Decision Aiding

Multiple criteria decision making with generalized DEA and an aspiration level method

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Received 30 April 2001; accepted 9 May 2003
Available online 13 August 2003

Abstract

In this paper, we suggest an aspiration level approach using generalized data envelopment analysis (GDEA) and genetic algorithms (GA) in multiple criteria decision making such as engineering design problems. It will be shown that several Pareto optimal solutions close to an aspiration level of decision maker can be listed up as candidates of a final decision making solution by the proposed method. Through the robust design problem, it will be proved also that the aspiration level method using GDEA is useful for supporting a decision making of complex system.

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Keywords: Generalized data envelopment analysis; Aspiration level method; Multiple criteria decision making; Pareto optimal solutions; Robust design

1. Introduction

Many decision making problems can be formulated as multi-objective optimization problems (MOP). There hardly exists the solution that optimizes all objective functions in MOP, and then the concept of Pareto optimal solution (or efficient solution) is introduced [12]. Usually, there exist a number of Pareto optimal solutions, which are considered as candidates of final decision making solution [8]. It is an issue how decision makers decide the final solution from the set of Pareto optimal solutions. Interactive multi-objective optimization methods have been developed to this end [6,12,15]. These methods search a decision making solution by processing the following two stages repeatedly: (1) solving auxiliary optimization problem to obtain a Pareto optimal solution closest to the given aspiration level, and (2) revising their aspiration levels by making trade-off analysis. Problems such as engineering design problems, however, criteria functions cannot be given explicitly in terms of design variables. Under this circumstance, values of criteria functions are
In this paper, we suggest an aspiration level approach to the GDEA method for supporting decision makers to choose a desirable solution among the Pareto optimal solutions. To this end, we firstly propose a method for showing some limited range of Pareto optimal solutions near decision maker’s aspiration level. Decision makers can choose a final solution from the shown Pareto optimal solutions. In this event, we also propose a method for making decision maker’s trade-off analysis easy. In order to help one’s understanding, the proposed method will be explained along an example of a robust design problem. The paper is organized as follows. The following section describes the proposed method and interprets it through a simple example. Section 3 applies our method to a robust design problem, and shows the effectiveness of it. Finally, our conclusions are given.

2. The proposed method

Consider a multi-objective optimization problem as follows:

\[ \text{(MOP)} \]

\[ \begin{align*}
& \text{minimize} & & \mathbf{f}(x) = (f_1(x), \ldots, f_m(x))^T \\
& \text{subject to} & & x \in X = \{ x | g_j(x) \leq 0, \ j = 1, \ldots, l \},
\end{align*} \]

where \( x = (x_1, \ldots, x_n)^T \) is a vector of design variables and \( X \) is the set of all feasible solutions.

For convenience, the following notations for vectors \( \mathbf{a} (= (a_1, \ldots, a_m)^T) \) and \( \mathbf{b} (= (b_1, \ldots, b_m)^T) \) in \( \mathbb{R}^m \) will be used:

\[ \mathbf{a} > \mathbf{b} \iff a_i > b_i, \ i = 1, \ldots, m, \]

\[ \mathbf{a} \succeq \mathbf{b} \iff a_i \geq b_i, \ i = 1, \ldots, m, \]

\[ \mathbf{a} \succeq \mathbf{b} \iff a_i \geq b_i, \ i = 1, \ldots, m \text{ but } \mathbf{a} \neq \mathbf{b}. \]

Generally, unlike traditional optimization problems with a single objective function, there seldom exists an optimal solution that minimizes all objective functions \( f_i(x) \ (i = 1, \ldots, m) \) simultaneously in the problem (MOP). Hence, the concept of an optimal solution based on the relation of Pareto domination is given as follows [12].

usually obtained by some analyses such as structural analysis, thermodynamic analysis, fluid mechanical analysis and so on. These analyses require considerably expensive computational time. Therefore, it takes too much time to obtain a Pareto optimal solution closest to a given aspiration level. This makes the interaction between decision makers and computer almost practically impossible.

In order to overcome this difficulty, multi-objective optimization methods using genetic algorithms (GA) have been studied by many authors [2,4,5,7,13,14,17]. GAs are useful for generating the set of Pareto optimal solutions. Ranking methods [5,7] can be applied to non-convex optimization problems, but need computing until a number of generations to converge to a good approximate efficient frontier. GAs are also effective for generating efficient frontiers, and then decision making can be easily performed on the basis of visualized efficient solutions than the existing methods in less computational time, and overcomes the shortcomings of existing methods as stated in the above. It has been proved particularly in problems with two-objective functions that GAs are well utilized for figuring efficient frontiers, and then decision making can be easily performed on the basis of visualized efficient frontier. GAs are also effective for generating Pareto optimal solutions. But even GAs like the other methods are difficult to visualize Pareto optimal solutions in cases in which the number of objective function is more than 4. Hence, it is troublesome to grasp the trade-off among many objective functions, and decision makers hesitate to choose a final solution from a number of Pareto optimal solutions.

In the circumstance, an important subject is to develop a new method by which computational time can be reduced and also good approximate efficient frontiers can be obtained. Since there is a conflict between these two requirements, it is quite difficult to attain those requirements simultaneously. Thus, we proposed the method using data envelopment analysis (DEA) that evaluates quantitatively each Pareto optimal solution [17]. GAs are useful for generating optimal solutions in cases in which the number of Pareto optimal solutions. But even GAs like the other methods are difficult to visualize Pareto optimal solutions near decision maker’s aspiration level. Decision makers can choose a final solution from the shown Pareto optimal solutions. In this event, we also propose a method for making decision maker’s trade-off analysis easy. In order to help one’s understanding, the proposed method will be explained along an example of a robust design problem. The paper is organized as follows. The following section describes the proposed method and interprets it through a simple example. Section 3 applies our method to a robust design problem, and shows the effectiveness of it. Finally, our conclusions are given.
**Definition 1** (Pareto optimal solution). A point \( \tilde{x} \in S \) is said to be a *Pareto optimal solution* to the problem (MOP) if there exists no \( x \in S \) such that \( f(x) \leq f(\tilde{x}) \).

A final solution to the problem (MOP) may be found from the set of Pareto optimal solutions by existing methods, for example, aspiration level techniques [9], if the value of objective function can be obtained easily. In cases in which it takes much computation time to evaluate objective functions, interactive methods become unsuitable due to time limitation. In the problem (MOP) with two or three objective functions, under this circumstance, roughly figuring efficient frontiers helps decision makers decide the final solution. In order to generate efficient frontiers, several approaches have been studied by many authors [2,4,5,7,13,14]. However, it is difficult in general to represent the whole Pareto optimal solutions even for two-objective function. Moreover, if the number of objective function is larger than three, it is almost impossible not only to visualize all Pareto optimal solutions in the objective space, but also to make a final decision from them.

In this paper, in order to find the most interesting part for decision makers (not the whole of efficient frontier), we combine an aspiration level approach to multi-objective programming and GDEA. For treating constraints, we introduce an augmented objective function using penalty functions imposed on constraints. Here, an augmented objective function of \( f_i \) (\( i = 1, \ldots, m \)) in the problem (MOP) is given by

\[
F_i(x) = f_i(x) + \sum_{j=1}^{l} b_j \times \{ [g_j(x)]_+ \}^a, \quad i = 1, \ldots, m,
\]

where \([y]_+ = \max\{y, 0\}\), \( b_j, \ j = 1, \ldots, l \), is a penalty coefficient and \( a \) is a penalty exponent. \(^1\)

Now, the original problem (MOP) can be converted into a problem to minimize the augmented objective function \((F_1(x), \ldots, F_m(x))\). \( x', \ j = 1, \ldots, p \), are individuals in GA which represent all alternative decision variables. First, we explain the GDEA method [17] for measuring the efficiency of an individual \( x' \in \{x', j = 1, \ldots, p\} \), where \( p \) is the number of individuals. The following problem (P) is to maximize the objective function with respect to \( \Delta \) and \( v \), where \( \Delta \) is a scalar and \( v \) is a vector.

\[
\text{maximize}_{\Delta, v} \quad \Delta
\]

subject to \( \Delta \leq \tilde{d}_j - \sum_{i=1}^{m} v_i (F_i(x') - F_i(x)), \)

\[
j = 1, \ldots, p,
\]

\[
\sum_{i=1}^{m} v_i = 1,
\]

\[
v_i \geq 0, \quad i = 1, \ldots, m,
\]

\[(\varepsilon : \text{sufficiently small number})\]

where \( \varepsilon \) is a sufficiently small number, and \( \tilde{d}_j \) (\( j = 1, \ldots, n \)) is the value of multiplying the maximal component of \((y_{10} - y_{1j}, \ldots, y_{p0} - y_{pj}, -x_{10} + x_{1j}, \ldots, -x_{mo} + x_{mj})\) by its corresponding weight. (For example, if \((y_{10} - y_{1j}, -x_{10} + x_{1j}) = (2, -1)\), then \( \tilde{d}_j = 2\mu_{1j} \) \( x \) is the value of a monotonically decreasing function with respect to the number of generation. \(^2\)

In the GDEA method, the fitness in GA uses the optimal value \( \Delta^* \) to the problem (P), which represents the degree how close an individual \( f(x') \) is to the efficient frontier. It has been shown in [17] that the GDEA method can generate non-convex efficient frontiers owing to the term \( \sum_{i=1}^{m} v_i (F_i(x') - F_i(x')) \). We have a good convergence to efficient frontiers by changing \( \varepsilon \), that is, taking a large \( \varepsilon \) can remove individuals which are located far from the efficient frontier, and also taking a small \( \varepsilon \) can generate non-convex efficient frontiers.

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\(^1\) The values are appropriately decided according to given problems. Since the theoretical estimate of these values is generally difficult, they are given empirically. In the examples of this paper, we set as \( b_1 = b_2 = \ldots = b_l = 1 \) and \( a = 2 \).

\(^2\) Practically, \( x \) is given by \( x(t) := \omega \cdot \exp(-\beta \cdot t), \quad t = 0, 1, \ldots, N \), where \( \omega, \beta \) and \( N \) are positive fixed numbers. \( \omega \) \((= x(0))\) is determined to be sufficiently large, e.g. 10, 10\(^2\) and 10\(^3\). \( N \) (the number of generations until the termination of computation) is given by the time limitation for decision making. For given \( \omega \) and \( N \), \( \beta \) is decided by solving the equation \( x(N) = \omega \cdot \exp(-\beta \cdot N) = 0 \).
It is usually not rational to compute the whole set of Pareto optimal solutions. Therefore, we propose a method for getting a part of Pareto optimal solutions needed for making a final decision. To this end, we combine an aspiration level approach to the GDEA method. Given the aspiration level of decision maker \( f := (f_1, \ldots, f_m) \) and the ideal point \( f^* := (f_1^*, \ldots, f_m^*) \), \(^3\) we formulate the problem (AP) to find Pareto optimal solutions close to the aspiration level:

(AP)

\[
\begin{align*}
\text{maximize} & \quad \Delta - \lambda H(W(f(x^0) - \bar{f})) \\
\text{subject to} & \quad \Delta \leq \bar{d}_j - \sum_{i=1}^{m} v_i(F_i(f^*) - F_i(x^0)), \\
& \quad j = 1, \ldots, p, \\
& \quad \sum_{i=1}^{m} v_i = 1, \\
& \quad v_i \geq e, \quad i = 1, \ldots, m, \\
& \quad (e \text{: sufficiently small number})
\end{align*}
\]

where \( H(y) = \max\{y_1, \ldots, y_m\} \), \( y := (y_1, \ldots, y_m) \) and the matrix \( W \) is diagonal with its elements \( w_i = \frac{1}{\bar{d}_i} \). \( e \) is an appropriately given positive number. \(^4\) For instance, \( \lambda \) is taken as a large number when we search as close Pareto optimal values to the given aspiration level as possible. As a result, the optimal value to the problem (AP) represents the degree how close \( f(x^0) \) is to the Pareto frontier and to the given aspiration level. Therefore, the optimal value to the problem (AP) can be used as the fitness in GA. For example, as is seen in Fig. 1, let \( \bar{f} \) be a given aspiration level and \( f^* \) be the ideal point in the objective space. Then individuals in the neighborhood of Pareto optimal value \( A0 \) have good fitness, and Pareto optimal region shadowed in Fig. 1 are considered to be the most interesting part of decision makers for the final decision which is described below in more detail.

From Pareto optimal solutions in the region, we select representative alternatives using the measures given by

\[
\begin{align*}
A0 &= f(x^0), \\
\bar{x}^0 &= \arg \min_{x^0} \max_{i=1, \ldots, m} w_i(f_i(x^0) - \bar{f}_i), \\
\bar{x}^i &= \arg \min_{x^0} w_i(f_i(x^0) - \bar{f}_i), \quad i = 1, \ldots, m,
\end{align*}
\]

where \( x^i \) is of the set of generated Pareto optimal solutions. Note that the \( \bar{x}^0 \) can be approximately given as the one which gives the maximum among optimal values to the problem (AP) for each \( x^i \). Fig. 1 illustrates the meaning of the alternative \( A0 \) given by the Eq. (1) and the alternatives \( A1 \) and \( A2 \) given by the Eq. (2).

When the aspiration level \( \bar{f} \) is infeasible, as seen in Fig. 1(a), the non-negative values of \( w_i(f_i(x^0) - \bar{f}_i) \), \( i = 1, \ldots, m \), mean the degree of regret for \( x^0 \). Then, solutions to the problem (AP) can be regarded as compromise solutions. If \( \bar{f} \) is feasible, as is in Fig. 1(b), the non-positive values of \( w_i(f_i(x^0) - \bar{f}_i) \), \( i = 1, \ldots, m \), mean the degree of satisfaction for \( x^0 \). Thus, the obtained Pareto optimal values by solving (AP) are satisfactory solutions.

Consider respectively the following cases when the aspiration level is located:

(i) close to the \( f^* \),
(ii) on the efficient frontier,
(iii) on the line the connects \( f^* \) with one of the edges of the Pareto optimal values.

In case of (i), \( \|\bar{f} - f^*\| \) has a non-zero value to some extent because human beings have some threshold for their recognition. Therefore, there is no serious problem in computation of \( w_i(\bar{f}_i - f^*_i) \). Although the obtained Pareto optimal region to this aspiration level is large, this implies that we have to compromise with a large region due to the given greedy aspiration level. In case of (ii), since the aspiration level itself is Pareto optimal, the
obtained solution is located at this level. The decision maker is naturally satisfied with this obtained solution in many cases. If not, however, revise the aspiration level and solve the problem (AP) again. We can treat the case (iii) in a similar way to case (i).

Summarizing our algorithm leads to the following:

**AP Algorithm**

**Step 1 (Initialization)**

Generate $p$-individuals randomly.

**Step 2 (Crossover · Mutation)**

Make $p/2$-pairs randomly among the population. Making crossover for each pair generates a new population. Mutate them according to the given probability of mutation.

**Step 3 (Evaluation of Fitness)**

Find an optimal solution to the problem (AP) for the aspiration level. Its optimal value is adopted as the fitness at the generation. Here, the aspiration level is given prior by decision makers.

**Step 4 (Selection)**

Select $p$-individuals from the current population on the basis of the fitness given by the optimal value to problem (AP). Here, the elitist preserving selection [7] is adopted.

For illustration of the proposed method, consider a simple example with two-objective functions as follows:

\[
\text{minimize } f(x) = (f_1(x), f_2(x)) = (x_1, x_2)
\]

subject to

\[
\begin{align*}
 x_1^3 - 3x_1 - x_2 & \leq 0, \\
 x_1 & \geq -1, \quad x_2 \leq 2.
\end{align*}
\]

To obtain Pareto optimal solutions for the above example, we set parameters; the size of population is 80, the probability of mutation is 0.05, $\lambda(t) = 10 \times \exp(-0.2 \times t)$, $t = 0, \ldots, 29$, $\lambda = 10^2$ and $\epsilon = 10^{-6}$, and the aspiration level is $(0.5, 1)$. Given the aspiration level, we perform the above procedure until 30 generation.

Then, the results are as shown in Fig. 2. Fig. 2(a) shows the true efficient frontier to this problem, which is non-convex and non-concave.  

The horizontal axis and the vertical axis indicate the values of objective functions $f_1$ and $f_2$, respectively. Fig. 2(b) shows the whole Pareto frontier obtained by the GDEA method only. The symbol $\bullet$ represents a Pareto optimal value, and the symbol $\circ$ does individuals except for Pareto optimal values at the generation. Fig. 2(c) shows a part of Pareto frontier generated by our proposed method. Here, the aspiration level is $(0.5, 1)$, and the ideal point in this example is $(-1, -2)$.

As is seen in Fig. 2, almost of all individuals are concentrated at the neighborhood of intersection of the efficient frontier and the line segment between the aspiration level point $(0.5, 1)$ and the ideal point $(-1, -2)$. Hence, the obtained Pareto optimal values are candidates for final decision making solution with respect to the present aspiration level, and hence decision makers choose one from them.

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5 Let $E$ be an efficient frontier set in $\mathbb{R}^n$ and let $\mathbb{R}^n_+$ be the non-negative orthant in the objective space. Then we say the efficient frontier to be convex if $(E + \mathbb{R}^n_+)$ is a convex set. Otherwise, the efficient frontier is said non-convex.
3. Robust design: Welded beam optimization problem

In this section, we apply it to an engineering design problem, which is cited from [10,11] and reformulated on the basis of Arakawa et al.'s Fuzzy Numbers method.

As was shown by Rao [11] and Arakawa et al. [1], the robust design can be formulated as multi-objective optimization problem with more than three objective functions. They used the weighted-sum method to generate Pareto optimal solutions, but did not perform a trade-off analysis. Chen et al. [3] showed that the set of Pareto optimal solutions is non-convex and utilized the weighted Tchebychef method instead of the weighted-sum method. Their method aims to explore some Pareto optimal solution for the given aspiration level, but not to grasp trade-off among objective functions. Since the trade-off analysis is an important task in multi-objective optimization, the above methods do not work sufficiently. Through this example, it will be shown that the proposed method can solve the problems rising from the existing methods.

3.1. Formulation

The Cantilever Beam, shown in Fig. 3, is supported at the end load \( P = 26689 \text{ N} \) at the tip other end with its extended length \( L = 0.3556 \text{ m} \). The depth of the weld \( h(m) \), the length of the weld \( l(m) \), the height \( t(m) \) and thickness \( b(m) \) of the beam, the error of the weld’s depth \( h^R(m) \) and the error of the weld’s length \( l^R(m) \) are treated as design variables. The constraints are the shearing stress \( g_1 \), the stress \( g_2 \), the deviation \( g_3 \) and the buckling load \( g_4 \). Under these conditions, the problem is to minimize the cost welding a beam \( f_1 \), to maximize the robustness with respect to the cost \( f_2 \), to maximize the robustness with respect to the fluctuation of the beam’s length \( f_3 \), to maximize the error with respect to the weld’s depth \( f_4 \) and to maximize the error with respect to the weld’s length \( f_5 \).

Then, we can summarize our problem as follows.

**Design variables**

\[ h(m), l(m), t(m), b(m), h^R(m), l^R(m). \]
Constraints
\[ g_1 := \tau \leq \tau_a, \]
\[ g_2 := \sigma \leq \sigma_a, \]
\[ g_3 := \delta \leq \delta_a, \]
\[ g_4 := P \leq P_c, \]
\[ g_5 := h^2 + h < b, \]
\[ g_6 := h^2 \leq h, \]
\[ g_7 := l^2 \leq l. \]

Objective functions
Minimize \( f_1 := 68216.1 h^2 l + 2970.8 tb(L + l), \)
Minimize \( f_2 := 136432.2 hlh^2 + (68216.1 h^2 \]
\[ + 2970.8 tb)l^2, \]
Minimize \( f_3 := 2970.8tbL^2, \]
Maximize \( f_4 := h^2, \]
Maximize \( f_5 := l^2. \]

Constants
\[ \tau = \sqrt{\tau^2 + \tau'^2 \frac{l}{R} + \tau'^2}, \quad \tau' = \frac{P}{\sqrt{2hl}}, \]
\[ \tau'' = \frac{MR}{J}, \quad M = P\left( L + \frac{l}{2} \right), \]
\[ R = \sqrt{\frac{l^2 + (h + t)^2}{4}}, \quad J = \sqrt{2hl}\left( \frac{l^2}{6} + \frac{(h + b)^2}{2} \right), \]
\[ \sigma = \frac{6PL}{t^2b}, \quad \delta = \frac{6PL^3}{Et^3b}, \]
\[ P_c = \frac{4.013 \sqrt{E l \pi}}{L^2} \left( 1 - \frac{t}{2l} \sqrt{\frac{E l}{\pi}} \right), \quad \tau = \frac{tb^3}{12}, \]
\[ \alpha = \frac{Gb^3}{3}, \quad E = 206.843 \text{ (GPa)}, \]
\[ G = 82.7371, \quad L^R = 0.01L(m). \]

Then, the procedure for deciding the final solution according to the present aspiration level is as follows.

**Stage one**
Generate Pareto optimal solutions by using the AP algorithm introduced in Section 2.

**Stage two**
Pick up several Pareto optimal solutions from the set of Pareto optimal solutions obtained in stage one as representative alternatives to the final decision making solution by the Eqs. (1) and (2). In this example, representative alternatives are as follows:
\[ A0 = (2.12170, 0.42431, 0.01413, 0.00134, 0.06519), \]
\[ A1 = (2.09579, 0.41857, 0.01386, 0.00128, 0.06526), \]
\[ A2 = (2.15548, 0.38580, 0.01436, 0.00108, 0.06076), \]
\[ A3 = (2.09679, 0.41975, 0.01385, 0.00128, 0.06527), \]
\[ A4 = (2.17270, 0.44531, 0.01441, 0.00152, 0.06623), \]
\[ A5 = (2.20449, 0.44352, 0.01444, 0.00110, 0.07026). \]

Fig. 4 represents the above alternatives normalized by
\[ (f_1', f_2', f_3', f_4', f_5') := \left( \frac{f_1 - f_1}{f_1}, \frac{f_2 - f_1}{f_2}, \frac{f_3 - f_3}{f_3}, \frac{f_4 - f_3}{f_4}, \frac{f_5 - f_5}{f_5} \right). \]

These values mean the degree how much \( x \) does not attain to the aspiration level of decision maker. The fact that the values are all negative in this example shows that the aspiration level is mild as a whole, and thus the representative alternatives achieve the given aspiration level.

![Fig. 4. Representative alternatives to the aspiration level.](image-url)
The alternative $A_0$ is the Pareto optimal value closest to the aspiration level, and is regarded as the first candidate of decision making solution. However, if decision makers are not satisfied with the alternative $A_0$, they search a more preferable alternative in a neighborhood of the alternative $A_0$. Suppose that they want to improve some of objective functions at the alternative $A_0$. Since Pareto optimal solutions cannot improve all objective functions simultaneously any more, decision makers have to agree with some sacrifice for some of other objective functions. How much some of objective functions should be improved and how much some of other objective functions should be sacrificed? This is the trade-off analysis.

Stage three

This stage decides the direction to which decision makers want to move. Suppose that decision makers want to improve $f_2$ at the alternative $A_0$, because the degree of attainment for $f_2$ is worst among those of all objective functions. Then, among the representative alternatives obtained in stage one, $f_2$ of the alternatives $A_1$, $A_2$ and $A_3$ are better than the alternative $A_0$. Since the value of $f_2$ at $A_3$ is not so improved, it is preferable to move to a direction between $A_1$ and $A_2$. Thus, list all Pareto optimal values for which the value of $f_2$ is between the alternative $A_1$. Fig. 5 illustrates those Pareto optimal values sorted in descending order of the value $f_2$.

It usually happens in multiple criteria decision making problems that improving an objective function makes other objective function much worse than expected. Hence, proceed to the next stage on the basis of the trade-off relation between objective functions.

Stage four

With the obtained result in stage three, ask decision makers’ demands for other objective functions.

For example, suppose that decision makers want to keep $f_4$ and $f_5$ within some allowable range:

(i) the degree of attainment for $f_4$ should be less than 0.2.
(ii) the degree of attainment for $f_5$ should be less than 0.25.

Then, pick up ones satisfying these requests of decision makers from the alternatives obtained in stage three. Fig. 6 represents these alternatives in

![Fig. 5. Alternatives to decision maker's requests.](image)
order of the value of $f_2$. Decision makers can finally choose a solution among them. If they cannot decide their final solution, we go back to stage one setting a new aspiration level.

4. Conclusions

In this research, we suggested a new method which utilizes aspiration level and generalized data envelopment analysis for supporting a multiple criteria decision making. Also, in order to show the effectiveness of our method, we applied it to an engineering design problem considering the robustness of design variables. As a result, it was shown that (i) the proposed method can provide the most interesting part for decision makers of Pareto optimal solutions close to the given aspiration level; (ii) it is possible to grasp the trade-off among objective functions in this area by selecting representative alternatives $A_0$ and $A_i$, $i = 1, \ldots, m$, by the Eqs. (1) and (2); (iii) the final solution can be made on the basis of the provided information. Our aim of this paper is to propose a method for supporting decision making by showing some range of Pareto optimal solutions near decision maker’s aspiration level. Decision makers can choose the final solution from the shown Pareto optimal solutions. In order to make one’s understanding easy, we explained the proposed method along an example in this paper. However, we made further applications, in which the effectiveness of the method was also observed. These results were omitted here due to page limitation. See [18,19] for more details.

Moreover, it is noted that the values of criteria functions and a weight to each criteria are normalize technically in practical computation, and thus, the selection of aspiration level does not influence on the proposed method. It is possible to consider more than two aspiration levels simultaneously. The representative alternatives can be selected among the intersection of the sets of near Pareto optimal solutions to an each aspiration level. In this case, however, much expensive computation is needed, because the number of solving (AP) increases in proportion to the number of aspiration levels. Therefore, it is desirable that decision maker should decrease the number of initial aspiration levels as much as possible. In many practical engineering design problems, the number of initial aspiration levels can be set as one or at most a few, because engineering designers know roughly about their aspiration levels in advance through their experiences.

The proposed method can be effectively applied to a wide range of complex optimization problems such as engineering design problems and reinforcement learning with unknown value functions in machine learning.

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