A New Reliability Prediction Model in Manufacturing Systems

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Abstract—Reliability prediction has been widely studied in many research fields to improve product and system reliability in manufacturing systems. Traditionally, to establish the prediction model, modelers would use all training data without preference. However, the prediction model based only on the most recent data may have better performance. In this paper, to realize an accurate prediction with the most recent data sets, we use the grey model to establish the reliability model. Then, the cubic spline function is integrated into the grey model to enhance the prediction capability of GM(1, 1). To further improve the prediction accuracy, the particle swarm optimization (PSO) algorithm is applied to 3spGM(1, 1). We call the improved version P-3spGM(1, 1). Finally, we validated the effectiveness of the proposed model using failure data sets of electric product manufacturing systems.

Index Terms—Cubic spline function, grey model, particle swarm optimization algorithm, reliability prediction model.

ACRONYMN

AGO accumulated generating operation
ARIMA auto regressive integrated moving average
GA genetic algorithm
GM grey model
IAGO inverse accumulative generating operation
PSO particle swarm optimization

NOTATION

\( c_1 \) cognitive parameter
\( c_2 \) social parameter
\( \frac{dc^{(1)}(t)}{dt} \) first order derivative
\( P_{max} \) the number of particles
\( k_{max} \) the maximum iterations
\( P^i_k \) the best individual particle position
\( P^g_k \) the best swarm position
\( r_1, r_2 \) random numbers between 0 and 1

\( V_i^0 \) particle velocities, \( 0 \leq V_i^0 \leq V_{max}^0 \) for \( i = 1, 2, \ldots, i_{max} \)
\( w^0 \) weighting
\( z^{(1)} \) background value

I. INTRODUCTION

With increasing complexity in manufacturing systems, failure impacts the system so strongly that there is a strong demand for reliability engineering to improve system reliability [1]. Major research areas of system reliability include software reliability, failure analysis, and reliability prediction. Researchers Dai et al. [2]–[7] studied the software reliability model, and grid computing system reliability model using the Bayesian approach, Markov models, heuristic algorithms, and regression methods.

Over the past few years, many researchers have used different kinds of methods to validate the reliability prediction model of electric products. Yao et al. [8] used time series models, and fuzzy methods to establish the failure prediction model of an electrical contact. Lolasa et al. [9] used neural networks, and expert systems to deal with reliability performance prediction during new product development. Gnanasambandam et al. [10] studied reliability prediction in electronics manufacturing using exert systems. Maisch et al. [11] used time series models, including exponential smoothing, regression, and the ARIMA model to predict the decreasing reliability trends during usage.

However, most existing models are established based on all available training data. They treat all data equally in the training process; however, if the prediction is for a time series, the most recent data may carry more information than those older data. Thus, when a prediction model is established based on all training data without preference, the resulting prediction may not be very accurate [12]. To realize an accurate prediction with the most recent data sets, Deng proposed the grey model based on grey system theory [13] in 1982. As the prediction model, the grey model [14] plays an important role to make accurate prediction in many fields, ranging from economics through physics to engineering. But the grey model has irrational problems concerning the calculation of its derivative, and its background value. Thus, the prediction accuracy is limited [15], [16]. In this paper, we propose an improved GM(1, 1) model which stands for the first order grey model with a single variable. Then it is applied to perform the failure prediction of electronic products in manufacturing systems. We integrate the cubic spline function into GM(1, 1) to enhance its prediction capability. The improved version is called the 3spGM(1, 1) model. To further improve the prediction accuracy,
the particle swarm optimization (PSO) algorithm is applied to the 3spGM(1, 1) model. The improved version is called P-3spGM(1, 1). Finally, the failure data of electronic products in manufacturing systems are used to validate the proposed model. The experimental results show that the proposed model has good performance for reliability prediction.

This paper is organized as follows. Section II describes the GM(1, 1), which denotes the first-order grey model with single variables. Section III discusses the improved 3spGM(1, 1) prediction model based on the cubic spline function. In this model, the calculations of derivative and background value are done using the cubic spline function. In Section IV, the P-3spGM(1, 1) model based on the PSO algorithm is discussed. In this model, the coefficients of 3spGM(1, 1) are optimized by the PSO algorithm. A case study, and simulation results are presented in Section V. For comparison, the well-known ARIMA model is also implemented to verify the superiority of the proposed model. Section VI gives the consideration. The prediction capability of the proposed model is satisfactory. From the results of the accuracy comparison, the P-3spGM(1, 1) model acquired the most accurate prediction. Finally, conclusions are drawn in Section VII.

II. GREY PREDICTION MODEL

If the system information is fully known, the system is called a white system; while if the system information is unknown, it is called a black system. A system with partial information known, and partial information unknown, is a grey system [17]. It avoids the inherent defects of conventional, large sample statistical methods, and only requires a limited amount of discrete data to evaluate the behavior of a system with incomplete information. As a prediction model, the grey model is based on the method of accumulated generating operation (AGO), rather than finding the statistics features to preprocess the original data so that the after-processed data will become regular. Based on the processed data, we can establish a suitable model to approximate the system dynamics. Here are a few differences between the grey model, and the ARIMA model [18].

- The grey model does not have a requirement around the distribution characteristic of the original data series. The data series used in the grey model are preprocessed based on AGO. In contrast, ARIMA not only requires a law for the distribution of the original series, but also requires a large amount of observed data so that the prediction can be satisfactory. But the grey model can be established if the original series contains more than four observations [14].
- ARIMA is a short-term prediction method, and the grey model can carry out the prediction of short, medium, or long-term problems.
- ARIMA cannot use multivariate to form a prediction model.

A. GM(1, 1) Model

For the original data series \(x(0)(t)\), \(t = 0, 1, \cdots, n\), a new series \(x(1)(t)\), \(t = 0, 1, \cdots, n\) can be generated by the AGO as

\[
x(1)(t) = \sum_{i=0}^{t} x(0)(i)
\]

From \(x(1)(t)\), we can form the grey prediction model GM(1, 1), which is expressed by one variable, and a first order differential equation as

\[
\frac{dx(1)}{dt} + ax(1) = b
\]

where the coefficients \(a\), and \(b\) are called the grey development, and grey input coefficients, respectively.

The grey derivative for the first order grey differential equation with AGO is conventionally represented as

\[
\frac{dx(1)}{dt} = \lim_{\Delta t \to 0} \frac{x(1)(t + \Delta t) - x(1)(t)}{\Delta t}
\]

Let \(\Delta t \to 1\), and obtain

\[
\frac{dx(1)}{dt} \approx x(1)(t + 1) - x(1)(t) = x(0)(t + 1),
\]

Then the discrete form of the GM(1, 1) differential equation model is expressed as

\[
x(0)(i) + ax(1)(i) = b
\]

where \(z(1) = \{z(1)(1), z(1)(2), \cdots, z(1)(n)\}\) is called the background value of \(\frac{dx(1)}{dt}\), and is calculated by

\[
z(1)(i) = \frac{1}{2} [x(1)(i - 1) + x(1)(i)]
\]

Using the least-square method, the coefficients \(a\) and \(b\) can be obtained as

\[
\hat{a} = \begin{bmatrix} a \\ b \end{bmatrix} = (A^T A)^{-1} A^T X_n,
\]

\[
A = \begin{bmatrix} -z(1)(1) & 1 \\ -z(1)(2) & 1 \\ \vdots & \vdots \\ -z(1)(n) & 1 \end{bmatrix},
\]

and

\[
X_n = \begin{bmatrix} x(0)(1) \\ x(0)(2) \\ \vdots \\ x(0)(n) \end{bmatrix},
\]

Then, the modeling value of (2) is obtained as

\[
x(1)(i) = \left(x(0)(0) - \frac{b}{a}\right) e^{-ai} + \frac{b}{a},
\]

By the inverse accumulative generating operation (IAGO), the prediction series for the original time series can be obtained as

\[
x(0)(i) = \xi(1)(i) - \xi(1)(i - 1)
\]

where the data series \(\{\xi(0)(0), \xi(0)(1), \cdots, \xi(0)(n)\}\) is called the fitting series, while the series \(\{\xi(0)(n + 1), \xi(0)(n + 2), \cdots, \xi(0)(n + k)\}\) is called the prediction series. Usually, the number of data used in GM(1, 1) is rather small because only two coefficients are required to be identified in (7). In other words, GM(1, 1) is often used as a short-term prediction.
scheme [19]. The GM(1, 1) model realizes the prediction based on only a set of the most recent data in a time series. Predictions of this kind are to establish a curve for the most recent data, and then make predictions based on the established curve.

B. Error Analysis

It is commonly believed that differential equations are only suitable for continuous differential functions. The advantage of the grey prediction model GM(1, 1) with differential equations may be established on the basis of the AGO transformation series for original discrete data. Hence, the grey model makes it possible to solve prediction problems based on limited data sets. However, this model has irrational problems concerning the calculation of derivatives, and the background value \( z \). The irrational problems are explained as follows.

- The derivative for the first order differential equation with AGO continuous series is calculated using (3). For discrete data series \( x^{(i)}(i) \), because its derivative does not exist, the first order derivative of GM(1, 1) is calculated using the difference method as
  \[
  \frac{dx^{(i)}(i)}{dt}|_{t=i} \approx \frac{x^{(i)}(i) - x^{(i)}(i - 1)}{i - (i - 1)} = x^{(0)}(i) \quad (12)
  \]

By the Lagrange mean value theorem, we can obtain

\[
\frac{x^{(i)}(i) - x^{(i)}(i - 1)}{i - (i - 1)} = \frac{dx^{(i)}(i)}{dt}|_{t=i-\xi} \quad (13)
\]

where \( 0 < \xi < 1 \). The derivative at time \( i \) is replaced by the derivative at time \( i - \xi \).

- \( z^{(1)}(i) \) is called the background value of the differential equation \( dx^{(i)}(i)/dt \) [14], and is the mean of \( x^{(i)}(i) \) and \( x^{(i)}(i) \). The background value \( z^{(1)}(i) \) is the unclear values in the interval \([i - 1, i]\), and the interval consists of uncertain information [20]. Here, the meaning of unclear value is that we only can get the values \( x^{(i)}(i - 1) \), and \( x^{(i)}(i) \), but we cannot accurately know the background value of the derivative in the interval \([i - 1, i]\).

By the above calculation methods, the prediction accuracy of GM(1, 1) is unsatisfying when the original data shows great randomness [21].

III. 3spGM(1, 1) MODEL

To enhance the prediction accuracy, we present a cubic spline function to calculate the derivative, and background value. The newly generated model is defined as 3spGM(1, 1). The spline function is a kind of interpolation function which has been widely applied to various engineering problems. In this paper, the cubic Hermite spline [22], which is based on the cubic Hermite polynomial, is presented to improve the prediction accuracy. The Hermite polynomial has the property that the spline can pass exactly through all the known points, and the first and second order derivatives of original data can be acquired accurately. Therefore, the difference calculation of the grey model by (12) can be transformed into a differentiation calculation. Thus the prediction performance of the grey model can be improved. But some other functions, such as the B spline function, or the quadratic function, have not the property that a curve can pass through all the known points. Thus the derivatives of the original data cannot be acquired accurately. Therefore, the prediction model 3spGM(1, 1), which is based on the cubic Hermite spline, can provide better prediction performance than a conventional GM(1, 1).

A. 3spGM(1, 1) Model

The coefficients \( a, b \) of the GM(1, 1) differential equation (2) can be calculated using a cubic Hermite spline. The newly generated model is defined as 3spGM(1, 1). The calculation method of coefficients \( a \) and \( b \) is given as

\[
\begin{align*}
\mathbf{A}_3 &= \begin{bmatrix} a \\ b \end{bmatrix} = (A_3^T A_3)^{-1} A_3^T X_n \\
\end{align*}
\]

where

\[
A_3 = \begin{bmatrix} -z^{(3)}(0) & 1 \\
-z^{(3)}(1) & 1 \\
\vdots & \vdots \\
-z^{(3)}(n) & 1 \end{bmatrix}
\]

\[
z^{(1)}(i) = x^{(1)}(i)
\]

\[
X_n = X'
\]

The algorithm for calculating \( X' \) is described in the next section. The prediction values are acquired by (10).

B. Algorithm of First Order Derivative \( X' \)

The first order derivative \( dx^{(i)}(i)/dt \) for AGO transformation series \( x^{(i)}(i) \) can be obtained by differentiation directly. The AGO series \( x^{(i)}(i), i = 0, 1, \ldots, n \) are viewed as knot values \( x(t_i) \) of a cubic spline function, that is \( x(t_i) = x^{(1)}(i) \). Then the cubic Hermite polynomial can be expressed as [23], [24]

\[
\begin{align*}
\mathbf{H}_3(t) = x(t_{i-1}) F_0(\lambda) + x(t_i) F_1(\lambda) + [d'(t_{i-1}) G_0(\lambda) + x'(t_i) G_1(\lambda)] \\
\lambda &= (t - t_{i-1})/h_{t_i}, \quad t_{i-1} < x(t_i) < t_i, h_{t_i} \text{ is called the interpolation distance. Some important properties of the Hermite polynomial are}
\end{align*}
\]

\[
\begin{align*}
\mathbf{H}_3(t_i) &= x(t_i) \\
\lim_{t \to t_i^-} \mathbf{H}_3(t) &= \lim_{t \to t_i^+} \mathbf{H}_3(t), p = 0, 1, 2 \\
\mathbf{H}_3(t_0) &= \mathbf{H}_3(t_0) = 0
\end{align*}
\]

Let the interpolation distance \( h_{t_i} = 1 \). By the above properties (19), we can obtain

\[
\begin{align*}
\frac{1}{2} x'(t_0) + 2x'(t_1) + \frac{1}{2} x'(t_2) = B(t_1) \\
\frac{1}{2} x'(t_1) + 2x'(t_2) + \frac{1}{2} x'(t_3) = B(t_2) \\
\frac{1}{2} x'(t_2) + 2x'(t_3) + \frac{1}{2} x'(t_4) = B(t_3) \\
\vdots \\
\frac{1}{2} x'(t_{n-2}) + 2x'(t_{n-1}) + \frac{1}{2} x'(t_n) = B(t_{n-1})
\end{align*}
\]
and

\[
\left\{ \begin{array}{ll}
2x'(t_0) + x'(t_1) = B(t_0) \\
x'(t_{n-1}) + 2x'(t_n) = B(t_n)
\end{array} \right.
\]

(21)

Then, the first order derivative \( \mathbf{X}' \) can be calculated by

\[
\mathbf{X}' = \begin{bmatrix}
x'(t_0) \\
x'(t_1) \\
\vdots \\
x'(t_n)
\end{bmatrix} = \mathbf{A}^{-1}\mathbf{B}
\]

(22)

where

\[
\mathbf{A} = \begin{bmatrix}
2 & 1 & 0 & \cdots & \cdots & 0 \\
1 & 2 & 1 & \ddots & & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & 1 & 2 & 1 & 0
\end{bmatrix}
\]

(23)

\[
\mathbf{B} = [B(t_0), B(t_1), \ldots, B(t_n)]^T
\]

(24)

\[
\left\{ \begin{array}{l}
B(t_0) = 3(x(t_1) - x(t_0)) \\
B(t_i) = \frac{3}{2}(x(t_{i+1}) - x(t_{i-1})), i = 1, 2, \ldots, n-1 \\
B(t_n) = 3(x(t_n) - x(t_{n-1}))
\end{array} \right.
\]

(25)

IV. P-3spGM(1, 1) MODEL

We propose the PSO algorithm [25] to adjust the values of \( \hat{a}_3 \) of 3spGM(1, 1) repeatedly until they reach optimal values, and the prediction error is reduced to a minimum. This proposed model is defined as P-3spGM(1, 1). The PSO algorithm is a promising new optimization technique which models the set of potential problem solutions as a swarm of particles moving about in a virtual search space. The procedure initializes a set of random particles, and searches for the superior value through iteration. Each particle is a point in the solution space, with an associated speed. In P-3spGM(1, 1), the initial particle positions are randomly taken according to parameters shown in (14). The particles’ speed, and all other parameters are randomly assigned. Then each particle adjusts its own flight track according to its flight experience, and that of its community, to draw close to a superior point. The superior point is also updated through iteration. The PSO algorithm is similar to the genetic algorithm (GA), and has a strong ability to find the most optimal parameter. The prime advantage of PSO over GA is its algorithmic simplicity. Furthermore, the capability of PSO to converge towards optimality or near optimality faster than GA [26] makes PSO preferable to GA.

The proposed P-3spGM(1, 1) algorithm is an iterative procedure that uses the most recent data for training to validate the one step ahead prediction accuracy of P-3spGM(1, 1). In our algorithm, the inputs to the P-3spGM(1, 1) model correspond to the subset of failure data. The previously failure data are used for training, and finding the optimal coefficients \( \hat{a}_3 \) as shown in (14), then the last failure data are predicted. We stop the calculation when \( k = k_{\text{max}} \), where \( k \) and \( k_{\text{max}} \) are the \( k \)-th, and maximum iterations, respectively. By comparing the prediction values with actual sample data, we can obtain the prediction accuracy. The optimal algorithm for coefficients \( \hat{a}_3 \) based on PSO is described as follows.

Algorithm:

1) Initialization
   a) Set the objective function vector, and approximate function vector, as

\[
\mathbf{X} = \begin{bmatrix} x^{(0)}(0) \\ x^{(0)}(1) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, \quad \hat{\mathbf{X}} = \begin{bmatrix} \hat{x}^{(0)}(0) \\ \hat{x}^{(0)}(1) \\ \vdots \\ \hat{x}^{(0)}(n) \end{bmatrix}
\]

(26)

where \( \mathbf{X} \) is the set of original data series, \( \hat{\mathbf{X}} \) is the approximate function, and it is calculated by (11), and (14).

b) Set the approximate parameter (particle position) as

\[
\hat{a}_{3i}^k = \begin{bmatrix} a_i^k \\ b_i^k \end{bmatrix}
\]

(27)

where \( \hat{a}_{3i}^k \) is the \( i \)-th particle position at the \( k \)-th iteration. Here, the initial values of \( \hat{a}_{3i}^0 \) are randomly taken according to the values of coefficients \( a_i \) and \( b_i \) of (14).

c) Set the evaluation function as

\[
\min : J = \mathbf{E}^T \mathbf{E}
\]

(28)

where

\[
\mathbf{E} = \mathbf{X} - \hat{\mathbf{X}}.
\]

(29)

\( \mathbf{X} \) and \( \hat{\mathbf{X}} \) are the objective function, and the approximate function, respectively. Here, the PSO algorithm is used to minimize the value of \( J \) through the adjustment of the particle position of \( \hat{a}_{3i}^k \) shown in (27) repeatedly.

2) Calculate the evaluation function \( J \).

\[
J^k = \left[ \mathbf{X} - \hat{\mathbf{X}} (\hat{a}_{3i}^k) \right]^T \left[ \mathbf{X} - \hat{\mathbf{X}} (\hat{a}_{3i}^k) \right]
\]

(30)

where \( \hat{\mathbf{X}} (\hat{a}_{3i}^k) \) is the approximation function for the \( i \)-th particle position at the \( k \)-th iteration.

3) Update particle positions.

The particle positions are updated by

\[
\hat{a}_{3i}^{k+1} = \hat{a}_{3i}^k + \mathbf{V}_{i}^{k+1}
\]

(31)

where

\[
\mathbf{V}_{i}^{k+1} = \alpha \mathbf{V}_{i}^{k+1} + \mathbf{F}_{i}^{k+1} + \mathbf{C}_{i}^{k+1}
\]

(32)

\[
\mathbf{F}_{i}^{k+1} = c_1 r_1 (\mathbf{P}_i^{k} - \mathbf{X}_i^{k}) + c_2 r_2 (\mathbf{P}_g^{k} - \mathbf{X}_i^{k}), \quad \alpha \in (0,1)
\]

(33)

The \( \mathbf{P}_i^{k} \), and \( \mathbf{P}_g^{k} \) are updated by

\[
\mathbf{P}_i^{k} = \min_{0 \leq j \leq k} J^j
\]

(34)

\[
\mathbf{P}_g^{k} = \min_{0 \leq j \leq k} J^j
\]

(35)
Fig. 1. The flow chart of optimization process of $\alpha^*_a$ by PSO algorithm.

Step 4) Detect stopping criterion. If $k = k^{\text{max}}$, goto Step 5; otherwise $k = k + 1$, and goto Step 2.

Step 5) Output results.

End of algorithm

The flow chart of the above optimization process is shown in Fig. 1. By the optimization process, the parameters $\alpha^*_a$ are updated for $k^{\text{max}}$ times, and the evaluation function $J$ as the convergent error is reduced. The vector $\hat{x}^{k^{\text{max}}}$ becomes the prediction generated data series as the result of approximated calculation.

Fig. 2. The number of failure, and prediction results of the GM(1, 1) model for case 1.

V. Case Study

We present two real cases of reliability prediction to validate the effectiveness of the proposed model with three evaluating criteria. They are the mean square error (MSE), absolute mean error (AME), and absolute mean percentage error (AMPE), which are calculated respectively as

\begin{align}
\text{MSE} &= \frac{1}{n} \sum_{i=1}^{n} e^2(i) \\
\text{AME} &= \frac{1}{n} \sum_{i=1}^{n} |e(i)| \\
\text{AMPE} &= \frac{1}{n} \sum_{i=1}^{n} \left| \frac{e(i)}{x^{(0)}(i)} \right| \times 100\%
\end{align}

where $e(i) = x^{(0)}(i) - \hat{x}^{(0)}(i)$. Three criteria are calculated using the data of final prediction output, and original input.

1) Case 1—Prediction of the Number of Failures in a Product Manufacturing Process: The data used for our study are the number of failure data of new electric products in a manufacturing process. Failures for 100 days have been collected. Because the grey model is developed using the most recent data available, the minimum modeling size is four [27]. We decide the number of data points used to form the grey model by trial and error. In this study, we have tried several data set sizes, and found that when the data set size is seven, the one step ahead prediction models have the best performance. We use the number of failure from the past seven days to form the GM(1, 1) model, and the number of failures for the next day is predicted. The prediction form is called one step ahead prediction sequentially. The one step ahead prediction results from GM(1, 1) based on seven data points are plotted in Fig. 2, from which we can find that GM(1, 1) cannot exactly match the system dynamics, and the prediction error always exists. But the GM(1, 1) model provides an excellent approach to model dynamic systems of product failure. The cubic Hermite spline is then adopted to improve the performance. Fig. 3 depicts the prediction results of 3spGM(1, 1). The prediction results are better than that of GM(1, 1). The extreme effect has been somewhat reduced, and the prediction
The initial values are errors, the PSO algorithm is employed to form P-3spGM(1, 1). The number of failures, and prediction results of the ARIMA model for case 1 are plotted in Fig. 4. From which we can find P-3spGM(1, 1) reduces the prediction error between the actual output, and prediction value. To compare the prediction results of the proposed model with the conventional model, we also present the most widely used ARIMA(1, 0, 0) model to do the same experiment. The prediction results are plotted in Fig. 5. The prediction curve cannot keep track of the original one. The comparative analysis of prediction errors, and the computation time are listed in Table I. The results validated the effectiveness of the proposed model.

2) Case 2—Prediction of the Number of Failures in the Product Operating Testing Process: We now study the failure data from new electronic products in long term operation testing. We use 200 days of operation data. In this study, we tried several data sizes, and found that when the data size is five, the prediction models have the best performance. Therefore, the number of failures from the past five days are used to form prediction models, and the number of failures for the next day is predicted. The prediction models of GM(1, 1), 3spGM(1, 1),
P-3spGM(1, 1), and ARIMA(1, 0, 0) are employed, respectively. For P-3spGM(1, 1), the settings of the initial values are $\varphi_{max} = 20$, $k_{max} = 50$, $c_1 = 0.4$, $c_2 = 0.4$, $\gamma_0 = 0.2$, and $u^{0} = 0.2$. The prediction results of GM(1, 1), 3spGM(1, 1), P-3spGM(1, 1), and ARIMA models are plotted in Figs. 6–9, respectively. Note that we compressed and expanded parts of the figures so that the results can be easy to see. The prediction results of 3spGM(1, 1) are better than those of GM(1, 1). Moreover, the P-3spGM(1, 1) model narrows the prediction error between the actual output, and prediction value of 3spGM(1, 1). The prediction results of ARIMA(1, 0, 0) provide larger prediction error. Table II shows the comparative analysis of prediction errors, and the computation time. The P-3spGM(1, 1) model acquired the most satisfactory prediction results.

### VI. Consideration

The major purpose of this paper is to develop the reliability prediction model based on an improved grey model. As the improved method, the cubic Hermite spline, which is based on the cubic Hermite polynomial, is first presented to enhance the prediction capability of the grey model. The Hermite polynomial has the property that the spline can pass exactly through all the known points, and their first and second derivatives are continuous. Thus, the irrational problems concerning the calculation of derivatives and background values can be resolved by using the 3spGM(1, 1) model. Then, the PSO algorithm is employed to acquire the most accurate prediction. The PSO algorithm is an iterative calculation method that has the strong ability to acquire the most optimal parameter, and reduces the convergent error to a minimum. Therefore, the coefficients of 3spGM(1, 1) can be optimized by the PSO algorithm. However, the drawback of P-3spGM(1, 1) is that the training time increased while GM(1, 1) does not require training. The two real cases of the number of failures of electronic products in manufacturing systems are used to validate the effectiveness of the proposed model. To compare the prediction power of the proposed model to alternatives, we also repeat the experiments using a conventional time-series ARIMA model. The ARIMA model’s successes rely on a law for the distribution of the original series, as well as a large number of data sets. In our study, as the prediction models are established with a small number of data sets, the prediction accuracies of time-series models are unsatisfying. On the contrary, the prediction capability of the proposed model is satisfactory. From the results of accuracy comparison shown in Tables I and II, the P-3spGM(1, 1) model was the most accurate.

### VII. Conclusions

In this paper, we use an improved grey model to develop the reliability prediction model of electronic products in manufacturing systems. The grey model can deliver accurate prediction with the most recent data in the time series. In today’s global market economy, the use of just-in-time (JIT) manufacturing has become one part of a company’s overall strategy [28]. However, when a new product type is developed, we need a large amount of time, manpower, and materials to implement the reliability testing of the product. Most existing reliability models are established based on either experience, or on all training data sets. Therefore, it is often difficult to implement the research, and sometimes the research is not even feasible due to cost consideration or time [29]. If a reliability prediction model can be established with the most recent data sets, we can get the one step ahead prediction values of the number of failures. We can then decide whether to stop or continue this test. Grey system theory, different from Fuzzy logic, is not a theory to establish the analysis model based on experience. It is also different from statistical theory which needs all training data sets to establish the model. The grey model is a very effective method for solving uncertainty problems under discrete data, and incomplete information. It uses the AGO to eliminate the fluctuation in the original data. Thus it can predict the development trend of the system accurately [30], [31]. For manufacturing systems, the proposed
reliability prediction model based on an improved P-3spGM(1, 1) model is very useful.

REFERENCES


