Abstract.
In this paper we deal with the problem of formalizing the notion of aesthetic judgment. Many aspects of the representation of beauty have been evaluated and studied in the Analytical Philosophy and in the Linguistic literature. However, to our knowledge, an approach that includes a complete formalization from a knowledge representation point of view has not yet been carried out.

In this paper we approach the problem from a Neo-Kantian perspective. We shall consider an aesthetic judgment as both an evaluation and a description. We model this conceptualization by means of a formalization in first-order logic, that incorporates the fundamental notions of a social theory of taste called the Theory of Distinction originally conceived for explaining Fashion phenomena and then provided as a model for taste in general [3, 35].

We presuppose five axiomatic sorts: objects, individuals, a subset of individuals named the elite, a collection of characteristics of objects named the properties and a collection of objects which can be assigned to some properties called the values. The elite individuals establish canons and acknowledge other individuals in terms of good taste, being this kind of judgment expressed as well among generic individuals.

1 Introduction
The purpose of this paper is to characterize the aesthetic judgment in formal terms, and to express it in first order logic. We adopt a Neo-Kantian perspective, namely we assume that such judgments are both the expression of an “a priori” classification, and an “a posteriori” evaluation. Objects classified as beautiful are considered such only by public judgment, that means that we should take in account sociological aspects [35].

The expression of judgments of an aesthetic nature is very frequent in common sense reasoning. This happens in various circumstances, and in general in commenting on events or things, even those of a rather common nature. While watching a movie, visiting an art gallery, shopping for personal items, browsing a web site, setting up preferences of an application, we judge things from an aesthetic point of view.

We believe that a system able to support aesthetic reasoning could be fruitfully employed in semantically-augmented design tools, databases, and collaborative workgroup systems, that is, applicative domains where human communication and knowledge sharing can be empowered by aesthetic judgements that are transparent for the user, and that can be made fully explicit, modifiable, quotable and re-usable.

A semantically-augmented aesthetic knowledge system would provide markup of aesthetic nature, so that we can imagine search features able to process the following requests:

- Look for all the documents that speak about authors of books with the same style of Umberto Eco;
- Look for all the web sites that have a graphic layout aesthetically similar to the Google’s one;
- Look for a document about films of a style approved by the French Avantgardists of the sixties.

These searches, from common to more exotic ones are not possible within currently existing Automated User Recommendation Systems, Automated User Profiling User Preference Settings of Computer Aided Design Systems, the sole applications which try to address problems of the representation of taste of the users. We believe that features based on a general theory of aesthetic judgments would be able to enhance the effectiveness of such applications and even others which are also sketched in below.

The fundamental motivations for the development of independent reasoning systems are: the need for autonomous axiom sets due to the specific nature of the domain; the existence of specific applications, where the domain is relevant.

The first motivation applies here for social aspects of aesthetic knowledge, which will be sketched in below; the second motivation has been discussed above.

The approach we introduce here is general. However we believe that it is not possible to think at a completely abstract conceptualization of aesthetics. We therefore restricted ourselves to the specific domain of collaborative Design, where the two aspects of the approach we adopted have clearly defined meaning. Our theory is based upon the notion of Judgment, and upon the notion of Distinction, which play an important role in the above mentioned theory.

We base our approach on the ideas expressed by Rodgers et al. in [28]. In these papers Rodgers et al. have addressed the problem of managing aesthetic knowledge based on the notion of explanation, and have interpreted the textual aspect of an application, originally CADET©, and now WebCADET©. Another interesting software tool is FIORES [6]. What is missing in these investigation, and is under consideration from our point of view is the ontological level of this knowledge management. An ontology of aesthetic aspects in Design would provide the right tool to integrate software tools like WebCADET© or FIORES, in practical cases, and form the base for the construction of a aesthetics-aware Semantic Web.
The notion of aesthetics as a conceptual category has been considered as formal since the fundamental analyses of Immanuel Kant [20]. However, the first deep attempt to embed Kant’s approach into a sociological theory can be attributed to Bourdieu [3].

[2] contains not only references to various philosophical, linguistic and proto-sociological investigations stressing formal difficulties in the applicability of Kant’s approach, but also direct criticisms of the approach per se. Bourdieu introduces the notion of elite, whose members perform the activity of imposing their taste choices on the mass individuals. Elites impose their tastes on the community as a way to distinguish themselves from the mass of individuals. Their choices tend to be imitated by the mass individuals for the simple reason that elite individuals also label mass individuals with judgments of “good taste” which is a socially desirable attribute. This theory has the name of Theory of Distinction.

The Theory of Distinction violates in a direct way the basic notion of aesthetic judgment as proposed in Kant’s theory, which is supposed to be disinterested. In particular, Kant defines taste as the exercise of judgment relative to the delight obtained by the relation with the object. There are three kinds of delight: the delight in the agreeable, the delight in the good and the delight in the beautiful. Only the latter is disinterested, being the first driven by the desire of satisfying the senses, and the second looking for approval by the community, being driven by the desire for others’ esteem. However, some further observations on the original Bourdieu theory are important, as in particular observed in [35]. One major point is that the belonging to an elite is not a static property of individuals, but is instead determined by the approval that the taste of that individual receives by the community.

A very interesting application domain of aesthetic judgments’ theories is Fashion. The most cited paper in the field is Simmel’s work [30]. For more recent investigation reports and deep literature analysis interested readers may refer to [17] for a sociological perspective and to the work of Commo and Jeanne [10] for a more specific economic analysis.

There are cases of unconscious application of the above mentioned sociological theory to collaborative creative design [6], though this model does not explicitly refer either to the theoretical framework or to its foundational issues. However one case of intentional application of the above cited theory exists in the computer science theory and concerns digital libraries [29]. In this paper the Theory of Distinction is employed directly to model the social behavior of the users of digital libraries and to settle the different activities and privileges of those users.

Though Kant’s theory is commonly evaluated as one of the most relevant basic analysis of the notion of aesthetic judgments, and the sociological theory of aesthetics of Bourdieu is generally acknowledged as a well-focused and very neat point of view others have studied the aesthetic problem from a point of view rather different, employing the language of Bourdieu, from the point of view of an elite, as in semiotics studies such as [21].

The analysis of the specific language of aesthetic communities, elites in particular, and of creativity as the fundamental aspect of elite definition, specifically in the context of Design, can be found in various papers [19, 13, 12, 32].

The notion of aesthetic canon is historically referred to classic architecture theory, and to the Science of Beauty so common in the XIX century. The current term, especially in industrial applications, including Fashion, is style. For a contemporary analysis of the nature of style from the Design point of view see [7, 26]. In particular in [16, 6, 15] readers may find mathematical approaches to the problem of the representation of styles.

A style should identify the nature of an organic but simultaneously disseminated coherence. This is a difficult concept to be captured by traditional top-level ontologies. Foundational issues are provided in [21] from a semiotic point of view, and in [7] from the point of view of design studies.

Recently an investigation has been carried out which attempted to apply the principles of collective creative design to the real world obtaining an application for such activities that was designed for standalone users initially [28], and further for the use on the Web [5]. Similar problems are solved in a more general case, including general design (not only aesthetic one) and based on a formal theory of component [4].

What is missing in the above mentioned literature is a formalization of the conceptualizations proposed, in Gruber’s terms, an ontology [18]. The rest of this paper is devoted to provide a first order theory of aesthetic judgments that incorporates Bourdieu’s social model of taste formation in terms of Distinction between elite and mass individuals, and the notion of aesthetic property as one of the fundamental notions to provide formal models of taste. We are convinced that such a theory will provide the basis for a complete model of the way in which humans reason about aesthetic judgments.

An attempt to provide guidelines for affordable interlaced approach between Artificial Intelligence and Aesthetics has been carried out by Williams [34]. However this author focused on the notion of surprise, which is which is interesting but not exhaustive.

We employ the general methodological recommendations suggested by Cocchiarella [8], and we superimpose, in particular, Occam’s razor, following the indications of Ushold and Gruninger [25]. More generally speaking we shall formalize the theory under Sowa’s schema [31]. Our commitment is to minimize the number of primitives for the theory and to employ axioms which denote all the intended models that represent our desiderata.

An important aspect of applications of artificial intelligence to various artistic contexts is the support to creativity by means of perceptual emulation. For instance in Computer Music or Computer Graphics we often find models based upon the Gestalt Psychology, a theory founded on the principle that humans consistently choose the simplest between the possible cognitive models which are compatible with the current sensory perception. For a general reference see [24].

3 Terminology and definitions

The theory we present is First-Order, with a signature formed by variables, that we denote with lowercase Latin letters like $x$, if needed with indices, predicates, that we denote by uppercase sans serif letters; connectives, quantifiers and parentheses as usual. Greek lowercase letters are used for formulae, functions and constants. The scope of a quantifier is determined by square brackets; we systematically avoid unnecessary explicit quantifications; therefore, the formula $\phi(x)$ means $\forall x (\phi(x))$. Sequences of quantifications $\exists x_1 \exists x_2 \ldots \exists x_n$ or $\forall x_1 \forall x_2 \ldots \forall x_n$ are synthesized in $\exists x_1, x_2, \ldots, x_n$ and $\forall x_1, x_2, \ldots, x_n$ respectively. Axioms are provided in lines starting with $\textbf{A}$, followed by a counter (unique in the
paper), and definitions in lines starting with D, again followed by an unique counter.

We assume the basic axioms for classic deductive calculi. The inference rules are Natural Deduction and Generalization.

We employ five axiomatic sortal predicates used for defining the types of individuals (I), its subtype elite-individuals (E), the type physical objects (O), the type properties (P) and the type values (V). One more sortal for mass individuals is defined based on the sortals I and E. Conceptually, an individual is able to express judgments only about physical objects, and we mean physical objects to be distinct from individuals. The reification of properties needs an attribution predicate A, that relates a property p and an object o, so that A(p, o) represents the predication p(o). The predicate A can also relate triples formed by a property, an object and a value as in A(p, o, v) whose meaning is that the property p has the value v for the object o. We do not settle types for the possible values. In Section 4 we presuppose the existence of the type real numbers.

The fundamental sortal axioms are in below. The operator ⊕ represents the “exclusive or” logical operation. The first axiom states that defined types are incompatible. The second axiom establishes that the sortal E identifies a subset of the objects identified by the sortal I.

\[ A.1 \ (x) \oplus O(x) \oplus Pr(x) \oplus V(x) \]

\[ A.2 \ E(x) \rightarrow I(x) \]

The notion of mass-individual (M) as an individual not in the elite is provided by the definition below.

\[ D.1 \ M(x) \equiv_{df} \ I(x) \land \neg E(x) \]

The attribution of properties by A both in binary and in ternary versions requires constraints to the sorts of the arguments.

\[ A.3 \ A(p, o) \rightarrow Pr(p) \land O(o) \]

\[ A.4 \ A(p, o, v) \rightarrow Pr(p) \land O(o) \land V(v) \]

We also employ the basic relation of beautiful (B), defined between individuals and objects. By B(i, o) we denote that i believes that o is beautiful.

In Section 4 we introduce some basic axioms constraining the behavior of B, relatively to the use by elite individuals. We moreover specify a defined predicate good-taste (G) attributed by elite individuals to individuals. One fundamental desideratum we propose here is that every elite individual has good taste.

4 A general theory of aesthetic judgment

In this section we shall provide an analysis of the content of an aesthetic judgment in three steps. In Subsection 4.1 we exploit the Bourdieu’s theory of Distinction in formal terms (for the part regarding the expression of judgments as a social fact); in Subsection 4.2 we depict the notion of judgment as a descriptive step, and in Subsection 4.3 we formalize the relation between these two aspects of the expression of an aesthetic judgment.

1 Humans can express judgments regarding other, but these kinds of judgments will not be considered in this first analysis for the sake of simplicity. The aspects involved in such judgments are many and of many different kinds, including constraints due to social aspects. The expression of judgments among individuals is a matter of further investigations.

2 The introduction of criteria does not aim at deploying an objectivity commitment with respect to aesthetics, which is totally against an approach à la Bourdier. Indeed our point here is to model appropriately the basic results of psychological research in perception, which have strongly stressed the commitment to harmony and symmetry as natural constraints of objects classified as beautiful by humans. The distinction here is between natural beauty and taste which is a product of the social environment.

4.1 The expression of judgments in social terms

The purpose of the axiomatization presented in this section is to provide a formal theory of aesthetic judgment which constitutes a formalization of the portion of Bourdieu’s Theory that defines the relation between elites and mass individuals. The first axiom provides constraints to the sorts of the arguments of the predicate B.

\[ A.5 \ B(i, o) \rightarrow I(i) \land O(o) \]

We may now provide a definition of good taste, as the coherence with one vision of the world by an elite individual, and a notion of good taste of a single individual. The predicate G(i, j) means that the individual i believes that the individual j has good taste. The predicate G(i) means that the individual i has good taste.

\[ D.2 \ G(i, j) \equiv_{df} B(j, o) \rightarrow B(i, o) \]

\[ D.3 \ G(i) \equiv_{df} \exists [E(j) \land G(i, j)] \]

A basic requirement for good taste judgment is the assertion that every individual believes to have good taste.

\[ A.6 \ G(i, i) \]

By the above definitions along with axiom 4.1 every elite individual has good taste.

4.2 Aesthetic criteria

The basic theory of judgment is expressed in social terms by the above defined predicate formalization. This is only the evaluation part of the expression of an aesthetic judgment. The description part is still missing.

Such a description will be provided by introducing the notion of style. Essentially a style is an aesthetic similarity, that has its natural expression, using Lakoff’s terminology, in a radial category.

A radial category is different from a class in the meaning in which this term is intended in Knowledge Representation. The strict qualification of a class is the existence of a single equivalence relation that defines the class in extensional terms, whose intensional counterpart is a list of properties the elements of the class have in common.

In a radial category, the extensional structure is identical, but the description at the intensional level is rather different. A radial category can be viewed as a graph in which a prototype is similar to other objects in one sense, and these objects are similar to other objects as well. Each step in the graph is justified by the sharing of a set of properties (or of the values of a shared property). However, two objects that are similar to the prototype do not necessarily share the same properties with each other. This representation can take place among single objects, or among categories.

There are several proposals for employing such categories in reasoning, from the point of view of description logic a good reference is [14], and similarly [23] approached the problem from the point of view of the representation of vagueness. What we need for our model is a simple notion of prototype, axiomatized by the predicate PT, and the relation of aesthetic similarity AS. The expression PT(x, y) means that x is a prototype for y, and the expression AS(x, y) mean that x is aesthetically similar to y.

\[ A.7 \ PT(x, y) \rightarrow [x \neq y \rightarrow AS(x, y) \land \neg PT(y, x)] \]

3 Two aesthetically similar objects tend to get the same kind of judgment from the elite population, so a style results defined as an aesthetic similarity exactly because similar things are approved by the elite.
Aesthetic similarity is the sharing of aesthetic properties\textsuperscript{4}. There exist two basic properties of predicates that set up their aesthetic nature, in our model: harmony and symmetry\textsuperscript{5}.

An unary predicate $p$ is harmonious when it implies that two measures of the objects predicated by $p$ are in golden proportion. Such a definition is vague and generic, since, in general, it is easy to find a pair of measures of a given physical object that are in golden proportion. However, the purpose of the axiomatic theory we present here is not to provide a complete model, but conversely a general one, which can be customized in order to capture specific needs of people using the system for representing peculiar aspects of a given domain.

The property of harmony is established by means of the operator $H$. We provide axioms for this in below. The constant $\gamma$ represents the golden section ($\frac{\sqrt{5} - 1}{2}$). A measure of an object is a property $\mu$ with two arguments $x$ and $v$, meaning that the real number $v^\mu$ is the value of the measure $\mu$ for the object $o$. The division operator $/\mu$ is implemented in the obvious manner.

\textbf{A.8} $H(p) \leftarrow [A(p, o) \rightarrow \exists \mu_1, \mu_2, v_1, v_2 [A(\mu_1, o, v_1) \land A(\mu_2, o, v_2) \land (v_1/v_2) = \gamma]]$

The semantic nature of the chosen measures is not explicitly stated in the above axiom, and relies on the correct choice of measures, which is left to the user of the system.

For instance, the shapes of Figure 1 are in golden proportion based on the correct choices of measures as expressed in the picture.

\begin{figure}[h]
  \centering
  \includegraphics[width=\textwidth]{shapes.png}
  \caption{Four shapes in golden proportion.}
  \label{fig:shapes}
\end{figure}

A unary predicate $p$ is symmetric when it implies that at least two supplementary parts of the objects predicated are roughly congruent (RC). Congruence is well defined in spatial reasoning, as in, for instance [2, 11]. In particular, in [2] there is an explicit axiomatization of a notion of rough congruence we can employ directly. The expression $RC(x, y)$ means that $x$ is roughly congruent to $y$.

We also employ a mereology, by means of the predicate $P$ which is supposed to be axiomatized in the form of the Region Connection Calculus [27, 33, 9]. Again $P(x, y)$ means that $x$ is a part of $y$.

We also assume the weak supplementation principle (every region $r$ has two parts whose union is $r$). The operator $S$ is used for stating symmetry of a property. The sum operator (SUM) is binary and computes the supplementation of the two objects.

\textbf{A.9} $S(p) \leftarrow [A(p, o) \rightarrow \exists \alpha_1, \alpha_2 [P(\alpha_1, o) \land P(\alpha_2, o) \land SUM(\alpha_1, \alpha_2) = o \land RC(\alpha_1, \alpha_2)]]$

For instance, the shapes of Figure 2 are symmetric, in the sense that two parts of them are roughly congruent. Multiple symmetry is obtained by repeated application of the above axiom.

\begin{figure}[h]
  \centering
  \includegraphics[width=\textwidth]{symmetry.png}
  \caption{Three symmetric shapes.}
  \label{fig:symmetry}
\end{figure}

We are now able to provide an axiom for aesthetic similarity.

\textbf{A.10} $AS(o_1, o_2) \leftarrow \exists p[A(p, o_1) \land A(p, o_2) \land (H(p) \lor S(p))]$

More complex aesthetic properties can be defined from simpler ones (down to atomic properties), by means of “semiotic” composition operators, denoting relations of iconic reference, indexical references, symbolic references, metaphoric relations, metonymic relations, and so on. Atomic properties appear to be tightly tied to specific biological features of the human cognition system, whereas complex properties can be used to model more indirect, culturally and socially constructed tastes.

### 4.3 Modelling Canons

Intuitively a canon is a class of objects that the elite individuals consider beautiful. When an elite settles a canon, the members of the elite will establish: a single prototype; a radial category, based on the definition of the pairs formed by the prototype and the objects related to it\textsuperscript{7}; the properties which relate in terms of aesthetic similarity the prototype to the other members of the radial category.

We introduce a definition of the unary operator $\text{Can}$ which establishes the existence of a single prototype (we shall make use of the quantifier $\exists$ meaning there is one and only one), and of the aesthetic similarity. The expression $C(o)$ means that $o$ is a canon. A canon is an object.

\textbf{D.4} $C(o) \equiv_{def} \exists \alpha [PT(o', o) \land \forall i, o'' [E(i) \land PT(o', o'') \rightarrow B(i, o'')]]$

The above defined relation is not the “establishment” of a canon, namely does not represent the very interesting event of the birth of a style. For doing such a representation we need an explicit representation of time, which is worth studying in further work.

### 5 Conclusions and further work

The notions we employ here do not take in account several aspects which can contribute to the expressivity of the system.

**Awfulness:** in our model objects are specified as beautiful, but there is no predicate for stating that an object is awful. The beautiful and awful objects do not cover entirely the domain, anyhow, since it is possible that some objects are neither beautiful nor awful;

**Fuzzyness:** judgments of taste are not crisp. We use expressions like rather nice, not so bad, acceptable, masterpiece, horrible;

**Attribution of specific aesthetic qualities:** we employ expressions that are different from the simple judgment. We say surprising, original, innovative, amazing, interesting, boring, unaffordable, incomprehensible. The meaning of these attributes is worth studying, and not captured by the notions introduced here;

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\textsuperscript{4} The relation of prototype, along with the aesthetic similarity provides a model of radial categories à la Lakoff. The axiomatization of Aesthetic Similarity is left to further work, but in general we can state that this is an equivalence relation.

\textsuperscript{5} The limitation to such particular properties might be debatable, and only has to be considered as a first stage in this investigation.

\textsuperscript{6} Tarski’s theory of Real Numbers is First-Order, and can be employed in our axiomatization.

\textsuperscript{7} We provide here only first-step radiality. The implementation of second-step radiality requires choices about the notion of prototype which have been discussed in [22].
Notion of Bad Taste: Individuals classify others based on judgments of good and bad judgments of bad taste. This is complex matter, involving, special things that one person thinks you must believe to be beautiful for being of good taste, things that cannot be retained beautiful for being of good taste; 

Competition: Elite individuals compete in the arena of good taste formation, while in the current model we simply have individuals in a generic elite. The notion of community within an elite should be captured; 

Dynamism: Time is central in the notion of aesthetics and also in the conceptualization of elite cooperation and expulsion. People of good taste tend to enter open elite communities and individuals of bad taste tend to exit them. Moreover people can change aesthetic judgments, probably even easier than they do for other kinds of evaluations; 

Elite differentiation: There are various ways in which an individual belongs to a community. People may have some right of being in the elite, and certain elite communities may have entrance and exit barriers. 

There are several ways in which this research can be taken further. First of all we may look for more specific, more refined or more complete models of aesthetic knowledge. We then are interested in evaluating the performance of these systems, including the one presented here, for reasoning. Finally we are interested in deploying an experimental evaluation of the ontology in practical domains, in particular Design and Fashion. Currently all the above mentioned paths have been started and show promising preliminary results. 

Acknowledgements

Authors gratefully thank ACP s.r.l. for funding. This work has taken place within the project D.A.V.A. “Tecniche di Decisione Automatica con criteri multipli e Valutazioni Arbitrarie” (Techniques of automated decision with multiple criteria and arbitrary evaluation). We would like to thank gratefully Roberta Cuel for her hints and suggestions, and for having deeply discussed with us some of the themes which emerged in this work. We also thank the reviewers of the sixteenth European Conference on Artificial Intelligence for their valuable comments which have helped us in improving the paper significantly. Last but not least we thank Tony Cohn for having read and commented a nearly final version of the paper. 

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