Flux footprints over a hilly surface: a case study
(Research Note)

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Abstract:

The flux footprint probability distribution (FPD) for the near-surface receptor over a series of sinusoidal hills is evaluated using a backward Lagrangian stochastic (LS) model. The wind and turbulence fields derived from large-eddy simulations under unstable conditions, in which the synoptic wind aligns with the surface-elevation-varying direction, are used to drive the LS model. The characteristics of the crosswind-integrated FPD (CIFPD) vary with the location of the receptor over the hilly surface. The widest CIFPD with the smallest peak value appears when the receptor is located in the crest area, while the narrowest CIFPD with the largest peak value appears in the wind convergent area. The peak value of CIFPD in the convergent area is larger than its counterpart derived from the experiment in which a flat surface and horizontally-homogeneous wind and turbulence fields are assumed, with the peak position being closer to the receptor. The case, however, is reversed for the divergent area. Horizontal heterogeneity in turbulence affects the magnitude of the peak and width of the CIFPD, compared with their counterparts over the flat surface, but has less impact on the peak location than that in the mean flow. Similar results are obtained when the CIFPD derived from an analytical footprint model is compared with that from the LS model over the hills. The analytical model fails to simulate CIFPD in the downwind area under weak wind conditions due to the longitudinal wind fluctuation not being considered.

Keywords: Complex terrain, Footprint function, Large-eddy simulation, Stochastic model
1. Introduction

The flux footprint probability distribution (FPD) function measures the relative contributions of the unit sources and sinks of a passive scalar over surface areas to the measured scalar flux at a height. Quantifying the FPD function is very useful to design field experiments and interpret flux measurements (e.g., Baldocchi 1997; Amiro 1998; Rannik et al. 2000a; Stoughton et al. 2000; Wang et al. 2006a; Wang et al. 2006b; Wang and Davis 2008; Chen et al. 2009, and references therein). The flux FPD is dependent on surface conditions, turbulence and wind fields over the footprint area, and the measurement height. It is difficult to directly measure and usually estimated by analytical models, Lagrangian stochastic (LS) models, ensemble average models (Reynolds-average closure models), and large-eddy simulation (LES). Reviews on footprint modeling can be found in the literature (Schmid 2002; Foken and Leclerc 2004; Vesala et al. 2008).

In practice, mean wind and turbulence fields are usually assumed to be horizontally homogeneous in many footprint modeling studies for two main reasons. First, analytical models, which have been widely used especially for long-term footprint calculations (because they are simple and fast), were developed under such an assumption (Schuepp et al. 1990; Horst and Weil 1992; Schmid 1994; Kormann and Meixner 2001; Kljun et al. 2004; Wang et al. 2006b). Second, it is extremely difficult to measure three-dimensional (3D) wind and turbulence fields in reality. For this reason, the horizontally homogeneous condition was still assumed even when LS models, which are valid theoretically under any atmospheric conditions, were applied (e.g., Leclerc and Thurtell 1990; Flesch 1996; Baldocchi 1997; Kljun et al. 2003; Markkanen et al. 2003; Rannik et al. 2003; Gockede et al. 2007; Cai et al. 2008). To acquire wind and turbulence fields under complex conditions, 3D LES models or Reynolds-average closure models need to be run; this is not yet practical especially for long-term simulations.

Studies, however, have shown that the horizontal homogeneity assumption may result in significant errors in footprint estimations. For example, Wang and Davis (2008) performed a LES study on the effects of a clearcut on carbon fluxes measured at a tall tower and found that using an analytical model may significantly underestimate the contribution of sources and sinks over the clearcut area to the measured fluxes. Similar studies have also been performed by a 1.5-order turbulence closure model (Sogachev and Lloyd 2004; Sogachev et al. 2005a; Sogachev et al. 2005b; Klaassen and Sogachev 2006). Recently, Steinfeld et al. (2008) and Markkanen et al. (2008) investigated flux footprints in a heterogeneously heated boundary layer using LES, suggesting that the direction and extension of footprints are significantly different from those derived under the horizontally homogeneous flow condition.

The present study extends previous studies on flux footprints under complex conditions. We focus on the influence of topography because many flux measurement sites are not located over flat terrain. It is noticed that Sogachev et al. (2004) have presented a case study on flux footprints over complex terrain using an Eulerian model with a 1.5-order turbulence closure scheme. In contrast, we will employ an LS model driven by LES-derived wind and turbulence statistics; this is advantageous over the turbulence closure model in terms of simulating turbulence and wind fields on the small spatial scale of footprint areas (e.g., < 1 km). Two simple wind scenarios are selected in this study as a starting point both due to the complexity of the impact of topography and
due to the availability of wind and turbulence datasets. Section 2 briefly describes the LS footprint model and the selected wind and turbulence fields over a series of model hills for this study. In Section 3, the validation of the LS footprint model is first presented, then flux FPDs at different locations over the model terrain are compared with each other and with those over flat terrain. Section 4 gives a summary.

2. Method and model

2.1 A model hilly surface

The ground is assumed to be flat in the south-north direction (y) but its height (H) varies according to a cosine function in the west-east direction (x),

\[ H(x) = H_0 + A \cos \left( \frac{2\pi x}{L} \right), \]

where \( H_0 \) is a reference height (e.g., the mean height of ground above the sea level), \( A \) is the amplitude of the variation of the ground height, and \( L \) is the length scale of the variation of the ground height. In this case study, \( A \) and \( L \) are prescribed to be 0.16 and 4 km, respectively; this corresponds to a slope of approximately 0.16 (9 degrees). Eq. (1) represents the surface elevation variation of a moderately steep hilly surface and not uncommon in carbon flux measurement sites (e.g. Lee and Hu 2002).

2.2 Mean wind and turbulence fields

Mean wind and turbulence fields over the above prescribed hilly surface have been simulated under various conditions using a LES model (Bryan and Fritsch 2002). Results from two archived simulations under convective conditions are selected to drive the LS model in this study. The model configurations for both simulations are identical except that the geotropic wind (\( U_g \)), which blows only in the \( x \) direction, is 5.0 m s\(^{-1}\) in one scenario (Scenario I) and 1.0 m s\(^{-1}\) in the other scenario (Scenario II) (see appendix for the LES model and set-ups). Figure 1 shows the contours of the streamlines from the two cases. Table 1 summarizes atmospheric parameters at the lee (A), trough (B), windward (C) (relative to the background wind direction, from left to right), and crest (D) areas (Figure 1) for Scenario I and Table 2 for Scenario II. In both tables, \( u^* \) is the friction velocity, \( w^* \) is the convective velocity scale, \( L \) is the Obukov length, \( U \) and \( W \) are the mean horizontal and vertical wind velocities at 20 m above the surface, respectively; \( u^2 \) and \( w^2 \) are the variances of the horizontal and vertical velocities at 20 m, respectively.

Mean horizontal and vertical wind velocities vary horizontally in both scenarios. A major difference in the wind feature between the two scenarios lies in the occurrence of local recirculation. The recirculation occurs in Scenario II due to the weak background wind. Strong wind convergence occurs in the lee and windward areas, while wind divergence occurs near the trough area. The direction of the mean horizontal wind in the lee area is opposite to that of the background wind. Such a wind direction change does not occur in Scenario I, though the mean horizontal wind is also divergent in the lee area and convergent in the windward area. Turbulence statistics vary in the horizontal as well. For example, differences in the wind velocity variances on 20 m above the surface among the four sites can reach 20% to 40% (Tables 1 and 2). In both cases, the local atmosphere is more unstable in the trough area than in other areas. The atmosphere in Scenario II is in general more unstable than that in Scenario I due to the weaker background wind. In
particular, the local atmosphere in the trough area in Case II is nearly under the freely
convective condition (−L is very small) due to the nearly-calm mean wind condition.

2.3 Backward LS model
The backward approach proposed by Flesch (1996) is used to derive the flux FPD
function. A number of passive particles are released from a receptor into the atmosphere
and tracked backward in time to their source locations on the surface (Flesch et al. 1995;
Flesch 1996; Kljun et al. 2002). For each particle, the initial release velocity, touchdown
location and velocity are recorded. Because the wind and turbulence statistics are
constant in the y direction and mean wind velocity in the y direction is zero in the
selected datasets, the flux FPD function integrated over the crosswind (y) direction is
evaluated. The crosswind-integrated FPD (CIFPD) at location x reads,

\[
f(x) = \frac{2}{N \Delta x} \sum_{i=1}^{N} \sum_{j=1}^{n_{ti}} w_{0}^{i,j},
\]

where \( N \) is the total number of particles released from the receptor. \( \Delta x \) is the horizontal
grid size within which statistics are calculated. \( n_{ti} \) is the total times of the \( i \)th particle
touching the surface (i.e., \( z = z_{0} \), where \( z \) is height and \( z_{0} \) is the roughness length) over the
range of \( x - \Delta x/2 \leq x < x + \Delta x/2 \), \( w_{0}^{i,j} \) is the touchdown vertical velocity of the \( i \)th
particle at the \( j \)th touchdown. Note that a particle may have multiple touchdowns on the
ground (i.e., \( n_{ti} \geq 1 \)). \( w_{int}^{i} \) is the initial vertical velocity of the \( i \)th particle released from the
receptor. The trajectories of the particles are evaluated by running an LS trajectory model
(see appendix) in the backward mode. In our experiments, 400,000 particles are randomly
released for each run from a box of 0.01×0.01 m\(^2\) centered at the receptor point. The
initial velocity components of each particle follow the Gaussian distributions with the
turbulence statistics at the receptor point in the Eulerian field. Each particle is tracked for
1 hour or 4 km in the horizontal. \( \Delta x \) is 10 m. Time step is taken as the smaller value
between 0.01s and \( dt_{max} \), where \( dt_{max} \) is the maximum allowed time step as proposed by
Rotach et al. (1996). To avoid a very small time step near the surface where the
turbulence length scale is very small, all turbulence parameters below 0.5 m are treated to
be equal to the values at 0.5 m. A perfect reflection condition is applied to the boundaries
at \( z_{0} \) and \( z_{i} \) above the surface, where \( z_{0} \) is the roughness length and \( z_{i} \) is the convective
boundary layer height. In other words, the vertical and horizontal fluctuating velocities
are changed to their negative counterparts (Wilson and Flesch 1993).

2.4 Experiments
Four hypothetical towers are assumed to be located at A, B, C and D (Figure 1),
respectively. In the control experiment (CNTL), flux footprints for a receptor on the
height of 20 m of each tower are calculated using the LS model driven by the LES-
derived wind and turbulence statistics over the hilly surface. Two additional experiments
are performed. The first experiment (EXP I) is designed to examine the effects of the
horizontal heterogeneity of wind and turbulence statistics on the FPD estimation. In EXP
I, two assumptions have been made to run the LS model for the receptor on each tower.
One is that surface is flat in any direction and the horizontal wind is neither divergent nor
convergent. The other is that the vertical profiles of wind and turbulence statistics at a
single site represent the entire domain. In other words, wind and turbulence statistics are
homogeneous in the horizontal. The second experiment (EXP II) is designed to examine how well results from an analytical model may approximate the FPD. The analytical model is driven by the local stability parameters (at the receptor site). This experiment is important because analytical models are still used in many applications to give approximate solutions to FPD under horizontally heterogeneous conditions, though they are invalid theoretically.

3. Results and discussion

In this section, the LS footprint model is validated by comparing its results with those reported in the literature. Then, the FPDs derived from the LS footprint model for the receptors on the four hypothetical towers are compared.

3.1 Validation of footprint calculation

The LS approach described in section 2 is used to simulate the flux FPD for the receptor at 10 m above a flat and horizontally-homogeneous surface with $z_0 = 0.14$ m under the meteorological condition of $L = -32$ m, $u_* = 0.27$ m s$^{-1}$, $z_i = 500$ m, $w_* = 0.93$ m s$^{-1}$. The flux FPD under the same condition has been investigated previously by numerical simulations and observations, which can be used to validate the LS approach. Steinfeld et al. (2008) employed a coupled LES-LS model with a 10 m resolution to simulate flux footprints under the identical condition. Leclerc et al. (1997) also derived the FPD from the Eulerian perspective under the same condition using LES with a resolution of 50 m. The flux FPD from field experiments under the same condition (Finn et al., 1996) was derived by Leclerc et al. (1997).

The LS model is run in two modes, with the turbulence only in the $z$ direction (1D turbulence) and turbulence in both $x$ and $z$ directions (2D turbulence) being considered, respectively. Given the atmospheric parameters, the vertical profiles of mean wind speed and turbulence statistics are determined by the similarity theory (references or appendix). For comparison, the analytical model by Horst and Weil (1992) (HW hereafter) is also used to estimate the flux FPD under the same condition.

Figure 2 shows the CIFPDs derived from different calculations and observations as a function of the upwind horizontal distance from the receptor. The CIFPD derived from the LS model with the 2D turbulence (thick line) is broader than that with the 1D turbulence (thin line). The former also has a larger peak value and the peak location is closer to the receptor location ($x = 0$). The CIFPD from the LS model with the 2D turbulence is in better agreement with that derived from the coupled LES-LS model (dashed line) than that with the 1D turbulence when the LS model results are compared with observations (symbols). These comparisons suggest (1) that the LS model with the 2D turbulence is capable to well simulate flux footprints and (2) that the longitudinal turbulence may not be neglected to accurately simulate flux footprints. It is also found that the CIFPD from the LS model with the 1D turbulence is close to that from the HW analytical model (dotted line) in which only the 1D turbulence is considered.

3.2 Flux footprints for receptors at different locations over the hilly surface – control experiment

Figure 3 shows the CIFPD for the receptor on 2 of each hypothetical tower varying with the horizontal distance from the tower location in case I. The $x$-axis aligns to
the mean background wind direction. The peak value (denoted by $f_{\text{max}}$) of the CIFPD, position ($x_f$) where $f_{\text{max}}$ appears (relative to the tower location), and the half-width ($x_w$) of the CIFPD curve (i.e., the distance between points where $f$ is equal to half of the peak value) are different among the four sites.

The CIFPD for the receptor in the crest area (D) is widest and has the smallest $f_{\text{max}}$. This is because the mean horizontal wind speed in the crest area is largest among the four sites (Table 1). A stronger horizontal wind favors particles from upwind surface sources farther from the tower to reach the receptor point as shown by the long tail of the CIFPD curve at site D. The CIFPD for the receptor on the windward side (C) has the largest $f_{\text{max}}$ and smallest $x_w$, with $x_f$ being closest to the tower. Although the mean horizontal wind speed on the receptor level at site C is not the smallest, the convergent flow brings more particles over the surface area closer to the tower to the receptor point (than the non-convergent flows in other sites), which moves $x_f$ closer to the tower and leads to the larger $f_{\text{max}}$. The CIFPD in the lee area (A) is slightly narrower than that in the trough area (B), despite the larger mean wind speed. This is due to the stronger turbulence in the lee area. Physically, stronger vertical turbulence makes more particles from sources closer to the tower to reach the receptor and, hence, the CIFDP becomes narrower, while stronger horizontal turbulence has an opposite effect. As shown in Table 1, the wind variance in the lee area is 50% stronger in the vertical but only 20% stronger in the horizontal than that in the trough area. As a result, the net effect leads to a narrower CIFDP in the lee area. Also, the divergent flow (negative $W$) on the lee side contributes to the longer tail of the CIFPD curve at site A.

Figure 4 shows the CIFPD under the weak background wind condition (Scenario II). Three features are evident, compared to those under the large background wind condition (Figure 3). First, the peak location at each site is closer to the tower due to smaller mean wind speed. Second, a majority of the footprints for the receptor in the lee area (A) are distributed on the right side of the tower (i.e., the downwind side of the tower relative to the mean background wind), while they are on the left side of the tower in Scenario I. This is also the case for the receptor at B. The reason is that the local horizontal wind direction is opposite to the mean background wind direction due to the effect of terrain (Figure 1). Third, surface sources over the area located at the downwind side (relative to the local wind direction at the receptor point) of the tower have significant contributions to the measured flux, while they have little in Scenario I. This is caused by the horizontal wind fluctuation (characterized by $\sqrt{u'}^2$) being close to or even larger than the mean horizontal wind speed. For example, the local mean wind in the trough area (B) is nearly calm.

Among the four sites, the CIFPD for the receptor on the windward slope (C) has the largest peak value and narrowest half-width. Although the mean horizontal wind speed values at C and D are close, the wind is less convergent and turbulence in the horizontal is stronger at D than at C, accounting mainly for the broader CIFPD curve and smaller peak value. In addition, it is found that the CIFPD at site D is nearly symmetric with respect to $x_f$, due to the strong horizontal wind fluctuation in comparison with the mean wind speed; this is the case for the CIFPD in the trough area (B) where local mean wind is nearly calm. Further, the comparisons of the CIFPDs at sites C and D suggest that the convergent flow tends to make the CIFPD less symmetric (about $x_f$). The mean wind
is convergent in the lee area A, similar to that at C. But the larger wind speed and less unstable local atmosphere (larger $|L|$) at site A lead to the wider FPD.

### 3.3 Horizontally-homogeneous wind and turbulence case – EXP I

In EXP I, the LS model is run for the receptor at each site under the same condition as that in the control experiment except that the wind and turbulence fields are forced to be horizontally homogeneous (section 2.4). Resulting CIFPD curves are shown by the broken lines in Figures 4 and 5. Differences in the CIFPD for the receptor on a tower from the two experiments are qualitatively related to whether the mean wind is convergent or not in the local upwind area of the tower.

Under the strong background wind condition, the CIFPD in the divergent area (i.e., site A in the lee area) from the control experiment has the larger $x_w$ and smaller $f_{\text{max}}$ than that from EXP I, with $x_f$ being closer to the receptor (Figure 3a). The case, however, is reversed in the convergent area (i.e., site C on the windward slope, Figure 3c). For sites B and C in the trough and crest areas with weak mean wind divergence or convergence, the CIFPD curves derived from the two experiments have similar $x_f$ and $x_w$ as shown in Figure 3 b & d; this also suggests that horizontal heterogeneity in turbulence may only have minor impact on CIFDP’s peak location, peak value, and half width compared with the mean flow convergence/divergence (because the horizontal variations in turbulence statistics (e.g. $u^2$ and $w^2$) are nearly maximal in the areas). The same conclusion can be drawn from the comparison under the weak background wind condition (Figure 4).

### 3.4 Flux FPD from the HW analytical model – Exp II

In EXP II, the flux FPD is calculated using the HW analytical model with local stability parameters at each site. Results are shown by the red lines in Figures 3 and 4. In the convergent area, the peak location of the CIFPD derived from the HW model is farther from the tower than that from the LS model in the control experiment, while closer in the divergent area. In the weaker convergent or divergent area, the peak locations are closer.

Under the weak wind condition, the peak values of the CIFDP from the HW model are larger than those from the control experiment. This is because the horizontal wind fluctuation, magnitude is larger than the mean wind speed, plays important roles in determining the particle trajectories (and hence footprints) but is not taken into account in the analytical model. In addition, the HW model simulates a much narrower CIFPD, suggesting that the analytical model may not be able to give reasonable approximations for the flux FPD under very unstable conditions. Under the strong background wind condition, the peak value is once to twice larger than that from the control experiment at all four sites except for the one on the windward side where the peak value and half-width are similar to those from the control experiment.

It is noticed that the CIFPD from the HW model is also different from that in EXP I, though the horizontally homogenous condition has been assumed in both experiments. This can be explained by two aspects. One is that the horizontal wind perturbation is not considered in the analytical model. For this reason, FPD from the HW model is wider and has the larger peak value. The other is that the vertical profiles of mean wind and turbulence statistics at a single tower (derived from LES over the hilly surface) may not follow the similarity theory that is used in the analytical model.
4. Concluding remarks

A LS model has been run in the backward mode to simulate flux footprints over a hilly surface. The model is driven by the LES-derived wind and turbulence statistics under unstable conditions. Four hypothetical towers are assumed to be situated at the lee, trough, windward, and crest areas, respectively. Footprints for the receptor on 20 m of each tower are evaluated under two atmospheric scenarios with different geotrophic winds ($U_g = 1$ and $5 \text{ m s}^{-1}$). For both scenarios, the CIFPD is the widest and has smallest peak value for the receptor in the crest area, while the narrowest CIFPD and the largest CIFDP peak value occur for the receptor in the windward area. Most of footprints are distributed in the local upwind area of each tower in the strong background wind scenario. This is, however, not true for the weak background wind scenario in which a significant part of footprints are distributed in the local downwind area of the tower due to the effects of the longitudinal turbulence whose magnitude is similar to or even larger than the mean wind speed.

To highlight the effect of horizontally-heterogeneous mean wind and turbulence fields on footprints, the LS model is run over the flat terrain of the identical surface properties with the assumption that the vertical profiles of mean wind and turbulence statistics at a single site represent the entire domain. Comparisons suggest that the convergent flow (e.g., in the windward area or recirculation area) makes the CIFPD’s peak location closer to the tower and the CIFPD’s peak larger than the horizontally-homogeneous flow does. The case is reversed for the divergent flow. Horizontal heterogeneity in turbulence has less impact on the CIFPD than that in the mean wind.

Footprints are also estimated using the HW analytical model at each site. Differences are similar to those between the control experiment and the flat terrain experiment. The analytical model fails to predict the downward footprints, especially in the weak wind scenario due to the neglected effect of the longitudinal turbulence, with the peak value of the CIFPD from the HW model being twice larger than from the LS model over the hilly terrain and the CIFPD curve being much narrower.

Finally, it should be noted that numerous factors may affect footprints over complex terrain. This study only presents the FPD under simple meteorological and topographic scenarios, in which the terrain is idealized and varies only in the synoptic wind direction. The influence of topography on mean wind and turbulence fields (and hence footprints), however, is dependent on atmospheric stability and the synoptic wind direction relative to the terrain orientation in addition to the magnitude of the synoptic wind. As a result, the distribution of footprints in the cross-wind direction may not be symmetric, which is significantly different from that over the flat terrain. In this case, the FPD needs to be examined in the $x$-$y$ plane and only examining the cross-wind integrated FPD is not sufficient to characterize the FPD. In addition, the real-world ground elevation usually varies in two dimensions with multiple energetic modes and the ground land cover may be heterogeneous. Therefore, more scenarios need to be examined, for example, with different meteorological conditions over more realistic terrain. Given that it might be too complicated to generalize the influence of topography on FPD, it is recommended to conduct advanced modeling to characterize footprints for measurements over complex terrain at a given site.
Appendix

A.1 LS trajectory model

The trajectories of particles can be evaluated as,

$$dx_i = u_i dt$$

where \( i = 1, 2, \) and 3 corresponding to the \( x, y, \) and \( z \) directions, respectively. \( u_i \) is the \( ith \) velocity component, which is derived from the stochastic differential equation (Thomson 1987),

$$du_i = a_i dt + (C_0 \varepsilon)^{1/2} d\xi_i$$

where \( C_0 \) is an assumed constant (3.0 in this study), \( \varepsilon \) is the mean turbulence dissipation rate, \( d\xi_i \) is the \( ith \) component of a Gaussian white noise with a mean of zero and a variance of \( dt \). \( a_i \) is determined from the Fokker-Planck equation under the well-mixed condition (Thomson 1987). For a 3-D Gaussian turbulence,

$$a_i = -\frac{C_0 \varepsilon}{2} \lambda_{ik} (u_k - U_k) + \phi_i \frac{\phi}{p_E}$$

where \( \lambda_{ik} \) is the inverse of the 3D ensemble-mean stress \( (\tau_{ik}) \), \( U_k \) is the \( kth \) component of the ensemble-mean Eulerian velocity and linearly interpolated from the nearest grid points of the LES output. For dispersion in two or three dimensions, no unique solution for \( a_i \) exists. For a stationary flow, the simplest choice for \( \phi_i / p_E \) is

$$\phi_i = 1 \frac{1}{2} \frac{\partial \tau_{ij}}{\partial x_i} + U_j \frac{\partial U_i}{\partial x_j} = \left[ \frac{1}{2} \lambda_{ij} \frac{\partial U_i}{\partial x_j} + \frac{\partial U_i}{\partial x_j} \right] (u_j - U_j)$$

Note that the impact of non-Gaussian and skewed statistical properties is not considered. Studies have shown that results were not improved when the effects of non-Gaussian turbulence were considered in an LS model with the 2D turbulence even within a forest canopy (Flesch and Wilson 1992; Wilson and Sawford 1996). In the surface layer where vertical gradients of turbulence are much more important, turbulence has been assumed to be a Gaussian distribution in nearly all LS footprint models reported in the literature (e.g., Flesch 1996; Baldocchi 1997). Mean wind and turbulence statistics from LES are used to drive the LS model (Weil et al. 2004; Weil 2008).

A.2 LES model

The model solves the Navier-Stokes equations under the terrain-following coordinate system. The turbulence kinetic energy scheme proposed by Deardorff (1980) is used to parameterize the subgrid scale (SGS) turbulence. The third-order Runge-Kutta time-integration scheme is used, with a third-order vertical advection scheme and fifth-order horizontal advection scheme. Details about the model can be found in the literature (Bryan and Fritsch 2002; Bryan et al. 2003; Bryan 2005). This model has been successfully applied to simulate atmospheric boundary layer turbulence under complex conditions (Kang and Davis 2008; Wang and Davis 2008; Wang 2009a; Wang 2009b).

The model configuration is same as that described by Wang (2009b) except that no canopy is presented and the surface roughness length is 0.1 m. A brief description is
given below. The simulation domain is $8 \times 4$ km$^2$ with a horizontal grid of 10 m. The vertical grid size is 2.5 m below 40 m and 10 m above 102.5 m, and varies linearly in between. The top of the domain is 2502.5 m. The time step used is 0.1 s. The surface sensible heat flux is taken to be 0.12 K ms$^{-1}$. The lateral boundary condition is cyclic. A rigid lid is used as the top boundary condition with a Rayleigh friction scheme (Durran and Klemp 1983) above 1600 m to absorb spurious gravity waves and reflection from the top. The atmosphere is initialized with a stable potential temperature gradient of 4 K km$^{-1}$ below 500 m and 9.8 K km$^{-1}$ above. Turbulence is triggered by randomly adding a temperature perturbation to temperature at the lowest two levels. A geostrophic wind ($U_g$), aligning with the direction in which the ground elevation varies (i.e., the west-east direction), is imposed to represent the large scale forcing. The Coriolis force is not considered for the relatively small scale simulations. The model is integrated for 4.5 hours, and results from the last hour are averaged over time and in the $y$ direction (to approximate the ensemble averages of atmospheric variables). Two scenarios are presented in this study. $U_g$ is taken as 1 m s$^{-1}$ in one scenario, and 5 m s$^{-1}$ in the other scenario.
Reference:


Wang WG, Davis KJ, Cook BD, Butler MP and Ricciuto DM (2006a) Decomposing CO2 fluxes measured over a mixed ecosystem at a tall tower and extending to a region:
A case study. J Geophys Res-Biogeosci. 111(G2). DOI:G02005
10.1029/2005jg000093
Caption:

Figure 1 The contours of streamline function \( \psi(x, z) = \int_0^x U(x, z) \, dz \) for (a) Scenario I, and
(b) Scenario II. \( U = \frac{\partial \psi}{\partial z}, \quad W = -\frac{\partial \psi}{\partial x} \), the flow direction is sketched by open arrows.

Figure 2 Comparisons of crosswind-integrated FPDs from the backward LS model with the turbulence only in the z direction (1D) (thin line) and with the turbulence in both x and y directions (2D) (thick line). Results from the coupled LES-LS model by Steinfeld et al. (2008) are showed by the dashed line. Symbols show the results derived by Leclerc et al. (1997) from field observations (Finn et al., 1996). Results from the HW analytical model are showed by the dotted line.

Figure 3 (a) The cross-wind integrated FPDs for the receptor on 20 m of the tower located in the lee area (A) (see Figure 1) in the strong wind scenario from the LS model driven by the LES-derived meteorological fields over the ridge (black solid line), from the LS model with the assumed horizontally-homogeneous meteorological fields (dotted line), and from the HW analytical model (red line). (b) trough area (B), (c) windward area (C), and (d) crest area (D).

Figure 4 Same as Figure 3, except for the weak wind scenario.
Figure 1 The contours of streamline function $\psi(x, z) = \int_0^z U(x, z) dz$ for (a) Scenario I, and (b) Scenario II. $U = \frac{\partial \psi}{\partial z}$, $W = -\frac{\partial \psi}{\partial x}$, the flow direction is sketched by open arrows. Four hypothetical tower locations are indicated by ‘A’, ‘B’, ‘C’, and ‘D’.
Figure 2 Comparisons of crosswind-integrated FPDs from the backward LS model with the turbulence only in the z direction (1D) (thin line) and with the turbulence in both x and y directions (2D) (thick line). Results from the coupled LES-LS model by Steinfeld et al. (2008) are showed by the dashed line. Symbols show the results derived by Leclerc et al. (1997) from field observations (Finn et al., 1996). Results from the HW analytical model are showed by the dotted line.
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Figure 4 Same as Figure 3, except for the weak wind scenario.
Table 1 Atmospheric parameters at four locations for Scenario I, $U_g = 5$ m s$^{-1}$

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<tr>
<th>variables</th>
<th>Lee area A</th>
<th>Trough area B</th>
<th>Windward area C</th>
<th>Crest area D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_*$</td>
<td>0.19</td>
<td>0.17</td>
<td>0.24</td>
<td>0.35</td>
</tr>
<tr>
<td>$w_*$</td>
<td>1.51</td>
<td>1.54</td>
<td>1.61</td>
<td>1.55</td>
</tr>
<tr>
<td>$L$ (m)</td>
<td>-5.4</td>
<td>-3.4</td>
<td>-11</td>
<td>-32</td>
</tr>
<tr>
<td>$z_i$ (m)</td>
<td>876</td>
<td>925</td>
<td>1067</td>
<td>955</td>
</tr>
<tr>
<td>$W m s^{-1}$</td>
<td>-0.61</td>
<td>-0.02</td>
<td>0.75</td>
<td>0.017</td>
</tr>
<tr>
<td>$U$ (m s$^{-1}$)</td>
<td>2.4</td>
<td>2.1</td>
<td>3.0</td>
<td>4.3</td>
</tr>
<tr>
<td>$\bar{w}^2$ (m$^2$ s$^{-2}$)</td>
<td>0.38</td>
<td>0.25</td>
<td>0.32</td>
<td>0.23</td>
</tr>
<tr>
<td>$\bar{u}^2$ (m$^2$ s$^{-2}$)</td>
<td>1.19</td>
<td>0.97</td>
<td>0.94</td>
<td>0.87</td>
</tr>
</tbody>
</table>

Note: $U$, $W$, $\bar{u}^2$, and $\bar{w}^2$ are the values at 20 m above the local surface.

Table 2 same as Table 1 except for Scenario II, $U_g = 1$ m s$^{-1}$

<table>
<thead>
<tr>
<th>Variables</th>
<th>Lee area A</th>
<th>Trough area B</th>
<th>Windward area C</th>
<th>Crest area D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_*$</td>
<td>0.19</td>
<td>0.06</td>
<td>0.16</td>
<td>0.155</td>
</tr>
<tr>
<td>$w_*$</td>
<td>1.51</td>
<td>1.4</td>
<td>1.5</td>
<td>1.59</td>
</tr>
<tr>
<td>$L$ (m)</td>
<td>-4.9</td>
<td>-0.2</td>
<td>-2.8</td>
<td>-2.7</td>
</tr>
<tr>
<td>$z_i$ (m)</td>
<td>876</td>
<td>706</td>
<td>861</td>
<td>1025</td>
</tr>
<tr>
<td>$W m s^{-1}$</td>
<td>0.3</td>
<td>-0.043</td>
<td>0.23</td>
<td>0.051</td>
</tr>
<tr>
<td>$U$ (m s$^{-1}$) at 20m</td>
<td>-1.24</td>
<td>-0.42</td>
<td>1.03</td>
<td>1.02</td>
</tr>
<tr>
<td>$\bar{w}^2$ (m$^2$ s$^{-2}$)</td>
<td>0.36</td>
<td>0.26</td>
<td>0.30</td>
<td>0.31</td>
</tr>
<tr>
<td>$\bar{u}^2$ (m$^2$ s$^{-2}$)</td>
<td>0.78</td>
<td>0.48</td>
<td>0.55</td>
<td>0.65</td>
</tr>
</tbody>
</table>