Two-Dimensional Autoregressive Model for MIMO Wideband Mobile Radio Channels

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Abstract—In this work, we propose the multichannel two-dimensional (2D) autoregressive (AR) model for multiple-input multiple-output (MIMO) wideband mobile wireless channels. The parameters of the proposed model can be estimated from the real-world measurement data. For this purpose, we suggest using a straightforward extension of the prediction error minimization (PEM) algorithm. We also address the problem of possible instability of the multichannel 2D AR model. A model stabilization procedure based on numerical optimization techniques is proposed. The performance of the multichannel 2D AR model has been evaluated based on the synthetic data generated using two different channel simulators.

I. INTRODUCTION

The effectiveness of a channel model in the design of MIMO wireless systems relies on the realistic representation of the statistical behavior of the real-world radio channels in time, frequency, and space. During the last decade, a number of radio channel models has been proposed for the development and optimization of wireless MIMO communication systems. A recent survey of the existing MIMO channel models can be found in [1].

The double-directional channel model has attracted a lot of interest in the recent past [1], [2]. The wireless multipath propagation channel is represented by a finite number of discrete, possibly clustered, specular waves. The diffuse wave component is regarded as the white noise and neglected. The parameters characterizing each of the specular waves are estimated from the measurement data using, e.g., the space-alternating generalized expectation-maximization (SAGE) algorithm [3]. However, the results presented in [4]–[6], demonstrate that the diffuse wave component cannot always be discarded.

In this paper, we propose the multichannel 2D AR model for MIMO wideband mobile radio channels. In contrast to the double-directional channel model mentioned above, the radio channel between each of the transmitting and the receiving antennas is modeled by the 2D rational transfer function. Our interest in the multichannel 2D AR model is motivated by the high level of flexibility intrinsic to the AR models, which has been extensively used in spectrum estimation and system identification, see, e.g., [7]–[9] and the multiple references therein. Some of the previous works related to the AR modeling and simulation of the wireless communication channels can be found, for example, in [10]–[12]. Spectral estimation for multiple 2D signals using the multichannel 2D AR model is discussed in [13]. To the best of our knowledge, no results related to the multichannel 2D AR modeling of the radio channels for the wireless communication systems have yet been published.

The parameters of the multichannel 2D AR model can be estimated from the real-world measurement data by using a straightforward extension of the well-known Yule-Walker algorithm or, alternatively, the PEM algorithm, which is presented below. None of these methods guarantees the stability of the resulting multichannel 2D AR model. Therefore, we propose a procedure, which is based on the multi-objective numerical optimization method, that can be used in order to stabilize the multichannel 2D AR model.

The performance of the multichannel 2D AR model has been evaluated on the two synthetic MIMO $2 \times 2$ channels. The first channel has been generated using a channel simulator based on the multichannel 2D AR model with known parameters. The second channel is constructed by means of a simulator based on the double-directional radio channel model.

The paper is organized as follows. In Section II, we describe the multichannel 2D AR model. The parameter estimation method is presented in Section III. In Section IV, we consider the stability of the multichannel 2D AR model. Section V shows the results of the performance evaluation. Finally, the concluding remarks are given in Section VI.

II. THE MULTICHANNEL 2D AUTOREGRESSIVE MODEL

We consider a MIMO wideband wireless channel, which contains $N_T$ antennas at the transmitter side and $N_R$ antennas at the receiver side. Let the matrix sequence $\mathbf{H}[m, n]$ be the channel time-variant frequency response (TVFR) sampled at discrete frequencies $f_m = -B/2 + m \Delta f \in [-B/2, B/2]$, $m = 0, \ldots, M-1$, and at discrete time instances $t_n = n \Delta t \in [0, T]$, $n = 0, \ldots, N-1$. We denote the frequency bandwidth and the observation time interval as $B$ and $T$, respectively. The matrix sequence

$$
\mathbf{H}[m, n] = \begin{pmatrix} H_{11}[m, n] & \cdots & H_{1N_R}[m, n] \\ \vdots & \ddots & \vdots \\ H_{N_T1}[m, n] & \cdots & H_{N_TN_R}[m, n] \end{pmatrix}
$$

(1)
can be equivalently represented in the vectorized form as

\[ \mathbf{h}[m, n] = \text{vec} (\mathbf{H}[m, n]) = \begin{pmatrix} H_{11}[m, n] \\ H_{21}[m, n] \\ \vdots \\ H_{N_f N_R}[m, n] \end{pmatrix} \]  

(2)

where \( \mathbf{h}[m, n] = [h_1[m, n], h_2[m, n], \ldots, h_{N_f N_R}[m, n]]^T \) and \( h_i[m, n], i = 1, \ldots, N_f N_R, \) is the sampled TVF of the \( i \)-th subchannel. The operator \((\cdot)^T\) denotes the transpose.

We assume that each subchannel \( h_i[m, n] \) of the vector sequence \( \mathbf{h}[m, n] \) is a complex zero mean 2D wide-sense stationary (WSS) random process (random field). Furthermore, the vector sequence \( \mathbf{h}[m, n] \) corresponds to the multichannel 2D AR process of the form

\[ \mathbf{h}[m, n] = - \sum_{[i_1, i_2] \in \mathcal{S}} \sum_{\{i_1, i_2\} \neq [0, 0]} \mathbf{A}^T[i_1, i_2] \mathbf{h}[m - i_1, n - i_2] + \mathbf{u}[m, n] \]  

(3)

where \( \mathbf{A}[i_1, i_2] \) are complex matrix coefficients of dimensions \( N_f N_R \times N_f N_R \). The vector sequence \( \mathbf{u}[m, n] \) is a complex multichannel 2D white noise with the matrix cross-correlation function (CCF) \( \mathbf{R}_{\mathbf{uu}}[k, l] \) defined as

\[ \mathbf{R}_{\mathbf{uu}}[k, l] = E \{ \mathbf{u}^*[m, n] \mathbf{u}[m + k, n + l] \} = \mathbf{P}_{\mathbf{uu}} \delta[k, l] \]  

(4)

where \( \delta[k, l] \) is the 2D Dirac delta function and \( \mathbf{P}_{\mathbf{uu}} \) denotes the noise delay-Doppler power spectral density (PSD) matrix, which is constant. The operators \( E\{\cdot\} \) and \( \cdot^* \) represent the statistical averaging and complex conjugate, respectively.

We also assume that the channel model (3) is recursively computable (causal) [14]. The two most commonly used support regions \( \mathcal{S} \) that guarantees the recursive computability of the vector sequence \( \mathbf{h}[m, n] \) are the finite nonsymmetric half-plane (NSHP) and the finite quarter plane (QP) supports [8]. In this article, we focus our attention on the multichannel 2D AR models (3) with the finite QP support region \( \mathcal{S}_{QP} \) defined as

\[ \mathcal{S}_{QP} = \{[i_1, i_2] : 0 \leq i_1 \leq p_1, 0 \leq i_2 \leq p_2 \}. \]  

(5)

where \((p_1, p_2)\) is the order of the multichannel 2D AR model, henceforth designated AR\((p_1, p_2)\).

Using the relationship between the input PSD and the output PSD of a linear shift invariant (LSI) multichannel 2D filter (see, e.g., [7], [13]) we define the delay-Doppler PSD matrix \( \mathbf{P}_{\mathbf{hh}}(\tau', f_d) \) of the MIMO wireless channel as

\[ \mathbf{P}_{\mathbf{hh}}(\tau', f_d) = \mathcal{H}^*(\tau', f_d) \mathbf{P}_{\mathbf{uu}} \mathcal{H}^T(\tau', f_d) \Delta f' \Delta t \]  

(6)

where

\[ \mathcal{H}(\tau', f_d) = (\mathbf{I} + \sum_{i_1=0}^{p_1} \sum_{i_2=0}^{p_2} \mathbf{A}[i_1, i_2] e^{-j2\pi (\tau' i_1 + f_d i_2 \Delta t)})^{-1} \]  

(7)

1The diagonal elements of the delay-Doppler PSD matrix are the delay-Doppler spectra of the individual sub-channels \( h_i[m, n] \) (2) at the certain delay and Doppler frequency. The off-diagonal elements correspond to the samples of the cross-subchannel delay-Doppler spectra.

and \( \tau' \) and \( f_d \) are the propagation delay and the Doppler frequency, respectively. The matrix \( \mathbf{I} \) is the identity matrix.

III. ESTIMATION OF THE MODEL PARAMETERS

Suppose that the sampled TVFR \( \mathbf{H}[m, n] \) of the MIMO channel is given to us. For example, the matrix sequence (1) can be obtained from a channel sounder during a measurement campaign.

Assume that the order \((p_1, p_2)\) of the multichannel 2D AR\((p_1, p_2)\) model (3) is known. The parameters of the model, i.e., the matrix coefficients \( \mathbf{A}[i_1, i_2] \) and the noise PSD matrix \( \mathbf{P}_{\mathbf{uu}} \), are to be estimated from the available matrix sequence \( \mathbf{H}[m, n] \) (1) or, equivalently, from the vector sequence \( \mathbf{h}[m, n] \) (2).

A. Prediction Error Minimization

The PEM method is based on the strong relationship existing between the AR modeling and the linear prediction problem [9].

The linear forward predictor of \( \mathbf{h}[m, n] \) is defined as

\[ \hat{\mathbf{h}}[m, n] = - \sum_{[i_1, i_2] \in \mathcal{S}_{QP}, [i_1, i_2] \neq [0, 0]} \mathbf{A}^T[i_1, i_2] \mathbf{h}[m - i_1, n - i_2] \]  

(8)

with the prediction error given by

\[ \mathbf{e}[m, n] = \mathbf{h}[m, n] - \hat{\mathbf{h}}[m, n]. \]  

(9)

Consequently, the prediction error power matrix can be written as

\[ \mathbf{\Sigma} = E \{ \mathbf{e}[m, n] \mathbf{e}^H[m, n] \}. \]  

(10)

For the finite-sample vector sequence \( \mathbf{h}[m, n] \) (2) the estimator of the matrix \( \mathbf{\Sigma} \) takes the form

\[ \hat{\mathbf{\Sigma}} = \frac{1}{(M - p_1)(N - p_2)} (\mathbf{Z} + \mathbf{YX})^H(\mathbf{Z} + \mathbf{YX}) \]  

(11)

where the matrices \( \mathbf{Z} \), \( \mathbf{Y} \), and \( \mathbf{X} \) are defined below

\[ \mathbf{Z} = \begin{bmatrix} \mathbf{h}^T[M - 1, N - 1] \\ \mathbf{h}^T[M - 2, N - 1] \\ \vdots \\ \mathbf{h}^T[p_1, p_2] \end{bmatrix} \]  

(12)

\[ \mathbf{Y} = \begin{bmatrix} h_f^T[M - 2, N - 1] & \cdots & h_f^T[M - p_1 - 1, N - p_2 - 1] \\ h_f^T[M - 3, N - 1] & \cdots & h_f^T[M - p_1 - 2, N - p_2 - 1] \\ \vdots & \ddots & \vdots \\ h_f^T[p_1 - 1, p_2] & \cdots & h_f^T[0, 0] \end{bmatrix} \]  

(13)

\[ \mathbf{X} = \begin{bmatrix} \mathbf{A}^T[1, 0] \\ \mathbf{A}^T[2, 0] \\ \vdots \\ \mathbf{A}^T[p_1, p_2] \end{bmatrix} \]  

(14)
The matrix coefficients $A[i_1, i_2]$ of the multichannel 2D AR($p_1, p_2$) model can be estimated by minimizing the sum of the estimated prediction error powers, i.e.,

$$\left\{ \hat{A}[i_1, i_2] \right\}_{i_1, i_2 \in \{0, 1\}} \left\{ trace \left( \Sigma \right) \right\}. \quad (15)$$

This is a linear least-squares estimation problem. The estimate of the matrix $X$ (14) that minimizes (15) can be written as

$$\hat{X} = -Y^\dagger Z \quad (16)$$

where $Y^\dagger$ is the Moore-Penrose pseudoinverse of the matrix $Y$ [15].

The estimated noise delay-Doppler PSD matrix $\hat{P}_{uu}$ is equal to the residual prediction error power matrix $\Sigma_{\text{est}}$, obtained by substituting the solution $X$ (16) into (11).

IV. MODEL STABILITY

In order to use the multichannel 2D AR($p_1, p_2$) model (3) as a channel simulator, the model must be stable. The channel model (3) is stable when the following condition is fulfilled [8]

$$\det \left( I + \sum_{i_1=0}^{p_1} \sum_{i_2=0}^{p_2} A[i_1, i_2] z_{i_1} z_{i_2} \right) \neq 0,$$

for all $\{(z_{1_1}, z_{1_2}) : |z_{1_1}| \leq 1, |z_{2_2}| \leq 1\} \quad (17)$

where $z_{1_1}$ and $z_{2_2}$ are complex variables.

The PEM method, described in Section III, does not guarantee the stability of the resulting multichannel 2D AR($p_1, p_2$) model. Additionally, the stability test (17) is almost useless in practice due to the heavy computational load.

A. State-space representation of the multichannel 2D AR model

In the past years, a number of stability tests has been proposed for 2D recursive filters in state-space form [16], [17]. An attractive feature of the state-space representation is that it is directly applicable to multichannel 2D recursive filters, i.e., to the multichannel 2D AR($p_1, p_2$) model (3).

In this article, we consider the 2D state-space model representation developed by Roesser [18]. The Roesser’s state-space model can be formulated as follows [14]

$$\begin{bmatrix} x_h[m+1, n] \\ x_v[m, n+1] \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_h[m, n] \\ x_v[m, n] \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u[m, n]$$

$$h[m, n] = C_1 x_h[m, n] + C_2 x_v[m, n] + Du[m, n] \quad (18)$$

where $x_h$ and $x_v$ are the model state variables. The model input $u[m, n]$ and the model output $h[m, n]$ in (18) are the same processes $u[m, n]$ and $h[m, n]$ as in (3).

The two possible candidates for the model stability test of the Roesser’s state-space model are presented below [17].

![Fig. 1. Flowgraph representing the multichannel 2D AR(1,1) model.](image-url)

The Roesser’s state-space model (18) is bounded input bounded output (BIBO) stable if

$$\begin{cases} A_{11} & \text{is stable} \\ A_{22} + A_{21} (z_1 - A_{11})^{-1} A_{12}, |z_1| = 1 & \text{is stable} \end{cases} \quad (19)$$

where a square matrix, e.g., $A_{11}$, is stable if the maximum magnitude of its eigenvalues is less than 1. The second possible group of stability criteria is defined as

$$\begin{cases} \|A_{11}\|_2 < 1 \\ \|A_{22}\|_2 + \|A_{21}\|_2 (1 - \|A_{11}\|_2^{-1}) \|A_{12}\|_2 < 1 \end{cases} \quad (20)$$

where $\|\cdot\|_2$ is equal to the largest singular value of the matrix.

The criteria in (19) are sufficient and necessary conditions for the BIBO stability of the model (18). On the other hand, the criteria in (20) are sufficient but not necessary [17]. The experimental results show that the BIBO stability conditions (19) are more suitable for the stabilization procedure presented below, in spite of the obvious computational advantages associated with the stability test implemented according to the criteria in (20).

To be able to apply the stability test (19), the multichannel 2D AR($p_1, p_2$) model (3) has to be converted to the Roesser’s state-space representation (18). The conversion between the model representations can be done in at least two ways. However, considering the computational load required to test the stability of the model, it is desirable to minimize the number of state variables, i.e., the length of the vectors $x_h$ and $x_v$ in (18). The minimal state-space realization of the model (18) can be obtained by assigning the state variables in $x_h$ and $x_v$ to the outputs of the shift operators in a signal flow graph [14]. As an example, the flowgraph of a simple multichannel 2D AR(1,1) model is presented in Fig. 1. The shift operators are indicated as $z_{1_1}$ and $z_{2_2}$.

The details related to the state-space realizations of the 2D filters can be found in [14].

B. Stabilization procedure

In this subsection, we briefly describe a procedure that resolves the possible instability of the multichannel 2D AR($p_1, p_2$) model (3).

Step 1. Estimate the matrix coefficients $\hat{A}^{(0)}[i_1, i_2]$ and the noise delay-Doppler PSD matrix $\hat{P}_{uu}^{(0)}$ by minimizing the
sum of the estimated prediction error powers in (15) (see Section III).

Step 2. Calculate the matrices $A_{11}$, $A_{12}$, $A_{21}$, and $A_{22}$ of the Roesser’s state-space representation (18). If the BIBO stability conditions in (19) are satisfied, skip the next steps.

Step 3. Formulate the minimization problem (15) under constrains (19) as a multi-objective optimization problem that can be solved by the goal-attainment method [19], i.e.,

$$\min_{\gamma \in \mathcal{R}} \gamma$$

subject to

$$\text{trace}\left(\Sigma\right) - w_1 \gamma \leq \text{trace}\left(\hat{P}_{uu}(0)\right)$$
$$\rho(A_{11}) - w_2 \gamma \leq 1$$
$$\rho(A_{22} + A_{21}(z_1 I - A_{11})^{-1} A_{12}) - w_3 \gamma \leq 1, |z_1| = 1$$

where $\rho(\cdot)$ denotes the spectral radius of a square matrix [15], $\{w_1, w_2, w_3\}$ are the weighting coefficients that signify the relative trade-off between the objectives, and $\gamma$ is a scalar parameter (see, e.g. [19], [20]). Note that the matrices $\Sigma$, $A_{11}$, $A_{12}$, $A_{21}$, and $A_{22}$ in (21) are functions of the matrix coefficients $A[x_1, x_2]$, $[x_1, x_2] \in S_{QP}$, $[x_1, x_2] \neq [0, 0]$. The solution to the multi-objective minimization problem formulated in (21) can be found by applying the `fgoalattain` function implemented in MATLAB. The estimates $\hat{A}[x_1, x_2]$ obtained at Step 1 can be used as the initial parameter values.

Step 4. The matrix coefficients $\hat{A}[x_1, x_2]$ obtained in Step 3 are substituted into (14) to get the estimate of the matrix $\hat{X}$. Finally, the estimate of the noise delay-Doppler PSD matrix $\hat{P}_{uu}$ is equal to the residual prediction error power matrix $\Sigma_{\min}$ calculated by substituting the matrix $\hat{X}$ into (11) (see Section III).

A note regarding Step 2 and Step 3 of the algorithm described above is required. For the second stability criterium in (19) the largest magnitude eigenvalue of the corresponding matrix has to be calculated at the infinite number of points $z_1$ along the unit circle, $|z_1| = 1$. The conducted simulations suggest that a limited number of points $z_1$ is sufficient to check the stability of the multichannel 2D AR($p_1$,$p_2$) model.

V. Simulation Results

In this section, we present two examples that illustrate the performance of the multichannel 2D AR models. In each of the examples, the role of the measured $2 \times 2$ MIMO channel is played by a channel simulator, in the following referred to as the original model. Our task is to estimate the parameters of the multichannel 2D AR($p_1$,$p_2$) model (3), the target model, from the TVFR matrix sequence $\hat{H}[m,n]$ synthesized by using the original model.

In the first example, the original model is the multichannel 2D AR($2,2$) model (3). Due to the lack of space and a relatively large number of parameters, we do not provide the matrix coefficients $\hat{A}[i_1, i_2]$ and the noise delay-Doppler PSD matrix $\hat{P}_{uu}$ of the original model.

The parameters of the target multichannel 2D AR($2,2$) model, i.e., the matrix coefficients $\hat{A}[i_1, i_2]$ and the noise delay-Doppler PSD matrix $\hat{P}_{uu}$, have been estimated from the training TVFR matrix sequence $\hat{H}_{uu}$, $1 \leq m \leq 193, 1 \leq n \leq 100$, by employing the PEM method (see Section III). The BIBO stability test (19) shows that the resulting target multichannel 2D AR($2,2$) model is stable.

To evaluate the performance of the target multichannel 2D AR($2,2$) model, a test TVFR matrix sequence $\hat{H}[m,n]$ has been generated using the resulting target model. Another test TVFR matrix sequence $\hat{H}[m,n]$, of the same length as $\hat{H}[m,n]$, has been obtained using the original model. The temporal cross-correlation functions (TCCF) $\hat{r}_{h_{h,j}}[k]$, $k \in \mathbb{N}$, $i,j = 1, \ldots, N_N$, and the frequency cross-correlation functions (FCCF) $\hat{r}_{h_{j,l}}[l]$, $l \in \mathbb{N}$, of the target multichannel 2D AR($2,2$) model are estimated from the test TVFR matrix sequence $\hat{H}[m,n]$. Similarly, the TCCFs $\hat{r}_{h_{h,j}}[k]$ and the FCCFs $\hat{r}_{h_{j,l}}[l]$ of the original model are estimated from the test TVFR matrix sequence $\hat{H}[m,n]$. Some estimated TCCFs of the target model and of the original model are shown in Fig. 2. The corresponding estimated FCCFs are shown in Fig. 3.

As can be seen in Figs. 2 and 3, the selected TCCFs and
the FCCFs of the target multichannel 2D AR(2,2) model approximate well their respective counterparts of the original model. Similar results can be observed for other estimated temporal and frequency cross-correlation functions.

The second example presents an extreme case in a sense that the generated TVFR matrix sequence (1) corresponds to the discrete multichannel 2D PSD, while the target model implies a continuous multichannel 2D PSD. The original model in this example, is a channel simulator based on the double-directional channel model [1]. In the double-directional model the wireless propagation channel is represented by a set of $L$ complex exponents (multipath components). Each of these complex exponents is characterized by the complex amplitude, Doppler frequency, propagation delay, direction-of-arrival, direction-of-departure, and, possibly, polarization matrix. In our double-directional model, the transmitter is stationary and the receiver is moving. The transmitter and the receiver are equipped with linear antenna arrays. Each of the antenna arrays consists of two ($N_T = N_R = 2$) omidirectional single-polarization antenna elements separated by a half wavelength distance. The radio waves propagate in the azimuthal plane. Several other parameters are specified below:

- Number of multipath components: $L = 530$;
- Time interval between snapshots: $\Delta t = 10$ ms;
- Signal carrier frequency: $f_c' = 5.2$ GHz;
- Interval between frequencies: $\Delta f' = 3.125 \times 10^5$ Hz;
- Frequency bandwidth: $B = 60$ MHz;
- Measurement noise SNR: 20 dB.

The multipath components of the original model in the delay-Doppler plane are shown in Fig. 4.

![Fig. 4. The multipath components.](image_url)

Again, as in the first example, the training TVFR matrix sequence $H[m, n]$, $1 \leq m \leq 193, 1 \leq n \leq 100$, has been generated using the original model and supplied to the PEM to estimate the parameters $A[i_1, i_2]$ and $P_{uu}$ of the target multichannel 2D AR($p_1, p_2$) model. In this case, the order of the target model is unknown. Therefore, the parameters of the multichannel 2D AR(1,1), AR(3,3), and AR(5,5) models have been estimated. All of the target models have been stabilized using the procedure described in Section IV-B.

The TCCFs $\hat{r}_{h_1 h_1} [\tau_q]$ of the resulting target models and the TCCFs $\tilde{r}_{h_1 h_1} [\tau_q]$ of the original model have been estimated from the generated test sequences $H[m, n]$ and $\tilde{H}[m, n]$, respectively, at the discrete time lags $\tau_q$. Similarly, the FCCFs $\tilde{r}_{h_1 h_1} [\nu_p]$ of the resulting target models and the FCCFs $\tilde{r}_{h_1 h_1} [\nu_p]$ of the original model have been estimated from the corresponding test sequences at the discrete frequency lags $\nu_p$.

In Figs. 5 and 6, we demonstrate the estimated TCCFs and FCCFs, respectively, for the first subchannel of the resulting target models and of the original model.

![Fig. 5. The TCCF $\hat{r}_{h_1 h_1} [\tau_q]$ of the target models and the TCCF $\tilde{r}_{h_1 h_1} [\tau_q]$ of the original channel.](image_url)

![Fig. 6. The FCCF $\hat{r}_{h_1 h_1} [\nu_p]$ of the target models and the FCCF $\tilde{r}_{h_1 h_1} [\nu_p]$ of the original channel.](image_url)

As expected, a slightly better approximation of the correlation characteristics of the original channel can be achieved by increasing the order of the target model.

The effect of applying the stabilization procedure to the target multichannel 2D AR(5,5) model can be observed in...
Figs. 7 and 8 where we depict the delay-Doppler PSDs for the first subchannel before and after stabilization, respectively. The delay-Doppler PSDs of the target multichannel 2D AR models have been calculated using (6) and (7).

The stabilization procedure proposed in this article can be used to stabilize the multichannel 2D AR model. However, due to the large number of the model parameters, the stabilization procedure might be time consuming even for the multichannel 2D AR models of the moderate order, say $p_1 > 5$, $p_2 > 5$ for $2 \times 2$ MIMO systems.

In this connection, it is important to study the performance and stability of the multichannel 2D AR models obtained from the real-world measurement data. The potential of the stabilization methods existing in the framework of the control system design, e.g., the model stabilization by means of the state or/and output feedback controllers, constitutes another topic for further investigation.

VI. CONCLUSION

In this article, we have presented the multichannel 2D AR model for MIMO wideband mobile radio channels. The parameters of the multichannel 2D AR model can be estimated from the measured TVFR matrix sequence of a real-world channel. The model parameter estimation algorithm, described in the paper, is a straightforward extension of the PEM method developed for estimating parameters of the multichannel 1D and 2D single-channel AR models.

The main problem associated with the multichannel 2D AR model is a possible instability of the resulting channel model.

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