Combined performance and stability optimisation via central transfer case active control in four-wheeled vehicles

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Abstract—New driveline architectures endowed with torque biasing devices, such as active differentials and active transfer cases, have made available a new generation of on-demand four-wheel-drive vehicles which allow designing control systems capable of altering the car behaviour dictated from its mechanical layout, e.g., under- and over-steering characteristic. This work proposes an active transfer case controller aimed at optimising vehicle performance in terms of amount of traction torque transmitted to the ground while guaranteeing vehicle stability and driveability by keeping the vehicle side-slip angle within safety bounds.

I. INTRODUCTION AND MOTIVATION

In four-wheeled vehicles, electronic stability control (ESC) was introduced in the recent past to improve passengers’ safety in critical driving conditions and it is now part of most commercial cars. Several solutions to active chassis stability control have been proposed in the scientific literature, whose common aim is to actively modify the vehicle dynamics by generating suitable yaw moments to restore vehicle stability when dangerous maneuvers occur, see e.g., [1], [2]. The capability of altering, via an electronic control system, the car behaviour which is in principle dictated from its mechanical layout has opened the way to the so-called global chassis control, where the combined management of the available actuators, such as brakes, steering and traction is used to control the global behaviour of the vehicle, see e.g., [3], [4] and references therein. In the field of yaw control systems for vehicle stability and handling, most of the available solutions are brake-based, see e.g., [5]; these approaches try to enhance both vehicle performance and stability during curves by imposing – via differential braking – an under- or over-steering behaviour to the vehicle. The brake-based systems, however, have been shown to deteriorate the longitudinal performance of the vehicle, especially during strong acceleration maneuvers, [6].

An alternative to brake-based solutions is provided by the use of a new generation of torque biasing devices (active differentials or active transfer cases) in the vehicle driveline, which can be controlled to actively distribute the driving torque between front and rear axle to improve stability and performance, see e.g., [7]. These torque-biasing devices allow to provide a sort of on-demand four-wheel-drive (4WD) vehicle, allowing to actively change the vehicle configuration and to make it closer to a full front-wheel-drive to pursue safety objectives, or to a four-wheel-drive to optimise performance via a more balanced torque distribution to the four wheels.

In this paper, a control strategy for combined performance and handling optimisation is proposed based on the active control of the central transfer case, which allows to modulate the amount of torque to be redirected to the rear axle.

From the control design view-point, a control system which aims at optimising vehicle performance while guaranteeing stability and driveability raises several issues. First of all, dealing with the considered problem from a genuine feedback perspective, one needs both to choose the best controlled variable and to generate an appropriate reference signal. Secondly, to approach the controller design with classical tools, a meaningful – yet tractable – dynamic vehicle model is needed. As for the selection of the controlled variable, the most meaningful one related with vehicle stability is the vehicle sideslip angle, i.e., the angle between the vehicle longitudinal axis and the direction of the vehicle speed. However, an estimate of the vehicle sideslip angle is quite difficult to obtain in all driving conditions and with the accuracy needed to employ it as controlled variable, [8], [9], [10]. Furthermore, the selection of a suitable reference trajectory is not a trivial task, due to its sensitivity to vehicle speed and – most importantly – to road friction conditions. On the vehicle modeling side, the driving conditions in which the considered controller must be active are very complex, combining strong accelerations and cornering on low-grip roads. As such, linearised models appear not expressive enough to be employed, and a full vehicle nonlinear model results too complex for control design purposes.

Thus, in this work a supervisory control architecture is proposed, which handles the performance vs stability trade-off via a switching strategy which either opens or closes the transfer case, thereby selecting the amount of torque to be redirected to the rear axle. The controller tuning phase is performed via optimisation of appropriate cost-functions in a simulation-based approach, where the multibody simulator CarSim® is used as vehicle model.

II. SYSTEM ARCHITECTURE

The considered vehicle is an on-demand 4WD vehicle, where engine torque can be redirected to the rear axle via an electronically-controlled transfer case. The total torque \( T_{tot} \) available at the transfer case can be computed as

\[
T_{tot} = T_{eng} \eta_{gear} \tau_{gear},
\]
where $T_{\text{eng}}$ is the engine torque, $\eta_{\text{gear}}$ is the gearbox efficiency and $\tau_{\text{gear}}$ the gearbox gain. The torque requested to the transfer case is proportional to the front-rear axle speed difference $\Delta \omega_f = \omega_f - \omega_r$, namely
\[
T_{TC} = k \Delta \omega_f,
\] (2)
where $k$ models the transfer case equivalent stiffness and the front and rear axle angular speeds are computed as
\[
\omega_F = \frac{\omega_{f1} + \omega_{f2}}{2} \quad \omega_R = \frac{\omega_{r1} + \omega_{r2}}{2},
\] (3)
with $\omega_{f1}, i = \{f, r\}, j = \{l, r\}$ being the four wheels angular speeds and $\tau_{FD}$ and $\tau_{RD}$ the front and rear differential ratios, respectively. By modulating a valve opening in the transfer case hydraulic circuit, one can actively modify the equivalent transfer case stiffness $k$ value, see Equation (2), which can vary between a minimum value $k_{\text{min}}$ and a maximum value $k_{\text{max}}$. To account for the valve dynamics, a low-pass filter with a settling time of 70 ms has been employed.

In what follows the equivalent transfer case stiffness $k$ will be regarded as the control variable. This is motivated by the fact that the current transfer case technology allows to command only one of these two extreme values of the transfer case equivalent stiffness by opening or closing the associated valve.

Summarising, in the considered driveline architecture the fraction of the torque $T_r$ directed to the rear axle is given by $T_r = T_{TC}$. Thus, the torque available at the front axle $T_f$ can be computed as $T_f = T_{\text{in}} - T_r$.

III. CONTROLLER DESIGN

This section illustrated the whole controller design process. First of all, the adopted stability and performance indexes are introduced, together with the selected test drive. Further, the controller structure is detailed and the tuning phase discussed. Finally, a sensitivity analysis is performed to investigate the controller robustness with respect to vehicle speed and tire-road friction conditions.

A. Stability and performance indexes

As the overall goal of the proposed control system is to optimally manage the trade-off between performance and stability in critical maneuvers, appropriate cost functions need to be defined to provide a quantitative description of such properties. Specifically, performance is regarded as the capability of transferring to the ground the largest possible amount of longitudinal and lateral forces. As such, the natural cost function to measure performance is
\[
J_p = \sum_{i=1}^{4} |F_{x_i}| + \sum_{i=1}^{4} |F_{y_i}|, \quad i = \{1, \ldots, 4\},
\] (4)
where $F_{x_i}$ and $F_{y_i}$ are the $i$-th wheel longitudinal and lateral force, respectively. Note that, as far as vehicle dynamic behaviour is concerned, maximising $J_p$ when cornering means implicitly to avoid reaching large values of longitudinal and lateral wheel slips, thereby improving the vehicle capability of correctly following the desired trajectory imposed by the steering command.

The concept of vehicle stability, on the other hand, is related to the driver feeling of safety and driveability. The dynamic variable which best accounts for stability is the vehicle sideslip angle $\beta$. A large value of $\beta$ does not necessarily imply that the vehicle dynamics is unstable; however, as $\beta$ increases, the driver loses the ability of safely managing the current maneuver. According to the opinion of professional drivers, a sensible threshold on the value of $\beta$ to guarantee driveability is $5^\circ$. [2].

As such, the cost function $J_S$ quantifying stability is
\[
J_S = \int_{|\beta(t)|>5^\circ} |\beta(t)| \, dt,
\] (5)
which penalizes — along the whole maneuver — vehicle sideslip values exceeding the safety level of $5^\circ$. Note that, with respect to existing literature (see e.g., [2]) where a hard, pointwise constraint $\beta < 5^\circ$ is incorporated in the controller design, an integral cost function for stability is herein proposed. This, in our view, better reflects the fact that driveability and handling are driver-dependent feelings and should thus be quantified in a continuous manner.

Note that the equivalent stiffness $k$ of the transfer case acts on the two performance indexes as follows: a high stiffness value makes the vehicle closer to a 4WD one, thereby optimising performance at the expense of stability. Conversely, as the stiffness decreases, the vehicle behaviour moves toward a front-wheel-driven one, thereby offering reduced traction capability on low grip surfaces but improved stability. It is therefore impossible to achieve both performance and stability optimisation at the same time; hence, a control strategy to manage the trade-off is needed.

![Fig. 1. The proposed controller architecture.](https://example.com/fig1.png)

For controller design, a specific test maneuver has been devised, tailored to consider critical driving situations where a significant trade-off between performance and stability requirements exists. The considered test drive is composed of the following four maneuvers performed sequentially, with initial forward speed of $v_0 = 25$ km/h: 1) a 10 s-long full throttle opening coupled with a sinusoidal steering angle input of amplitude $90^\circ$ and frequency 0.5 Hz; 2) a 20 s-long full throttle opening coupled with a sinusoidal steering angle input of amplitude amplitude $20^\circ$ and frequency 0.2 Hz; 3) a 10 s-long full throttle opening with no steering; 4) a 15 s-long full throttle opening coupled with a sinusoidal steering angle input of amplitude $30^\circ$ and frequency 0.1 Hz.
angle input of different amplitudes, namely 30°, 60° and 90° and frequency 0.25 Hz.

The four parts of the test are separated by 10 s each, within which the throttle is closed from 100% to 10% in 1.5 s so as to slow down the vehicle before commencing the next acceleration phase. The test is simulated on low friction conditions (μ = 0.4), which is the most critical situation for stability. The proposed test drive was designed so that the first and fourth maneuvers are the most critical ones as far as stability is concerned, whereas the second and the third are more common and less dangerous maneuvers useful to show how the control algorithm acts to optimise vehicle performance.

\[ \beta^*(k) = \begin{cases} 
\frac{|\beta(k)|}{k_{\text{max}}} & \text{if } |\beta(k)| \geq k_{\text{max}} \\
\frac{|\beta(k)|}{k_{\text{max}}} - \frac{|\beta(k)|}{k_{\text{max}}} & \text{if } |\beta(k)| < k_{\text{max}} \\
0 & \text{otherwise}
\end{cases} \]

where \( \beta_F(k) \) and \( \beta_F(k) \) are low-pass filtered versions of the first and second time derivative of the input signal \( \beta(t) \), \( \gamma, \nu \) and \( \varepsilon \) are positive constants and the number of samples \( N \) is chosen so that a desired time-window elapses before resetting the data processing output provided that no other maxima occur (see Figure 2). The main advantage offered by this type of processing is that it allows to safely manage the fact that oscillations in \( \beta \) and \( \dot{\beta} \) are common in the considered maneuvers, and they cause these signals to take on low values which do not indicate that the vehicle behaviour is currently stable.

The control algorithm, as described in Figure 1, acts as follows. The default setting, used when no acceleration maneuvers occur, is to set the transfer case torque at \( T_{TC_{\text{max}}} = k_{\text{max}} \Delta \omega_{fr} \) (see also (2)), i.e., selecting the lowest equivalent stiffness for the transfer case. Conversely, when an acceleration occurs, i.e., when \( b_{\text{acc}} = 1 \), and no potential loss of stability is detected, i.e., when \( b_{\text{stab}} = 1 \), the transfer case torque is set at \( T_{TC_{\text{max}}} = k_{\text{max}} \Delta \omega_{fr} \) to optimise performance. If \( b_{\text{stab}} = 0 \) the transfer case torque is set to \( T_{TC_{\text{min}}} \) to avoid dangerous situations. Note that, if the acceleration maneuver continues and \( b_{\text{stab}} = 1 \) over a time window of 2 s, then this accounts for recovered stability and \( T_{TC} = T_{TC_{\text{max}}} \).

The acceleration supervisor, active for vehicle speeds higher than 2 m/s, computes the numerical derivative of the gas pedal position \( p_g \) and monitors its value. An acceleration is detected when \( p_g \) exceeds a predefined threshold, and \( b_{\text{acc}} = 1 \). Similarly, if \( p_g \leq 0 \) over a whole pre-specified time window \( b_{\text{acc}} = 0 \).

Finally, the stability monitor receives as input \( \beta(t) \), \( \dot{\beta}(t) \) which are processed to obtain meaningful signals for stability monitoring. The data processing algorithm provides as output the processed version of \( \beta \) and \( \dot{\beta} \), which will be referred to as \( \beta^* \) and \( \dot{\beta}^* \), respectively. For the computation of \( \beta^* \) (the same reasoning applies to \( \dot{\beta}^* \)), at the \( k \)-th sample, one has

\[ \beta^*(k) = \left\{ \begin{array}{ll}
|\beta(k)| & \text{if } |\beta(k)| \geq \beta^*(k-1) \\
|\beta(k)| - |\beta(k)| & \text{if } |\beta(k)| < \beta^*(k-1)
\end{array} \right. \]
In our approach, this is done via an ad-hoc optimisation procedure. Specifically, the performance and stability indexes are evaluated for different threshold values chosen over a predefined grid in the \((\beta^*, \hat{\beta}^*)\) parameters space by simulating the controlled system using the test drive described in Section III-A.

As such, the first step is to define the upper bounds on \(\beta^*\) and \(\hat{\beta}^*\) which can be associated to vehicle instability: the upper bound on \(\beta^*\) is set to 5\(^\circ\) (see (5)); the upper bound on the safety level for \(\hat{\beta}^*\) is less intuitive. It has been set to 15\(^\circ\)/s based on professional drivers considerations and on the analysis of experimental results.

The grid in the parameters space \((\beta^*, \hat{\beta}^*)\) is the shaded region shown in Figure 3: as can be seen, the final goal of the tuning phase is to find the best combination of the two parameters defining the point labeled A in Figure 3. This allows to uniquely set the threshold values to signal instability and cause the controller to switch between the two limiting values of the transfer case equivalent stiffness \(k_{\min}\) and \(k_{\max}\) and managing the performance/stability trade-off. To take into account the combined effect of the two parameters on vehicle stability the grid is more conservative with respect to the two upper bounds previously defined, which were set considering one single parameter at a time.

Hence, the optimisation procedure will explore \(\beta^*\) values from 1\(^\circ\) to 4.5\(^\circ\) with a resolution of 0.5\(^\circ\), and \(\hat{\beta}^*\) values from 4\(^\circ\)/s to 12\(^\circ\)/s with a resolution of 1\(^\circ\)/s.

Each simulation test provides as output two numerical values: the relative performance measure

\[
P(\beta^*, \hat{\beta}^*) = \frac{J_P(\beta^*, \hat{\beta}^*)}{J_{P_{\max}}},
\]  

where the performance cost function \(J_P\) (see Equation (4)) is normalized with respect to the best possible performance \(J_{P_{\max}}\), i.e., that obtained with the largest value of the transfer case equivalent stiffness. Conversely, the relative stability measure is given by

\[
S(\beta^*, \hat{\beta}^*) = \left(1 - \frac{J_S(\beta^*, \hat{\beta}^*)}{J_{S_{\min}}}\right),
\]

where the stability cost function \(J_S\) (see Equation (5)) is normalized with respect to the highest achievable stability level \(J_{S_{\min}}\), i.e., that obtained with the smallest value of the transfer case equivalent stiffness.

The tuning phase is initially performed simulating the test drive with initial vehicle speed \(v_0 = 25\) km/h and a friction coefficient \(\mu = 0.4\). Section III-D will be devoted to discuss the results of a sensitivity analysis performed with respect to both vehicle speed and friction coefficient.

The obtained results for all the explored grid points are graphically represented in Figure 4(a)-4(b), where a three-dimensional view of the relative stability and performance index values as functions of the tuning parameters \(\beta^*\) and \(\hat{\beta}^*\) are shown. The gray plane represents the results obtained with a fixed value of the transfer case equivalent stiffness \(k = k_{\min}\), i.e., with a vehicle configuration which is as close as possible to front-wheel-drive. By inspecting Figure 4(a) note that \(k = k_{\min}\) provides indeed the highest stability level, and that \(S(\beta^*, \hat{\beta}^*)\) significantly decreases as we move toward the boundaries of the grid. Conversely, Figure 4(b) shows that \(k = k_{\min}\) provides the worst performance measure, and that \(P(\beta^*, \hat{\beta}^*)\) significantly increases as we allow less conservative threshold values to manage the switch between \(k_{\max}\) and \(k_{\min}\).

A graphical representation which highlights the trade-off between performance and stability is given in Figure 5, where a plot of the relative performance versus relative stability index is provided. The cross in the lower left corner represents the result obtained with \(k = k_{\min}\), which gives the best stability and worst performance measures. Similarly, the square in the top right corner represents the result obtained with \(k = k_{\max}\), which gives the worst stability and best performance measures. The asterisks represent the results obtained for different but fixed values of \(k \in (k_{\min}, k_{\max})\), whereas circles are those obtained with the proposed controller.

As can be seen, the dark gray region is the portion of the combined \((S, P)\) plane which can be reached with a passive transfer case setting, whereas the light gray one is the one which can be reached thanks to the proposed active controller. The advantage is clear, as the proposed control strategy allows to significantly enhance performance while guaranteeing an only slightly suboptimal stability level.
Based on industrial requirements, which asked to privilege stability objectives, the triangle in Figure 5 is the selected best tuning case.

\[ \beta = \arctan\left(\frac{v_y}{v_x}\right) \]  

\[ (9) \]

**Fig. 6.** Performance versus stability index with initial speed \( v_0 = 35 \text{ km/h} \).

**D. Sensitivity Analysis**

As passengers vehicles control systems need to be robust with respect to the wide set of possible driving conditions, the robustness and consistency of the results obtained in the tuning phase need to be further investigated. The two most important sources of uncertainty in the considered setting are tire-road friction conditions and vehicle speed. Specifically, having tuned the controller with a friction coefficient of \( \mu = 0.4 \), i.e., a snowy road, we need to verify that, when moving to higher grip roads, the performance of the closed-loop system does not undergo a significant degradation due to a too conservative instability detection. When moving to vehicle speed, we expect stability issues to become more and more critical as speed increases, and to witness a possible performance loss as speed decreases below the value \( v_0 = 25 \text{ km/h} \) used in the tuning phase.

Hence, we first repeated the tuning phase over all the previously defined grid points with friction coefficient \( \mu = 0.4 \) and initial vehicle speed \( v_0 = 10 \text{ km/h} \) and \( v_0 = 35 \text{ km/h} \) (the latter has been found to be the upper bound on initial speed guaranteeing that the whole driving test can be safely performed with the front-wheel-drive configuration, that is with \( k = k_{\min} \)). The final results for \( v_0 = 35 \text{ km/h} \) are shown in Figure III-C, where the triangle represents the best tuning found for \( v_0 = 25 \text{ km/h} \), the plus the best for \( v_0 = 10 \text{ km/h} \) and the star the best for \( v_0 = 35 \text{ km/h} \) (and the other symbols have the same meaning as in Figure 5). As expected, at lower speed the stability issue becomes less critical and the proposed controller is able to very closely achieve the global optimum. The opposite is true for increased vehicle speed, where the gained advantage obtained with the proposed controller (light gray plane in Figure III-C) grows smaller than in the previous cases. Note also that for \( v_0 = 35 \text{ km/h} \) the optimal tuning obtained for \( v_0 = 10 \text{ km/h} \) (the plus in Figure III-C) causes a severe stability loss. However, the optimal tuning found for \( v_0 = 35 \text{ km/h} \) (star in Figure III-C), can be regarded as appropriate also for the other considered speed values. As such, fixed threshold values can be used at different vehicle speeds.

\[
\begin{array}{|c|l|l|l|l|}
\hline
\text{Friction condition} & \text{\( k_{\min} \)} & \text{\( k_{\max} \)} & \text{Controlled} & \text{\( \text{max}(\beta) \)} \\
\hline
\mu = 0.7 & 97.4 & 100 & 99 & 6.09 \\
\hline
\mu = 1 & 97.4 & 100 & 99 & 6.10 \\
\hline
\end{array}
\]

**TABLE I**

**PERFORMANCE AND STABILITY INDEXES ON \( \mu = 0.7 \) AND \( \mu = 1 \).**

Finally, a sensitivity analysis with respect to friction conditions has been performed, considering the values \( \mu = 0.7 \) and \( \mu = 1 \), which represent wet and dry road, respectively. Table I summarises the obtained results on \( \mu = 0.7 \) and \( \mu = 1 \), showing the percentage relative performance and stability measures compared to those obtained with \( k = k_{\min} \) and \( k = k_{\max} \). An absolute measure of the largest sideslip angle value \( \text{\( \beta \)} \) is also provided to ease the comparison.

Overall, the analysis reveals that a fixed threshold selection, properly optimised, allows us to correctly manage the performance/stability trade-off in all the considered situations.

**IV. SIDESLIP OBSERVERS AND EXPERIMENTAL RESULTS**

To implement the proposed controller on the target vehicle, the sideslip observer (see also Figure 1) to estimate both \( \dot{\beta} \) and \( \beta \) needs to be implemented and validated.

To this end, consider that, on the test vehicle, the following variables can be measured: vehicle speed \( v \), longitudinal acceleration \( a_x \), lateral acceleration \( a_y \), yaw rate \( \psi \), gas pedal position \( p_g \) and steering angle \( \delta \). For testing purposes, the vehicle was also equipped with a bi-axial optical sensor to accurately measure longitudinal and lateral velocities \( v_x \) and \( v_y \), so that \( \beta \) can be computed as

\[ \dot{\beta} = \arctan\left(\frac{v_y}{v_x}\right) \]  

\[ (9) \]

and \( \dot{\beta} \) is obtained as the numerical derivative of the measured signal. The employed \( \dot{\beta} \) observer is based on a kinematic
model of the vehicle lateral dynamics, i.e.,
\[ a_y = \dot{v}_y + \dot{\psi} v_x, \]  
(10)
where \( a_y \) is the lateral acceleration, \( v_x \) and \( v_y \) are the longitudinal and lateral vehicle speed, respectively, and \( \psi \) is the yaw rate. Recalling that \( v_x = v \cos \beta \) and \( v_y = v \sin \beta \) and assuming that \( \beta \) is small, one has \( \dot{v}_y = \dot{v} + v \dot{\beta} \approx v \dot{\beta} \), and \( v_x \approx v \). Therefore, the vehicle sideslip rate \( \dot{\beta} \) can be estimated from measured variables as
\[ \dot{\beta} = (a_y - \psi v)/v. \]  
(11)

The results obtained with the \( \dot{\beta} \) observer on experimental data are shown in the top plot of Figure 7, and can be regarded as accurate enough to be employed in the proposed threshold-based stability supervisor.

![Fig. 7. Performance of the sideslip observers on experimental data. Top plot: \( \dot{\beta} \); bottom plot: \( \beta \). Measured (dashed line) and estimated (solid line).](image)

For the side-slip angle estimation, the observer proposed in [10] has been employed, which has the advantage of being based on a kinematic model which contains no physical parameters of the vehicle, and, as such, is robust with respect to parametric variations. The results obtained with the \( \beta \) observer on experimental data are shown in the bottom plot of Figure 7. As can be seen by inspecting these results, the estimated \( \dot{\beta} \) can be regarded as accurate enough to be employed in the proposed threshold-based stability supervisor.

The experimental tests of the overall controller have been carried out on heavy-wet road, performing a strong acceleration maneuver with combined steering and cornering. The top plot of Figure 8 shows (scales are omitted for confidentiality reasons) the measured transfer case torque in open loop with \( k = k_{\text{min}} \) (dashed line) and with the proposed controller (solid line), while the bottom plot shows the measured wheel speeds in the same two conditions. The solid vertical lines indicate the initial and final time instants of the acceleration maneuver as signaled by the acceleration supervisor subsystem, while the dashed lines mark the bounds of the time interval during which a loss of stability was detected. As can be seen, in the time interval during which a safe acceleration was occurring a significantly larger amount of torque was transmitted to the rear axle via the transfer case (recall that in this phase the controller set \( k = k_{\text{max}} \), thereby enhancing vehicle performance). This can be appreciated also looking at the wheel speeds time history: when the system is active the wheel slip is kept much lower than in the open loop case, confirming that performance is being optimised. On the other hand, once the loss of stability is detected, the transfer case torque steadily decreases and aligns with the \( k = k_{\text{min}} \) case and, correspondingly, an increase in the wheel slip (especially on the inner front wheel, i.e., that at the curve interior, which is the less loaded one) is experienced. These results favorably witness the effectiveness of the proposed controller.

**REFERENCES**


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