Abstract—In this paper, we investigate the performance of adaptive beamforming while using subcarrier permutation PUSC in WiMAX cellular network. In the literature, it has been shown that frequency reuse 1 is possible for beamforming capable WiMAX networks but with partial resource utilization or base station coordination. In this paper, we show however that using distributed subcarrier permutation (PUSC) offers sufficient diversity to allow full resource utilization without the need of coordination. We study an IEEE 802.16e cellular network employing adaptive beamforming per PUSC Major Group. Performance is evaluated in terms of radio quality parameters and system throughput. Results are based on Monte Carlo simulations performed in downlink. The simulation results show that even with reuse 1 and full load conditions, outage probability can be reduced to an acceptable value.

Keywords: OFDMA, PUSC, IEEE 802.16e, WiMAX, SINR_{eff}, MIC, beamforming.

I. INTRODUCTION

Mobile WiMAX, a broadband wireless access (BWA) technology, is based on IEEE standard 802.16-2005 [1]. Orthogonal frequency division multiple access (OFDMA) is a distinctive characteristic of physical layer of 802.16e based systems. The underlying technology for OFDMA based systems is orthogonal frequency division multiplexing (OFDM). With OFDM, available spectrum is split into a number of parallel orthogonal narrowband subcarriers. These subcarriers are grouped together to form subchannels. The distribution of subcarriers to subchannels is done using three major permutation methods called: partial usage of subchannels (PUSC), full usage of subchannels (FUSC) and adaptive modulation and coding (AMC). The subcarriers in a subchannel for first two methods are distributed throughout the available spectrum while these are contiguous in case of AMC. Resources of an OFDMA system occupy place both in time (OFDM symbols) and frequency (subchannels) domains thus introducing both the time and frequency multiple access [2].

Adaptive beamforming technique is a key feature of mobile WiMAX. It does not only enhance the desired directional signal but also its narrow beamwidth reduces interference caused to the users in neighboring cells. Resultant increase in signal to interference-plus-noise ratio (SINR) offers higher capacity and lower outage probability, which is defined as the probability that a user does not achieve minimum SINR level required to connect to a service. Adaptive beamforming can be used with PUSC, FUSC and AMC (refer Tab.278 of [1]). Performance improvement of the most desired frequency reuse 1x3x1, because of adaptive beamforming, has already been demonstrated in [3]. The effect of frequency diversity (i.e., distributed subcarrier permutations) on adaptive beamforming is yet to be analyzed.

Authors of [4] study the power gain, because of adaptive beamforming, of a IEEE 802.16e based system. Results presented by authors are based on measurements carried out in one sector of a cell with no consideration of interference. Measurements are carried out using an experimental adaptive beamforming system. Reference [5] discusses the performance of WiMAX network using beamforming in conjunction with space division multiple access (SDMA). The simulations are carried out for OFDM (not OFDMA). Hence frequency diversity, because of distributed subcarrier permutations, is not taken into account. In [6] and [7], author has analyzed the performance of beamforming capable IEEE 802.16e systems with AMC. Unlike distributed subcarrier permutations (PUSC and FUSC), subcarriers in an AMC subchannel are contiguous on frequency scale. Hence PUSC/FUSC offer more frequency diversity as compared to AMC. Suggested interference coordination technique allows reuse 1 at the cost of reduced resource utilization. In [3], we have carried out system level simulations for WiMAX networks. The analysis was focused on comparison of different frequency reuse patterns. Adaptive beamforming gain was also considered. We have shown that reuse 1 is possible with partial loading of subchannels.

In this paper, we extend the previous results by investigating the effect of frequency diversity (as a result of subcarrier permutation PUSC) on adaptive beamforming performance in downlink (DL). The performance is analyzed in terms of cell throughput, SINR_{eff} and probability of outage. Reduction in interference, owing to adaptive beamforming, is also exhibited. In addition, we study ways of reducing computational load of beamforming simulations. The comparison of uniform spatial distribution of users and uniform angular distribution of radiation beams is carried out as well.

Rest of the paper is organized as follows: section II gives relative details of IEEE 802.16e PUSC subcarrier permutation method. SINR, beamforming, physical abstraction model MIC and simulator details are introduced in section III. Simulation results have been presented in section IV. Finally section V discusses the conclusion of this analysis.
II. SYSTEM DESCRIPTION

A. Partial Usage of Subchannels (PUSC)

In this section, we present the salient features of DL PUSC permutation. One slot of PUSC DL is two OFDM symbols by one subchannel while one PUSC DL subchannel comprises of 24 data subcarriers. Subchannels are built as follows:

1) The used subcarriers (data and pilots) are sequentially divided among a number of physical clusters such that each cluster carries twelve data and two pilot subcarriers.

2) These physical clusters are permuted to form logical clusters using the renumbering formula on p. 530 in [1]. This process is called outer permutation. This permutation is characterized by a pseudo-random sequence and an offset called \( DL\_PermBase \).

3) Logical clusters are combined together in six groups called the Major Groups. The even groups possess more logical clusters as compared to odd Major Groups. Throughout this paper, we shall refer these Major Groups as groups only.

4) The assignment of subcarriers to subchannels in a group is obtained by applying Eq.111 of [1]. This process is known as inner permutation. The assignment in inner permutation is also controlled by \( DL\_PermBase \). Pilot subcarriers are specific to each group. Since number of logical clusters is different in even and odd groups, the number of their respective subchannels is also different.

The above process of subchannel formation is explained with the help of examples in [8].

By using different \( DL\_PermBase \) in network cells, subcarriers of a given subchannel are not identical in adjacent cells. In this case, it has been shown in [9] and [10], that the above process is equivalent to choosing subcarriers using uniform random distribution on the entire bandwidth in every cell. During our simulations, we consider the same assumption. For simulations, network bandwidth is considered to be 10 MHz. With this bandwidth, the associated PUSC parameters are given in Tab.I.

B. PUSC and Beamforming

Pilot subcarriers are used for channel estimation. In case of beamforming, dedicated pilots are required for each beam in the cell. Since each PUSC group has its own set of pilot subcarriers, beamforming can be done per PUSC group.

As subcarriers of a subchannel are chosen randomly, a subcarrier of a slot may experience the interference from any one of the beams of a given interfering cell with a certain probability (cf. section III-D). In this way each subcarrier of a slot may experience different interfering array-plus-antenna gain since the colliding subcarrier may belong to any of six interfering beams in the neighboring cell. This diversity brought by PUSC is usually not considered in the literature pertaining to beamforming in WiMAX networks.

C. Modulation and Coding Scheme (MCS)

One of the important features of IEEE 802.16 based network is assignment of MCS type to a user depending upon its channel conditions. We have considered six different MCS types in our simulation model: QPSK-1/2 (the most robust), QPSK-3/4, 16QAM-1/2, 64QAM-2/3 and 64QAM-3/4 (for the best radio conditions). SINR threshold values for MCS types are given in Tab.II and have been referred from [11]. If SINR of a mobile station (MS) is less than the threshold of the most robust MCS (i.e., less than 2.9 dB), it can neither receive nor transmit anything and is said to be in outage.

III. NETWORK AND INTERFERENCE MODEL

A. Subcarrier/Effective SINR

SINR of a subcarrier \( n \) is computed by the following formula:

\[
SINR_n = \frac{P_{n,Tx}a_{n,Sh}^{(0)}a_{n,FF}^{(0)}K}{N_0W_{Sc} + \sum_{b=1}^{B} P_{n,Tx}a_{n,Sh}^{(b)}a_{n,FF}^{(b)}K/d_n^{(b)}} \]

(1)

where \( P_{n,Tx} \) is the per subcarrier power, \( a_{n,Sh}^{(b)} \) and \( a_{n,FF}^{(b)} \) represent the shadowing (log-normal) and fast fading (Rician) factors for the signal received from serving BS respectively, \( B \) is the number of interfering BS, \( K \) is the path loss constant, \( \alpha \) is the path loss exponent and \( d^{(0)} \) is the distance between MS and serving BS. The terms with superscript \( b \) are related to interfering BS. \( W_{Sc} \) is the subcarrier frequency spacing, \( N_0 \) is the thermal noise density and \( d^{(b)} \) is equal to 1 if interfering BS transmits on \( n^{th} \) subcarrier and 0 otherwise.

Slot is the basic resource unit in an IEEE 802.16 based system. We compute \( SINR_{eff} \) over the subcarriers of a slot. The physical abstraction model used for this purpose is MIC [9]. For computation of \( SINR_{eff} \), log-normal shadowing is drawn randomly for a slot and is same for all subcarriers of a slot. Since subcarriers of a subchannel (hence a slot) are not contiguous, fast fading is drawn independently for every subcarrier of a slot (Fig.1). For fast fading, Rice distribution has been considered in simulations. Rician K-factor has been referred from [12] (scenario C1).

### TABLE I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total available bandwidth ( BW_T )</td>
<td>10 MHz</td>
</tr>
<tr>
<td>FFT size</td>
<td>1024</td>
</tr>
<tr>
<td>No. of subchannels ( N_{Sc,h} )</td>
<td>30</td>
</tr>
<tr>
<td>No. of subchannels per even group ( N_e )</td>
<td>6</td>
</tr>
<tr>
<td>No. of subchannels per odd group ( N_o )</td>
<td>4</td>
</tr>
<tr>
<td>No. of PUSC groups</td>
<td>6</td>
</tr>
</tbody>
</table>

### TABLE II

<table>
<thead>
<tr>
<th>Index</th>
<th>MCS</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>QPSK</td>
<td>QPSK</td>
<td>16QAM</td>
<td>16QAM</td>
<td>64QAM</td>
<td>64QAM</td>
<td></td>
</tr>
<tr>
<td>SINR_{eff} ( [dB] )</td>
<td>2.9</td>
<td>6.3</td>
<td>8.6</td>
<td>12.7</td>
<td>16.9</td>
<td>18</td>
<td></td>
</tr>
</tbody>
</table>

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One of three sectors (Array) of the sector serving MS2

Axis of ULA (Uniform Linear Array) of the sector serving MS2

In a slot $\alpha^{(0)}_{BH}$ and $\alpha^{(b)}_{BH}$ are constant.

For one subcarrier, there is a unique value of $\alpha^{(0)}_{PP}$, $\alpha^{(b)}_{PP}$ and $\beta^{(b)}$.

Fig. 1. Shadowing and fast fading over a PUSC slot.

B. Beamforming Model

The beamforming model considered in our simulation is the delay and sum beamformer (or conventional beamformer) with uniform linear array (ULA). The power radiation pattern for a conventional beamformer is a product of array factor and radiation pattern of a single antenna. The array factor for this power radiation pattern is given as [13]:

$$AF(\theta) = \frac{1}{n_t} \left| \frac{\sin\left(\frac{n\pi}{2}(\cos(\theta) - \cos(\phi))\right)}{\sin\left(\frac{n\pi}{2}(\cos(\theta) - \cos(\phi))\right)} \right|^2,$$

where $n_t$ is the number of transmit antennas at BS (with inter-antenna spacing equal to half wavelength), $\phi$ is the look direction (towards which the beam is steered) and $\theta$ is any arbitrary direction. Both these angles are measured with respect to array axis at BS (see Fig.2).

The gain of single antenna associated with array factor is given by Eq.3 [9]:

$$G(\psi) = G_{max} + \max \left[-12 \left(\frac{\psi}{\psi_{3dB}}\right)^2 - G_{FB}\right],$$

where $G_{max}$ is the maximum antenna gain in boresight direction, $\psi$ is the angle MS subtends with sector boresight such that $|\psi| \leq 180^\circ$, $\psi_{3dB}$ is the angle associated with half power beamwidth and $G_{FB}$ is the front-to-back power ratio.

C. Path Loss Model

Line-of-sight (LOS) path loss ($PL$) model for suburban macro (scenario C1) has been referred from [12]. It is a three slope model described by following expressions:

$$PL(d) = \begin{cases} \text{free space model} & \text{if } d \leq 20m; \\ C(f_c) + 23.8\log_{10}(d) & \text{if } 20m < d \leq d_{BP}; \\ C(f_c) + 40\log_{10}(d/d_{BP}) & \text{if } d > d_{BP}, \\ +23.8\log_{10}(d_{BP}) & \end{cases}$$

where $f_c$ is the carrier frequency in Hz, $C(f_c)$ is the frequency factor given as: $33.2 + 20\log_{10}(f_c/2 \cdot 10^9)$, $d_{BP}$ is the breakpoint distance computed as: $4h_{BS}h_{MS}/\lambda_c$ and $\sigma_{SH}$ is the standard deviation of log-normal shadowing. The value of $\sigma_{SH}$ associated with above model is 4 dB for $d \leq d_{BP}$ and is equal to 6 dB beyond $d_{BP}$.

D. Simulator Details

The frequency reuse pattern considered in simulations is 1x3x1 (Fig.3). The number of cells in the network is nineteen (i.e., eighteen interfering BS). To speed up the simulation process and to include the effect of an infinite network, wraparound technique has been employed. A significant number of snapshots are being carried out for Monte Carlo simulations. Locations of MS in a sector are drawn using uniform random distribution and beams are steered according to these locations. At BS, four transmitting antennas have been considered while MS is supposed to possess one receiving antenna. All simulations are carried out with full loading of subchannels (i.e., 30) per sector.

As explained earlier, there can be up to six beams per sector when PUSC is used, i.e., one beam per group. In simulations, we have considered three cases with 1, 3 and 6 adaptive beams respectively. For first case, all six PUSC groups are used by one beam. In second case, each beam uses one odd and one even group. In the last case, each beam uses a distinct group. It is to be noted that number of channels per even and odd group are different (see Tab.I). To find the direction of adaptive beams, equivalent number of MS are drawn in a cell using spatial uniform distribution.

For the first case, one MS is drawn per sector and all subcarriers of a slot experience the same interfering beam pattern from a neighboring sector. On the other hand, in the second case, three MS are dropped in a sector and hence there are three interfering beams per sector. For each subcarrier used by a MS, the interfering beam is chosen with equal probability.

When there are six beams in a sector, the selection of interfering beam per subcarrier is no more equally probable.
The reason being that beams are associated to even or odd groups and thus have different number of subchannels. Hence, for a subcarrier, the probability of interfering with an even beam is given as:

\[ p_e = \frac{N_e}{N_{Sch}} \]

and with an odd beam it is:

\[ p_o = \frac{N_o}{N_{Sch}} \]

Considering a subcarrier, six MS are drawn per interfering sector. Respective beams are steered, three of them are odd and the others three are even. In a given interfering sector, the chosen beam is drawn according to the above discrete distribution.

During every snapshot, \( SINR_{eff} \) of a MS is calculated using MIC model. Cell space around BS is divided into twenty rings. Since MS is dropped using uniform random distribution, during a snapshot, it might be located in any of these twenty rings. \( SINR_{eff} \) and throughput are averaged over each of these rings and over complete cell as well. The former is used to study the effect of change in the values of \( SINR_{eff} \) and throughput w.r.t. distance from the BS. If \( SINR_{eff} \) value of a MS during a snapshot is less than 2.9 dB (threshold value being referred from [11]), it is considered to be in outage. Throughput of a MS during a snapshot, depends upon the MCS used by it.

Simulation parameters are given in Tab.III. The parameter values are mainly based on [9].

### IV. Simulation Results

In this section we present the simulation results. Average global throughput and average \( SINR_{eff} \) with respect to distance from BS are presented in Fig.4 and 5 respectively. All three beamforming scenarios discussed in section III-D are shown and compared to the without beamforming case. In addition, a scenario assuming beamforming only in the serving cell is presented.

A clear difference can be observed between beamforming and without beamforming cases. We can observe about 7 to 8 dB gain. The gain for “beamforming in the serving cell only” scenario is about 2 dB less. The difference shows the effect of beamforming on interference reduction. The difference in terms of \( SINR_{eff} \) and global throughput is not much with varying number of interfering beams.

However, it can be clearly seen in Fig.6 that outage probability significantly decreases when we take full advantage of diversity offered by PUSC. When increasing the number of beams, outage probability decreases from an unacceptable 9% (with one beam) to a reasonable 2% (with six beams). To show difference numerically, results are also presented in Tab.IV. It is interesting to note that average throughput and \( SINR_{eff} \) are not affected by the gain in outage probability.

Aside from above study, we have investigated the way of reducing the computational load of beamforming simulations. For this purpose, instead of considering spatial distribution of MS in the interfering sectors, uniform angular distribution of beams is considered.

To calculate array-plus-antenna gain because of an interfering beam, we only require the angle that beam subtends at ULA and boresight of the serving sector. For this, we need a two step procedure. In the first step, MS are uniformly distributed in the interfering sector and then, during second step, angles are calculated to find out their beam patterns. If we draw these angles in a sector using uniform angular distribution, first step can be avoided and simulation time can be saved. However, since the shape of cell is hexagonal (not circular), spatial distribution is not exactly equal to angular distribution.

The comparison of uniform spatial distribution of MS and

![Graph](image-url)

**Fig. 4.** Average \( SINR_{eff} \) (DL) vs distance with reuse 1x3x1.
uniform angular distribution of beams is depicted in Fig.7. The results of two are almost the same. Though average throughput and MCS probabilities are not presented here, they were also in agreement with the results of average $SINR_{eff}$ vs distance from BS curve presented here.

V. CONCLUSION

In this paper, we have analyzed the performance of beamforming capable WiMAX network by exploiting the frequency diversity of distributed subcarrier permutation PUSC. We have illustrated that, by employing beamforming per PUSC Major Group with reuse 1, outage probability can be reduced considerably. It can be reduced to 2%, even without partial loading of subchannels and base station coordination. This reduction neither affects $SINR_{eff}$ nor cell throughput. We have shown how beamforming not only enhances the desired signal but also reduces interference caused to MS in neighboring BS. It is also demonstrated that while simulating beamforming, uniform angular distribution of beams and uniform spatial distribution of MS give the same results. As a consequence, using the former method in simulations can reduce the computational load and hence the time of simulation.

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