A New Approach for Mobile Localization in Multipath Scenarios

Nadir Castañeda, Maurice Charbit and Eric Moulines

CNRS LTCI & GET,
46 rue Barrault, 75634 Paris cedex 13, France
{castaned, charbit, moulines}@tsi.enst.fr

Abstract—In this paper we consider the localization of a mobile station (MS) in time division multiple access (TDMA) based communication systems. We use joint angle and delay measurements of the emitted signals, impinging on an antenna array at different base stations (BSs). Contrary to previously reported work, our technique takes into account not only the measurement noise, but also multipath propagation and possible BSs reporting only non line of sight (NLOS) measurements. Data are processed using the maximum likelihood (ML) approach based on an implementation of the Expectation Maximization (EM) algorithm. We illustrate the proposed approach using simulated data.

I. INTRODUCTION

This paper is devoted to the problem of Mobile Station (MS) localization in TDMA-based communication systems. To this purpose, we use joint angle and delay measurements of the emitted signals by the MS, which is assumed to be at a fixed position during the localization procedure. We consider that these signals impinge via multiple paths on an antenna array situated at different BSs. Hence, the main issue of our approach is to determine which one of the estimated multipath parameters corresponds to the line of sight (LOS) path (if any).

The benefits and potential applications of MS localization and tracking for various commercial and public safety operations have already been well documented in the literature [1], [2]. The most popular techniques of locating mobile users involve measurements of the time of arrival (TOA), time difference of arrival (TDOA) and angle of arrival (AOA) of radio signals transmitted by the MS and received by a number of BSs (see [3], [4] for an overview). However, in order to meet the U.S. Federal Communications Commission (FCC) requirements on the accuracy of MS location for emergency calls [5], heterogenous measurements (i.e. TOA and AOA) may be used to improve such accuracy [6], [7]. Furthermore, heterogenous data based techniques are especially useful in hearability-restricted conditions when the number of BSs for location purposes is low [1], [6].

The accuracy of such location techniques depends on the LOS condition between MS-BS. Unfortunately, wireless communication systems are characterized by multipath propagation of the emitted signal [2], i.e. the signal received by the BS may be composed of LOS and NLOS propagation paths [8]. Moreover, emitted signals may propagate in a NLOS regime only producing severe accuracy degradation on the MS location when using the estimated TOA and/or AOA of these signals. Therefore, a method to distinguish LOS paths (if any) from the detected multiple paths at different BSs becomes mandatory to improve location accuracy [9]. Because the use of antenna arrays at the BS provides for the means to perform joint heterogenous measurements of the multipath signals, whose advantages have already been discussed, and at the same time permit to reduce the number of BSs needed for localization purposes, we adopt such an approach.

Several location techniques based on antenna arrays to estimate the TOA and/or AOA have already been proposed [8], [10]–[13]. Authors in [10] and the first two approaches in [11] propose algorithms to localize a source from the AOA of an emitting radio source. They consider the first arrival path or cluster of paths as the LOS paths, respectively. Jativa and Vidal in [12] propose a generalized likelihood ratio test to detect the first arrival path in a lag window before the first RAKE finger in CDMA based communication systems. The main disadvantage of these approaches is evident: NLOS propagation of the emitted signals will severely bias the location estimates. Reference [13] and the second two algorithms proposed in [11] select the path with the highest power as the LOS one. However, as pointed out by [12], most powerful paths do not necessarily include the first arrival path in a NLOS regime. Boujemaa and Marcos in [8] consider the problem of source localization from joint estimation of AOA and angular spread of the received signals. However, in order to achieve this, prior knowledge about angular spreading is assumed.

Herein we present a new approach to localize the position of an MS from joint measurements of AOA and TOA performed at different BSs. The statistical approach we propose takes into account the presence of multipath propagation and possible BSs reporting NLOS observations only. Collected data are processed using the maximum likelihood (ML) method based on an implementation of the expectation maximization algorithm [14]. The resulting location algorithm is able to select the LOS paths from the multiple paths arriving at the participating BSs and at the same time remove the measurements delivered by the BSs being in NLOS regime.

For completeness of this paper we include the theory of joint angle and delay estimation (JADE) [15], [16], which has been selected as the parametric method for channel estimation. The asymptotic behavior of JADE-MUSIC and JADE-ML based
estimators are also reviewed [15], [17]. However, other parametric channel models and algorithms of multipath parameter estimation can be used (see [13], [18], [19] as alternatives and the references therein). The analysis of the proposed approach through simulated data, by using both JADE-MUSIC and JADE-ML methods, is an original contribution of this paper.

The reminder of this paper is organized as follows. In sections II, III and IV we show the basis of the adopted method to estimate the space-time channel parameters. In section V, we present the proposed approach and we derive the algorithm to perform the localization of the source. Section VI presents the simulation results and performance of the proposed algorithm. Finally, in section VII, we present the conclusions.

II. DISCRETE CHANNEL MODEL

Following [16], joint estimation of AOA and TOA for a multipath propagation channel consists in two steps: (i) estimating the global channel response using learning sequences and observed data, (ii) exploiting the expression of the channel response as a function of AOAs and TOAs. As shown in [16], the discrete channel model capturing the effects of the array response $a(\theta)$ in direction $\theta$, delay $\tau$, symbol waveform $g(t)$ and path fading $\beta$ takes the form

$$ H = A(\theta)DG^T(\tau) $$

where $A(\theta)$ is an $M \times Q$ matrix whose columns are the $M$-element antenna-array response $a(\theta_q)$ to the $q$th path arriving from angle $\theta_q$, with $q \in \{1, \ldots, Q\}$ and $Q$ being the number of the propagating paths. $D$ is a diagonal matrix whose elements are the path fadeings. $G(\tau)$ is a $LP \times Q$ matrix whose columns are the $LP$ column-vectors $g^T(\tau_q) = [g(-\tau_q) g(T\tau_q) \ldots g(T(L - \tau_q) - \tau_q)]$ which contain the samples of the waveform delayed of $\tau_q$. The samples are taken at a rate of $P$ times the symbol rate $T$ and it is considered that the modulation waveform has finite support and that the channel length $L$ is also finite.

III. JADE METHOD

The strength of the JADE method is that of exploiting the stationarity of the AOA and TOA over a successive number $S$ of channel estimations of the form

$$ H^{(s)}_{est} = H^{(s)} + V^{(s)}_{est} $$

where $V^{(s)}_{est}$ is the zero-mean complex Gaussian estimation noise matrix at time slot $s$. Applying the vector operator to the above yields

$$ y^{(s)} = \text{vect}(H^{(s)}_{est}) = U(\theta, \tau)b^{(s)} + v^{(s)}, \text{ where } s = 1, \ldots, S $$

where $U(\theta, \tau) = [u(\theta_1, \tau_q) \ldots u(\theta_Q, \tau_q)]$ is the $MPL \times Q$ space-time matrix, with $u(\theta_q, \tau_q) = a(\theta_q) \otimes g(\tau_q)$ (where $\otimes$ is the Kronecker product). Vector $b_s$ contains the $Q$ fading coefficients for the $s$-th channel estimation and $v^{(s)} = \text{vect}(V^{(s)}_{est})$.

IV. ALGORITHMS FOR PARAMETER ESTIMATION AND ASYMPTOTICAL BEHAVIOR OF THE ESTIMATORS

Among the great variety of algorithms we can find in the literature to obtain the multipath parameters $\{\theta, \tau\}$ from equation (2), we will focus on ML and MUSIC [15]. Here, we are also interested in the asymptotical distribution of their estimates $\{\hat{\theta}, \hat{\tau}\}$. To simplify mathematical notations, we let $\eta = [\eta^1, \eta^2]^T = [\theta^T, \tau^T]$, proceeding for the same estimate $\eta$, and we drop the dependence of $U$ on the parameter $\eta$.

A. JADE-ML Estimates

We assume that both the estimation noise $\nu^{(s)}$ and the path fadeings $\beta^{(s)}$ are stationary Gaussian random processes. The channel estimates $\hat{y}^{(s)}$ are complex Gaussian random vectors with zero mean and covariance matrix $\hat{R} = E[\hat{y}^{(s)}\hat{y}^{(s)H}] = U \hat{R}_\beta U^H + \sigma^2 I$, where $\hat{R}_\beta = E[b^{(s)}b^{(s)H}]$. Thus by employing stochastic maximum likelihood techniques, it is well known that this is a separable optimization problem [20] that reduces to

$$ \hat{\eta} = \max_{\eta} \left\{ -\log |\Phi \hat{R} \Phi + \frac{1}{MPL - Q} \text{Tr}(\Phi^\perp \hat{R} \Phi^\perp) \right\} $$

where $\hat{R} = S^{-1} \sum_{s=1}^{S} \hat{y}^{(s)}\hat{y}^{(s)H}$ is the estimated covariance matrix, $\Phi = U(U^H U)^{-1}U^H$ is an orthogonal projector which projects any vector onto the space spanned by columns of $U$, $\Phi^\perp = I - \Phi$ is the orthogonal complement projector.

Now, applying classical limit theory it may be proved that, as $S$ goes to infinity, $\sqrt{S}(\hat{\Psi} - \Psi)$, where $\Psi = [\eta, \sigma^2, \hat{R}_\beta]$, is asymptotically a zero-mean Gaussian random vector, with covariance matrix given by the inverse of the Fisher Information Matrix (FIM). FIM’s elements can be determined from [17]

$$ f_{i,j} = S \text{ Tr} \left( R^{-1} \frac{\partial R}{\partial \psi_j} R^{-1} \frac{\partial R}{\partial \psi_i} \right) $$

where $\psi_i$ is the $i$-th component of $\Psi$.

B. JADE-MUSIC Estimates

Using (2) and MUSIC approach, $\hat{\eta}$ is given by the $Q$ minima of the cost function

$$ J(\eta) = U^H(\eta)\hat{\Pi}U(\eta) $$

where $\hat{\Pi}$ is the estimated orthogonal projector onto the noise subspace obtained from the eigendecomposition of $\hat{R}$.

Applying the same principle as in the JADE-ML case, it can be shown that, when $S$ goes to infinity, $\sqrt{S}(\hat{\eta} - \eta)$ is asymptotically a zero-mean Gaussian random vector with a $(2 \times 2)$ covariance matrix $\Gamma_{\theta,\tau}$ whose entries are given by

$$ \gamma_{\nu \nu} = \sum_{j,j'=1}^{2} C_{\nu j}^{(-1)} C_{\nu j'}^{(-1)} $$

$$ \sum_{l,p,l',p'}^{MPL} \frac{\partial R}{\partial \nu_l} \frac{\partial R}{\partial \nu_{l'}} \text{cov}(\hat{\nu}_{lp}, \hat{\nu}_{l'p'}) $$

(6)
where \( v, w \in \{1, 2\} \), \( C^{-1}_{ivw} \) denotes the four entries of the inverse matrix of
\[
C(\eta) = \begin{bmatrix}
\frac{\partial K^1}{\partial \eta^1} & \frac{\partial K^1}{\partial \eta^2} \\
\frac{\partial K^2}{\partial \eta^1} & \frac{\partial K^2}{\partial \eta^2}
\end{bmatrix}
\]
with
\[K^i = \frac{\partial J(\eta)}{\partial \eta^i}
\]
and where the covariance between the elements of \( \hat{\Pi} \) is given by
\[
\text{cov}(\hat{\pi}_{ij}, \hat{\pi}_{kl}) = S^{-1} [(\hat{\Pi} R \hat{\Pi})_i R^{-1} R \hat{\Pi}^T]_{kj} + (R \hat{\Pi})_i R^{-1} (R \hat{\Pi})_j
\]
where \( \Pi \) is the orthogonal projector onto the noise subspace and \( R = U R_q U^H \).

Obviously the limit covariance matrix \( \Gamma_{\theta, \tau} \) depends on the unknown true parameter values, \( \eta, R, \beta \) and \( \sigma^2 \). In practical situations, these values may be replaced by consistent estimates, as for example MUSIC estimates for \( \eta \) and ML estimates for \( R, \beta \) and \( \sigma^2 \) [21].

### V. Proposed Approach

Let us assume that we collect \( Q \) joint angle-delay measurements, corresponding to the \( Q \) paths seen for each of the \( I \) participating BSs (for simplicity we consider the same number of paths at each BS). Thus, according to (4) and (6), we further assume that the distribution of the LOS measurements may be approximated asymptotically by a normal distribution
\[
\hat{\eta}_{i,q} \sim \mathcal{N}(\eta_{i,q}; \hat{\eta}_{\theta, \tau}^{(i,q)})
\]
with mean located at the “true” value of the parameters vector \( \eta_{i,q} = [\theta_{i,q}, \tau_{i,q}]^T \) and covariance matrix given by \( \Gamma_{\theta, \tau}^{(i,q)} \), where \( q \) denotes the index of the \( q \)-th path at position \( i \), for \( i \in \{1, 2, \ldots, I\} \) and \( q \in \{1, 2, \ldots, Q\} \). Moreover, these measurements are directly related to the cartesian coordinates \((x_i, y_i)\) of the MS and the cartesian coordinates \((x_i, y_i)\) of the \( i \)-th BS by the following expressions
\[
\begin{align*}
\tan(\theta_{i,q} + \alpha_i) &= \frac{x - x_i}{y - y_i} \\
\tau_{i,q} &= c^{-1} \sqrt{(x - x_i)^2 + (y - y_i)^2}
\end{align*}
\]
where \( \alpha_i \) denotes the angle between the normal’s array at BS \( i \) and the geographic north of the cartesian system and where \( c \) denotes the speed of light. It is assumed here that both the BS locations \((x_i, y_i)\) and \( \alpha_i \) are known without error.

From the above, it follows that, in a cartesian coordinates system, the asymptotical distribution of the MS position measurements \( \hat{X}_{i,q} \) may be approximated by
\[
\hat{X}_{i,q} \sim \mathcal{N}(X, \Gamma_{i,q})
\]
with
\[
\Gamma_{i,q} = J \left( \begin{array}{c} x, y \\ \theta_{i,q}, \tau_{i,q} \end{array} \right) \Gamma_{\theta, \tau}^{(i,q)} \left( \begin{array}{c} x, y \\ \theta_{i,q}, \tau_{i,q} \end{array} \right)^H
\]
where \( X = [x \ y]^T \) is the “true” MS position vector and \( J(\cdot) \) is the Jacobian matrix allowing to go from \((\theta, \tau)\) to \((x, y)\) domain.

On the other hand, NLOS measurements are considered in this paper as outliers, from which no information about the position of the MS can be obtained [22], [23]. Thus, for simplicity, we assume that for these measurements all values within a delimited region \( \mathcal{R} \) are equally likely, that is
\[
\hat{X}_{i,q} \sim \mathcal{U}(\mathcal{R})
\]
where \( \mathcal{U}(\mathcal{R}) \) stands for the uniform distribution in the region \( \mathcal{R} \). In practice this region may be delimited by the area containing the BSs participating in the localization process.

#### A. Algorithm Derivation

For the \( i \)-th BS, we consider a sequence \( X_{i,1:Q} = \{X_{i,1}, \ldots, X_{i,Q}\} \) of \( Q \) MS position observations. Referring to (12) and (13), we assume that these observations are independent random variables distributed as
\[
P(X_{i,1:Q}; X, \gamma^i) = \sum_{k=0}^{Q} \gamma_{k} f_k(X_{i,1:Q}; X)
\]
where \( \gamma^i = \{\gamma^i_0, \ldots, \gamma^i_Q\} \) are the weighting coefficients for the probability functions \( f_k(X_{i,1:Q}; X) \), at the \( i \)-th BS, given by
\[
\left\{ \begin{array}{ll}
\prod_{l=1}^{Q} v(\hat{X}_{i,l}^{\text{LOS}}), & \text{for } k = 0 \\
\phi(\hat{X}_{i,k}^{\text{LOS}}; X, \Gamma_{i,k}) \prod_{l=1, l \neq k}^{Q} v(\hat{X}_{i,l}^{\text{NLOS}}), & \text{for } k \in \{1, \ldots, Q\}
\end{array} \right.
\]
where \( v(\hat{X}_{i,l}^{\text{NLOS}}) \) and \( \phi(\hat{X}_{i,k}^{\text{LOS}}; X, \Gamma_{i,k}) \) stand respectively for the pdfs of an uniform distribution and a Gaussian distribution with mean vector \( X \) and covariance matrix \( \Gamma_{i,k} \).

Because direct maximization of the likelihood of the observations is intractable, we suggest to use the EM approach. The EM algorithm [14] is a very popular tool for maximum-likelihood (or maximum a posteriori) estimation. The common strand to problems where this approach is applicable is a notion of incomplete-data, which includes the conventional sense of missing data but is much broader than that. The EM algorithm demonstrates its strength in situations where some hypothetical experiments yield (complete) data that are related to the parameters more conveniently than the measurements are.

According to the model introduced above, we may write the joint probability density of the complete data as
\[
P(X_{i,1:Q}, Z_i; X, \gamma^i) = \sum_{k=0}^{Q} \gamma_{k} f_k(X_{i,1:Q}; X) \mathbb{1}\{Z_i = k\}
\]
where \( Z_i \) is a discrete hidden random variable taking its values from the set \( \{0, 1, \ldots, Q\} \).

The EM algorithm is an iterative algorithm to compute maximum likelihood estimate. Each iteration may be formally
decomposed in two steps: an E-step and an M-step. The E-step consists in evaluating the conditional expectation of the complete data likelihood

\[ Q(\Theta, \tilde{\Theta}) = \sum_{i=1}^{I} \mathbb{E}\{ \log(P(X_{i,1:Q}, Z_i; X, \gamma^i)) | X_{i,1:Q}, \tilde{\Theta} \} \]  

where \( \Theta = \{ X, \gamma^i \} \) denotes, for all \( i \), the full parameter vector and where the expectation is taken w.r.t. the probability distribution associated with the value \( \tilde{\Theta} \) of the parameter. In the (generalized) M-step, we compute a new parameter estimate, \( \Theta \), which is chosen in such a way that \( Q(\Theta, \tilde{\Theta}) \geq Q(\tilde{\Theta}, \tilde{\Theta}) \) with equality if and only if \( \tilde{\Theta} \) is a stationary point of the likelihood function. This two step process is repeated until convergence is apparent. The essence of the EM is that increasing \( Q(\Theta, \tilde{\Theta}) \) forces an increase of the incomplete data likelihood.

Thus plugging (15) in (16) we obtain

\[ Q(\Theta, \tilde{\Theta}) = \sum_{i=1}^{I} \sum_{k=0}^{Q} \log(\gamma_k f_k(X_{i,1:Q}; X)) g_k(X_{i,1:Q}; \tilde{X}, \gamma^i) \]  

where

\[ g_k(X_{i,1:Q}; \tilde{X}, \gamma^i) = \frac{f_k(X_{i,1:Q}; \tilde{X})}{P(X_{i,1:Q}; \tilde{X}, \gamma^i)} \]

We then optimize (17) w.r.t. the coefficients \( \gamma_k^i \) and the source location \( X \), which gives

\[ \gamma_k^i = g_k(X_{i,1:Q}; \tilde{X}, \gamma^i) \]  

\[ X = \left( \sum_{i=1}^{I} \sum_{k=1}^{Q} \gamma_k^i \Gamma^{-1}_{i,k} \right)^{-1} \left( \sum_{i=1}^{I} \sum_{k=1}^{Q} \gamma_k^i \Gamma^{-1}_{i,k} X_{i,k} \right) \]

In practice, in order to avoid keeping local solutions, the best approach consists in initiating the algorithm for a number of departing points for the position only. These initial guesses may be taken from a grid of equidistant points situated over the region of interest. The weighting coefficients may initially be considered as equally likely, i.e. \( \gamma_k^i = 1/(Q + 1) \), for all \( i \) and all \( k \). Hence, the MS position will be given by the most likely estimated parameter vector among those estimated for each point of the grid.

VI. SIMULATION RESULTS AND REMARKS

Two simulations were performed in a system which approximates GSM: where the symbol period is \( T = 3.7 \mu s \), the channel is estimated at each time slot via least square s using 26 training bits. The binary sequence is modulated by a raised-cosine pulse with roll-off 0.35, assumed to be zero outside the interval \([-3,3]\). The sampling rate is considered to be twice the symbol rate. Data are collected over 26 time slots using an uniform linear array (ULA) with two sensors.

The MS is considered to stay at \((2,2)\)Km, in a 2-D cartesian system. The BSs equipped with an antenna array are placed at \((x_i, y_i) = \{(1,0.5), (3.5,1.5), (2,3.5)\}\)Km. In each position the number of paths (assumed to be known) is \( Q = 3 \). The angles between the normal arrays and the geographic north were \( \alpha = [45,-45,-179] \). We also consider that BS1 observes only NLOS paths, viewed as possible sources picked out randomly from the region \( R = \{(x,y) | x \in [0,4000] m, y \in [0,4000] m\} \). To the rest of the BSs that observe the LOS path two more paths were added, randomly chosen from \( R \). The path gains at each BS position are \( | \beta_1 |^2 = [0,8,1,0,9] \), \( | \beta_2 |^2 = [0,9,0,8,1] \), \( | \beta_3 |^2 = [0,7,0,8,1] \), respectively. The signal-to-noise ratio (SNR) is taken as the ratio of the variance of the strongest path to the variance of the noise \( \sigma^2 \) [15].

In the simulations we mean by ML-based approach and by MUSIC-based approach the proposed approach using the limit covariance matrices and the angle-delay measurements computed from JADE-ML and JADE-MUSIC models, respectively.

A. Simulation I

In this simulation we used the AOA and TOA measurements as well as the asymptotic covariance matrix, for the MS position, given by JADE-ML model (ML-based approach). The SNR was considered 10dB. Figure 1 depicts the top view of two superimposed likelihood surfaces for two cases: 1) “Initial Likelihood” is the likelihood surface as a function of the MS position with equal weighting coefficients \( \gamma_k^i = 0.25 \) for all \( i \) and for all \( k \) (as seeded to the algorithm in order to start), and 2) “Final Likelihood” is the likelihood surface as a function of the MS position using the weighting coefficients computed by the proposed approach. It should be noticed that both surfaces present two intersecting “beams”, due to the contribution of the BSs 2 and 3 which see the LOS paths among the detected total number of paths (compare it to figure 2 for the three BSs seeing the LOS paths). Second, the “initial likelihood” surface presents a maximum (intersection of two beams) severely biased from the true MS position (2,2)Km. This is due to the multipath propagation and measurements taken in a NLOS regime in BS 1. Better accuracy of the source position estimation is obtained after the proposed algorithm has correctly chosen the optimal weighting coefficients for the functions \( f_k(X_{i,1:Q}; X) \).

Estimated coefficients for this simulation at BS 1, 2, and 3 were respectively \( \gamma^1 = \{1,0,0,0\} \), \( \gamma^2 = \{0,0,1,0\} \) and \( \gamma^3 = \{0,1,0,0\} \). Which means that BS 1 reported only NLOS measurements, while at BSs 2 and 3, paths number 2 and 1, respectively, were chosen as the LOS ones. The estimated MS position is at \((193,2002)\)m.

B. Simulation II

To assess the performance of the proposed algorithm, we computed the variance and the bias per component on the estimation of the MS position from a 500 Monte-Carlo runs. We compute the variance as the sum of \( \hat{x} \)’s and \( \hat{y} \)’s variances and the bias per component as \( \text{BIAS}(x) = \frac{1}{\text{M}} \sum_{t=1}^{\text{M}} (\hat{x}_t - x) \). Figures 3 and 4 show respectively the experimental standard
deviations and bias on the MS position estimation using the proposed approach for different SNRs. As expected, the proposed approach based on ML presents better performance than MUSIC-based approach for low SNRs. As a matter of fact, MUSIC-based approach does not give satisfactory results in the range of 0-8dBs, because MUSIC algorithm is not able to distinguish all the minima in its cost function. Therefore, the associated asymptotic covariance matrices are bad conditioned and no valid result is produced by the algorithm.

VII. CONCLUSIONS

We proposed a new algorithm to locate a MS from joint measurements of TOA and AOA of the emitted signals in a multipath environment. We used an antenna array at different BSs to perform such measurements. Collected data were processed using the maximum likelihood method, based on an implementation of the EM algorithm. We did not consider the first arrival path or the path with the highest power as the LOS path. Instead we proposed a statistical approach which handles the presence of LOS paths and rejects the NLOS ones. The resulting algorithm is able to locate the MS with at least two BSs “seeing” multiple paths, comprising the LOS paths, while at the same time is able to remove information delivered by BSs being in NLOS regime only.

ACKNOWLEDGMENT

The authors would like to thank the financial support of the “Consejo Nacional de Ciencia y Tecnología” (CONACYT) and the “Secretaría de Educación Pública” (SEP), the Mexican organs concentrated on technology and public education.

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Fig. 2. Top view of the likelihood surface as a function of the MS position using the weighting coefficients computed by the proposed approach. It is considered here that the three BSs see the LOS paths among the total number of detected paths. The true MS position is at (2,2)Km.

Fig. 3. Experimental standard deviations of the MS position estimation obtained from 500 Monte-Carlo runs.

Fig. 4. Experimental BIAS for each cartesian component $(x, y)$ of the MS position estimation obtained from 500 Monte-Carlo runs.