep concern and have come to the attention of the European Union, since they may lead to award the contract to a supplier that could end up not providing the goods/services. Such anomalous bids have to be detected and subject to further investigation, possibly leading to their rejection.

A method, based on the average submitted bid, is considered here to detect such anomalous bids and aid the auctioneer in the possible rejection decision. We set a threshold $T$ equal to some fraction $\alpha$ of the sampling average $\overline{X}$ of the bids, and compare each bid with this threshold. The $i$-th bid $X_i$ is declared anomalous (and therefore subject to further examination) if $X_i < T = \alpha \overline{X}$. This criterion has been officially endorsed by at least two institutional bodies, in Spain and Italy.

We evaluate the performances of the average bid criterion by assuming a Gaussian model for the probability distribution of bids. Such model, in addition to being supported by statistical fitting to empirical data, reflects a common cost structure, with differences among the bids related either to errors in the cost estimation and quotation, or to small competitive advantages.

For the purpose of performance evaluation we consider two scenarios, named Scenario A and B. In Scenario A all the bids are regular (i.e. not anomalous). In Scenario B instead just $N - 1$ bids are regular, following the same Gaussian probability distribution as in Scenario A; beside these regular bids there is one
an anomalous bid, which follows again a Gaussian distribution with identical standard deviation as the regular bids but with a (rebated) mean value, lowered by the rebating factor $\beta$.

We consider two performance indices: the probability of detecting the anomalous bid when present (Detection Probability, evaluated under Scenario B), and the probability of declaring a bid anomalous when all the bids are instead regular (False Alarm Probability, evaluated under Scenario A).

The performances heavily depend on the number of tenderers and on the dispersion of bid values. Both performance indices improve as the number of tenderers grows and generally degrade as the dispersion grows (see Fig. 1). A larger dispersion of bids contributes to lower the detection probability as long as it is larger than 0.5. The range of values of the detection probability heavily depends on the rebating factor (it is always lower than 0.5 when $\beta = 0.9$, but larger than 0.5 when $\beta = 0.8$), and the sign of the slope of the curve depends on the rebating factor (again changing when passing from $\beta = 0.8$ to $\beta = 0.9$). The false alarm probability is quite large and even larger than the detection probability if the rebating factor is larger than 90% (see Fig. 2). In addition, if multiple anomalous bids are present (even just three) the performance worsens significantly. On the other hand, the presence of courtesy bids (bids that are submitted with the aim of showing interest in the auction but without the intention of winning) increases both the detection probability and the false alarm probability.

The use of the average-bid criterion, though officially endorsed in national legislations, is therefore recommended as a strongly precautionary criterion, i.e., when the need to avoid anomalous bids is considered much more relevant than the costs associated to deeper investigation of anomalous bids or to the erroneous rejection of regular bids.

![Fig. 1. Detection probability under an 80% rebating factor ($\alpha = 0.9$)](image1)

![Fig. 2. False alarm probability estimated by simulation ($\alpha = 0.9$)](image2)