2DOF Control of an Electrodynamic Shaker using Explicit Receding Horizon Control for Feedforward Term

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Abstract—This paper presents the application of a two-degree-of-freedom (2DOF) control of an electrodynamic shaker. The characteristics of a shaking system are considered to be nonlinear and variable because of the influence of the test piece. In order to compensation for this problem, the influence of the disturbance force needs to be suppressed. The controller is designed using $\mu$-synthesis by considering the uncertainty of the shaker. Furthermore, since the control performance is improved, an explicit receding horizon control controller is introduced to the 2DOF controller using Multi-Parametric Toolbox. A reference signal which is limited by the system specifications can be partly employed. Finally, a good performance can be realized, as confirmed by experiments conducted using actual equipment.

I. INTRODUCTION

Because the durability of specimens is confirmed, shakers are used in vibration tests. The specimen is test piece, and it is exposed to vibration environment under actual field conditions in the vibration test. Recently, shaking systems that can more accurately replicate an actual situation have been widely employed for vibration testing. The shaking systems are popularly used in the automotive industry such as Fig. 1. For other instance, they are used in the civil and architectural engineering fields. An electrodynamic shaker has some worthwhile features such as good linearity and wide frequency response. The shaking system needs to be controlled for not only exercising stable control but also better replication of the given reference waveform.

In applications in regard to robust vibration control, there are a lot of successful cases, like examples [1], [2], [3]. However, if the interaction between the shaker and the test piece cannot be ignored, the control of the shaking system becomes difficult. Further, if the characteristic of a controlled plant is nonlinear because of the influence of the test piece, the control becomes difficult in a similar manner. Because of this problem, the system stability can be compensated, although the control performance cannot satisfy the test specifications.

In regard to the realization of above problems, there have been some instances [4], [5], [6] in which electrohydraulic shakers have been successfully employed. In previous our study [7], let us similarly attempt to suppress the influence of the disturbance force. For the stable control, the perturbation of parameters, which are used in the estimator of the disturbance force, is considered in the robust design. The estimator is also added to a generalized plant, and the controller is then designed by $\mu$-synthesis. The disturbance-force compensator can suppress the influence of the nonlinear characteristics but cannot replicate the reference signal.

A 2-degree-of-freedom (2DOF) control is required because the response signal tracks to the reference signal. For instances [8], [9], there are a lot of successful cases owing to improving the performance of the feedforward controller. On the other hand, the vibration test using the electrodynamic shaker is generally executed at higher frequency band in order to its advantage, and the most test is required that the acceleration waveform is employed as the reference. However, the rated displacement of the electrodynamic shaker is not bigger than that of the other test equipments. Due to the displacement limitation, there is the case that the acceleration reference signal cannot be executed.

Therefore, in this study, 2DOF control system is constructed because of tracking to the acceleration reference waveform. A coprime factorization of a plant model is considered, the purpose of selecting a reference model is interpreted as the problem of obtaining the state feedback controller. Furthermore, for improving the replicate performance, it is considered that the state feedback controller is replaced by a switched controller. Under a constrain of the displacement limitation, an explicit receding horizon control (ERHC) controller, which has applied example such as [10], is designed such that the acceleration response tracks to the reference signal. Owing to the advantage of using the disturbance-force compensator, it is considered that the proposed control employing the ERHC controller obtain a good control performance.

In this paper, the disturbance-force compensator and the feedforward controller using the ERHC for electrodynamic shakers is presented. The rest of the paper is organized as...
follows: A mathematical model and an uncertainty weighting function are introduced in Section 2; A feedback controller is designed using $\mu$-synthesis in Section 3; A 2DOF controller with the ERHC controller is constructed in Section 4; Section 5 provides experimental results; and finally the paper is summarized in Section 6.

II. ELECTRODYNAMIC SHAKER MODEL

A. Nominal model

The plant to be controlled is an electrodynamic shaker depicted in Fig. 2. The electrodynamic shaker is based on the principle that an electrodynamic force is generated in the magnetic field. Assuming that the magnetic flux density is constant, a drive coil can be shown as a linear equivalent circuit. The schematic model of the shaker and the equivalent circuit are depicted in Fig. 2. The force $F_s$ and the reverse electromotive force $E_c$ can be represented as

$$F_s = Bli_1, \quad E_c = Bl\dot{x}_s,$$

where $x_s$ denotes the displacement of the armature, $i_1$ denotes the current of the drive coil; $l$, the length of the drive coil; and $B$, the magnetic flux density, respectively. From Fig. 2, the following (2) can be obtained

$$G_a u_m = R_d i_1 + L_d \dot{i}_1 + E_c$$

where $L_d$ denotes the inductance; $R_d$, the resistance; $G_a$, the amplifier gain; and $u_m$, the input voltage to the amplifier, respectively. The dynamic equation of the armature can be obtained as

$$F_s = m_s \ddot{x}_s + C_d \dot{x}_s + K_d x_s,$$

where $K_d$ denotes the stiffness coefficient of the suspension, and $C_d$ denotes the damping coefficient of the suspension. The transfer function from the input voltage $u_m$ to the acceleration $\ddot{x}_s$ can be represented as the following state-space form

$$\dot{x}_a = A_a x_a + B_a u_m, \quad y_m = C_a x_a,$$

$$A_a = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_d}{m_s} & -\frac{C_d}{m_s} & \frac{B_i}{L_d} & -\frac{R_d}{L_d} \end{bmatrix},$$

$$B_a = \begin{bmatrix} 0 & 0 \\ 0 & \frac{C_d}{L_d} \end{bmatrix},$$

$$C_a = \begin{bmatrix} \frac{K_d}{m_s} & -\frac{C_d}{m_s} & \frac{B_i}{m_s} \end{bmatrix}.$$
of the uncertainty weighting function $W_m$ selected to cover all the perturbations can be expressed as follows:

$$W_m = 0.009 \cdot \frac{s + 3 \cdot 2 \pi}{s + 7 \cdot 2 \pi} \cdot \frac{s + 300 \cdot 2 \pi}{s + 7 \cdot 2 \pi}.$$

(8)

III. FEEDBACK CONTROLLER DESIGN

A. Control objectives

The design of the disturbance-force compensator of an electrodynamic shaker is carried out using MATLAB. A controller is designed to maintain a robust stability against the uncertainty model. Moreover, in spite of the uncertainty which is considered above section, it is desired that a disturbance force is suppressed well. The robust controller for this system is then designed with $\mu$–synthesis, which has many applications such as [11]. The control error is increased when the difference between the model of the plant and the actual characteristic exists. Therefore, this mismatch needs to be reduced by using the disturbance-force compensator. In order to replication of the reference waveform, a 2DOF controller is described in the next section.

B. $\mu$–synthesis

Let us consider the feedback structure shown in Fig. 5, where the block $K_f$ denotes the disturbance-force compensator. The weighting function $W_s$ for the suppression performance of the disturbance force is considered. For improving the performance, the gain of the control frequency band is required to be enlarged within obtaining robust stability. According to the above points, $W_s$ is now chosen as

$$W_s = \frac{2.9 \cdot 14 \cdot 2 \pi}{s + 14 \cdot 2 \pi} \cdot \frac{70 \cdot 2 \pi}{s + 70 \cdot 2 \pi} \cdot \frac{140 \cdot 2 \pi}{s + 140 \cdot 2 \pi} \cdot \frac{s + 2.4 \cdot 2 \pi}{s + 0.56 \cdot 2 \pi}.$$

(9)

Since the signal-to-noise ratio of the displacement response signal is low at high frequency, the magnitude of the controller is required to be made small at the high frequency band. A weighting function $W_{uh}$ is then chosen as

$$W_{uh} = 2000 \cdot \frac{s + 2.5 \cdot 2 \pi}{s + 4000 \cdot 2 \pi}.$$

(10)

On the other hand, since the signal-to-noise ratio of the acceleration response signal is low at low frequency, the magnitude of the controller is required to be made small at the low frequency band. A weighting function $W_{ul}$ is then chosen as

$$W_{ul} = 0.9 \cdot \frac{s^2 + 2.5 \cdot 2 \pi \sqrt{2} s + (2.5 \cdot 2 \pi)^2}{s^2 + 0.002 \cdot 2 \pi \sqrt{2} s + (0.002 \cdot 2 \pi)^2}.$$

(11)

The design objective for stability and control performance is formalized as requirement for a closed-loop transfer function with weighting functions. Therefore, the generalized plant $P$ is constructed to treat the control objectives of the $\mu$–synthesis framework. Here, the block structure of the uncertainty $\Delta$ is defined as

$$\Delta := \{ \text{diag} (\delta_g, \delta_b, \Delta_d, \Delta_m, \Delta_p), \delta_g \in C^{1 \times 1}, \delta_b \in C^{1 \times 1}, \Delta_d \in C^{1 \times 1}, \Delta_m \in C^{1 \times 1}, \Delta_p \in C^{1 \times 3} \},$$

(12)

where $|\delta_g| \leq 1$, $|\delta_b| \leq 1$, $|\Delta_d| \leq 1$, $|\Delta_m| \leq 1$, $|\Delta_p|_{\infty} \leq 1$, and $\Delta_p$ is a fictitious uncertainty block for considering robust performance.

Since the controller satisfies this requirement, the D-K iteration procedure is employed. The controller is obtained after 2 iteration. The degree of this controller has been reduced from 47 states to 15 states.

IV. FEEDFORWARD CONTROLLER DESIGN

A. Overview of feedforward control

The 2-DOF controller shown in Fig. 6 is introduced to improve its transient response to the reference signal [12]. $F_d$ represents the reference model; $P_m$, the nominal model of the plant; and $P_r$, the actual plant, respectively. This control system has an advantage in that the transfer function $P_{yr}$ from the reference $r$ to the response displacement $y_a$ can be determined by $F_d$ irrespective of the choice of the
feedback controller $K_{fb}$. If the nominal model is perfect, i.e., if $P_m = P_r$, then the transfer function becomes $P_{yr} = F_d$. In addition, if the feedback control works precisely, $P_{yr}$ becomes equal to $F_d$.

It is consider that $P_m$ is written as the ratio of coprime factorization, which is given by

$$P_m = \frac{N_m}{M_m}, \quad N_mX_m + M_mY_m = 1,$$  \hspace{1cm} (13)

where $N_m,M_m,X_m$ and $Y_m$ are stable and proper respectively. If $F_d = N_m, F_d/P_m$ is replaced by $M_m$. After that, it is considered that the state-space model of $P_m$ is represented as follows:

$$x_m = A_m x_m + B_m u_m, \quad y_m = C_m x_m,$$  \hspace{1cm} (14)

where $A_m = C_a^{-1} A_a C_a, B_m = C_a^{-1} B_a C_m = C_a^{-1} C_a$. Here, it is known that $N_m,M_m,X_m$ and $Y_m$ are solved as follows: [13]

$$N_m = C_m (sI - (A_m - B_mK_m))^{-1} B_m,$$

$$M_m = 1 - K_m (sI - (A_m - B_mK_m))^{-1} B_m,$$

$$X_m = K_m (sI - (A_m - H_mC_m))^{-1} H_f,$$

$$Y_m = 1 + K_m (sI - (A_m - H_mC_m)^{-1} B_m,$$

where $K_m$ is real matrix, such that $A_m - B_mK_m$ is stable and $H_m$ is real matrix, such that $A_m - H_mC_m$ is stable. Therefore, the purpose of selecting $F_d$ is interpreted as the problem of obtaining the stable feedback controller of a block diagram in Fig. 7 because of $F_d = N_m$. As the diagram, $M_m$ is represented as the transfer function from $r$ to $u_m$ and $N_m$ is the transfer function from $r$ to $y_m$.

As to a function of this feedforward controller, it is able to consider that resulting signals, which is calculated in ideal simulation using $P_m$, are input to feedback control system with the actual plant.

### B. Design of ERHC using MPT

Moreover, for improving the control performance, the state feedback controller $K_m$ is replaced by a switched controller. Hence, the reference signal which is limited by the system specifications can be partly employed. In this study, the ERHC controller is applied to the plant model $P_m$ and the prediction model is shown in Fig. 8. As this figure, under constrains of the control input and the displacement response, the ERHC controller is designed such that the acceleration response tracks to the reference signal. Here, because the acceleration response is suppressed in low frequency band, the weighting function $W_f$ is introduced and its function is selected as

$$W_f = \frac{s^2 + \sqrt{2} \cdot 2\pi s + (2\pi)^2}{s^2 + \sqrt{2} \cdot 2\pi \cdot 0.01s + (2\pi \cdot 0.01)^2}.$$  \hspace{1cm} (16)

Let us consider the constrained finite-time optimal control problem,

$$J = \min_{u_{mk}(0), \ldots, u_{mk}(H-1)} \sum_{k=0}^{H-1} (||Q_y (y_{mk}(k) - r_k(k))||_2 + ||R_x \Delta u_{mk}(k)||_2 + ||Q_x x_{mk}(k)||_2)$$  \hspace{1cm} (17)

where $Q_x$ and $R_u$ denote the weighting coefficient for the state parameter and the control input; and $R_u$, the weighting coefficient for the tracking performance. However, since the reference signal is time varying, the ERHC controller is designed using the extended state-update equation as follows: [14]

$$\begin{bmatrix}
x'_{mk}(k+1)

u_{mk}(k)

r_k(k+1)
\end{bmatrix}
\begin{bmatrix}
A'_{mk}

B'_{mk}

0

0

0

I

I

I
\end{bmatrix}
\begin{bmatrix}
x_{mk}(k)

u_{mk}(k-1)

r_k(k)
\end{bmatrix} +
\begin{bmatrix}
B'_{mk}

I

0

0
\end{bmatrix} \Delta u_{mk}(k),$$  \hspace{1cm} (18)

where it is noticed that each parameter is discretized and $W_f$ is contained. The constraints of the displacement response and control input are given by

$$-0.003 \leq x_{mk1}(k) \leq 0.003,$$

$$-5 \leq u_{mk}(k) \leq 5.$$  \hspace{1cm} (19)\hspace{1cm} (20)

Using MATLAB and MPT 2.6.2, the ERHC is designed on the basis of above things. Each parameter of $Q_y,Q_x$ and $R_u$ is adjusted so that the reference signal can be replicated, and the resulting parameters are then set at $Q_y = 1000, Q_x = \text{diag}(1,1,1)$ and $R_u = 1$, respectively. In the prediction horizon $H$, the smaller value is chosen to $H$ within achieving the control performance, and $H = 20$.

### V. CONTROL RESULT

#### A. Experimental setting

The control performance is confirmed by experiments using the system showed in Fig. 9. A covered container with water is employed as the specimen for the excitation.
experiment, and a sloshing, which is liquid vibration, is occurred by executing the container. The size of rectangular-type container is 170 mm × 170 mm × 150 mm (length, width, height). The disturbance-force compensator is discretized via the Tustin transform at the sampling frequency of 512 Hz.

It is difficult that the disturbance force is directly measured. In this study, the disturbance force is estimated using the displacement and acceleration responses of the armature and the current response of the drive coil. Due to reduce the influence of noise and obtain the accurate estimate values, the order of the estimator is needed to be low. Therefore, the three variables are employed. As shown in this figure, the estimated disturbance force \( \hat{d}_f \) is given as follows,

\[
\hat{d}_f = B I_1 - K_d \ddot{x}_s - \left( m_s + \frac{C_d}{s + \omega_p} \right) \ddot{x}_s, \tag{21}
\]

where \( \omega_p \) denotes the adjustment parameter to keep the gain low in a low frequency band.

Because this estimate construction is employed, it is considered that these value mismatch are included in the estimated disturbance force. Then, in the case of compensating the estimated disturbance force, this difference can be small.

**B. Experimental results**

This experiment is executed by an excitation using a measured waveform data as the reference signal. The control performance is evaluated by the result of tracking the acceleration reference signal within achieving an operation in range of the rated displacement. In this experiment, it is assumed that the range of the rated displacement is set from -3 mm to 3 mm such as (19). The original waveform of acceleration reference is measured at the moving vehicle. Because effect of the proposal controller is showed, the level of the reference is slightly bigger than the rated displacement. Let us show the block diagram of the experimental system in Fig. 10, and \( G_f \) represents a scalar gain since the displacement response is suppressed within the limitation. It is noticed that \( M_m \) is only introduced as the feedforward controller because of employing the disturbance-force compensator.

First, in order to considering a conventional feedforward controller the ERHC is simply designed with extending the constraint of displacement. In this case, because the displacement response is suppressed within the limitation, \( G_f \) is set at 0.8. The control results of the acceleration response are shown in Fig. 11 and the displacement response is shown in Fig. 12. For Fig. 12, the displacement response achieves the rated displacement because of setting \( G_f \) at less than 1. However, the acceleration response is smaller than the reference in all experimental period.

Next, the control results with are shown in Fig. 13 and Fig. 14 similarly. \( G_f \) is set at 1 in this case. As the results, the displacement response is within the limitation because of using the ERHC controller with the constraint of the displacement. Furthermore, the acceleration response is consistent with the reference signal except for the period that the reference overshoots the displacement limitation.
VI. CONCLUSION

In this paper, a 2DOF controller using an ERHC controller was presented for an electrodynamic shaker. Similar to an actual scenario, a controlled plant with nonlinear characteristics was considered. In order to compensation for this problem, the influence of the disturbance force was suppressed, and the controller was designed using $\mu$-synthesis. Furthermore, since the control performance was improved, the ERHC controller was introduced to the 2DOF controller. Experiments were performed using the electrodynamic shaker with a reference waveform of which a level was slightly bigger than a displacement limitation. Because the disturbance-force compensator suppressed the nonlinear characteristic, satisfactory replication of the reference signal was achieved.

REFERENCES