Virtual Coupling Control for Dynamic Bipedal Walking

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Abstract

In our previous works, we have proposed some system augmentation methods for dynamic bipedal walking in order to realize variable walking patterns in real-time, however, the robust performance of the methods have not been discussed yet. Based on the observation we propose a new coupling control law considering flywheel’s effect for robustness. In this paper the validity of the methods is analyzed and investigated by numerical simulations and experiments.

1 Introduction

It has been established that a suitably designed unpowered biped robot can walk down a gentle slope utilizing only gravity effect and generates a stable periodic gait. It is called “passive dynamic walking” originally studied by McGeer[7]. The passive walker utilizes its physical dynamics and creates energy effective walking pattern automatically. We have studied some application methods of gait synthesis for active walkers on the floor based on the passive dynamic walking. Our main purposes about biped robot’s gait synthesis and motion planning are the following:

1. Realization of safe control against human-being and outside environment,
2. Energy-effective gait design based on passive dynamic walking,
3. Real-time computation and synthesis of variable walking pattern w.r.t. robot’s energy level.

In order to solve above problems, in our previous work, some application methods of Passive Velocity Field Control(PVFC) for passive walking robots have been proposed[1, 8]. PVFC is a mathematical control law for mechanical systems so that the augmented system becomes passive and the velocity of system converges to a desired velocity field. The previous work reported that PVFC is effective for the bipedal walking, however, some problems have been left yet. The passive velocity field controller requires very heavy amount of calculation and has many tuning parameters. Furthermore, due to the tracking control nature in PVFC the walking speed was not able to be changed easily by external forces, in other words, the controller makes very large reaction force against environment to track the desired velocity fields.

Based on the observation, in this paper we propose a virtual coupling control law via system augmentation. By the effect of the proposed method, the robot system combined with a virtual flywheel can be regarded as an impact-less and smooth dynamical system, and variable walking pattern is also able to be realized. Furthermore, tuning the control parameter we can make the active walking robust for disturbances. The validity of above proposed methods are examined by numerical simulations and experiments.

2 The Compass Gait Biped Model

Fig. 1 shows the model of compass-gait biped robot. The robot consists of two rigid bars and 3 point mass. The robot with suitable parameters can walk down without any control forces on a gentle slope[5, 6] and this is also able to walk on the level ground by the effect of “virtual gravity field" toward the horizontal direction (Fig. 1). The robot can exhibit passive dynamic walking virtually on the level ground utilizing only virtual gravity effect.

The dynamic equation of the robot is given by

\[ M(\mathbf{\theta})\ddot{\mathbf{\theta}} + C(\mathbf{\theta}, \dot{\mathbf{\theta}})\dot{\mathbf{\theta}} + g(\mathbf{\theta}) = \mathbf{\tau} + \mathbf{\tau}_e \]  

(1)

where \( M(\mathbf{\theta}) = [2 \times 2] \) is the inertia matrix, \( C(\mathbf{\theta}, \dot{\mathbf{\theta}}) = [2 \times 2] \) is the colisir matrix and \( g(\mathbf{\theta}) = [2 \times 1] \) is the gravity term. And \( \mathbf{\theta} = [\theta_1 \ \theta_2]^T \) is configuration of the robot.
The virtual energy is defined as
\[ E_V(\theta, \dot{\theta}, \phi) = \frac{1}{2} \dot{\theta}^T M(\theta) \dot{\theta} + P_V(\theta, \phi) \]  
(2)
where \( P_V \) is the virtual potential energy and given by
\[ P_V(\theta, \phi) = (m_H l + m l + m a) \cos \theta_1 \frac{g}{\cos \phi} - m b \cos \theta_2 \frac{g}{\cos \phi} . \]

\( E_V \) is kept constant in the case without any external energy sources. In the simulations, we set the physical parameters in Fig. 1 as \( m \) = 5.0, \( m_H = 10.0 \) [kg], \( a = 0.50 \), \( b = 0.50 \) and \( l = a + b = 1.0 \) [m]. Please see [2, 3] for the detail.

![Figure 1: Model of the compass-gait biped](image)

### 3 Virtual Coupling Control

In this section, we introduce a coupling control law via system augmentation for variable walking pattern w.r.t. the energy levels. The basic concept is to regard hybrid dynamical systems as impact-less (smooth) dynamical systems virtually and the energy information is used for the control. Fig. 2 shows the concept of the coupling control. In order to make the robot system passive and smooth, we introduce a virtual flywheel in computer. Then the biped robot combined with the flywheel becomes passive and smooth as an augmented mechanical system.

#### 3.1 Augmented mechanical system

The dynamic equation of the biped robot is given by (1) and that of flywheel is given by
\[ M_f \ddot{\theta}_f = \tau_f . \]  
(3)

The augmented mechanical system is defined by the following equation.
\[ \begin{bmatrix} M(\theta) & M_f \\ M_f & 0 \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\theta}_f \end{bmatrix} + \begin{bmatrix} C(\theta, \dot{\theta}) \\ 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\theta}_f \end{bmatrix} = \begin{bmatrix} \tau \\ \tau_f \end{bmatrix} \]  
(4)

We denote the augmented mechanical system as:
\[ M^a(q) \ddot{q} + C^a(q, \dot{q}) \dot{q} + g^a(q) = \tau^a + \tau^a_e \]  
(5)

where \( \dot{q} = [ \theta^T \ \theta_f ]^T \) is the configuration of the augmented mechanical system. It is clear that the augmented mechanical system is a decoupled one, and its coupling control inputs are determined as a function of states by the virtual coupling controller explained in the following section.

![Figure 2: Virtual coupling control](image)

#### 3.2 The coupling control law

The control input for the augmented mechanical system is given by
\[ \tau^a = \begin{bmatrix} \tau(\theta, \phi) \\ 0 \end{bmatrix} + S \ddot{q} \]  
(6)

where \( \tau(\theta, \phi) \) is transformed torque of the virtual gravity effect and given by
\[ \tau(\theta, \phi) = \begin{bmatrix} (m_H l + m l + m a) \cos \theta_1 \\ -m b \cos \theta_2 \end{bmatrix} \frac{g}{\cos \phi} . \]
Using this method, we can get constant-like and flat torques (See Fig. 6 (c)). This property is very important for biped robots from the ZMP point of view. And the structure of the coupling control term $S\dot{q}$ is given by:

$$S\dot{q} = \begin{bmatrix} -s^T S \end{bmatrix} \dot{q}$$

(7)

where $S = [3 \times 3]$ and $s = [2 \times 1]$. Please notice that the matrix $S$ has a skew-symmetric structure and the control does not affect the kinetic energy. Our proposed method is based on this structure. The vector $s$ is given by:

$$s = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$

(8)

where $s_1$ and $s_2$ are assumed to be constant in this paper though it can be determined by introducing a proper Riemann metric for the augmented system. The basic concepts of the virtual coupling control is explained in the following.

### 3.3 Rationale and properties of VCC

By the effect of the proposed method, a hybrid dynamical system can be regarded as an impact-less dynamical system by resetting $\theta_f$ after the impact which reduces the total energy. The augmented virtual energy is defined as:

$$E_v^\alpha(q, \dot{q}, \phi) = E_V(\theta, \dot{\theta}, \phi) + K_f$$

$$= E_V(\theta, \dot{\theta}, \phi) + \frac{1}{2} M_f \dot{\theta}_f^2$$

(9)

and this is kept constant value without any external forces $\tau_e^a$, because of the skew-symmetric structure of $s$, that is,

$$\frac{d}{dt} E_v^\alpha(q, \dot{q}, \phi) = \dot{q}^T S \dot{q} = 0 \quad (\text{if } \tau_e^a = 0)$$

(10)

holds. Even if the augmented mechanical system exhibits passive property against outside environment, its energy is dissipated due to collisions to the floor. In order to compensate for the effect, the velocity of the flywheel is reset by

$$\dot{\theta}_f^+ = \sqrt{\frac{2}{M_f} (E_d - E_v^\alpha)}$$

(11)

where $E_d$ is a desired augmented virtual energy which is saved just before the transition instant and $E_v^\alpha$ is the virtual energy of the robot just after the impact. By the effect of this reset algorithm, the dissipation of energy can be canceled and the closed system is regarded as an impact-less and smooth system. The virtual energy of the augmented system is the total of robot’s and flywheel’s, and varies while the external force exists. This feature can be used for variable pattern walking explained in the next section.

### 4 Variable Gait Synthesis

As one of methods to use the property in the previous section, we propose a virtual slope driven control w.r.t. energy levels of the system. The virtual slope is given by

$$\phi = \phi^* + \alpha \Delta E_v^a$$

(12)

where $\Delta E_v := E_V - E_V^0$. $\alpha > 0$ is the feedback gain. $\phi^*$ and $E_V^0$ are target virtual slope and energy respectively. Then the virtual slope $\phi$ is driven in real-time w.r.t. its energy levels.

Fig. 3 shows the simulated results of virtual coupling control. The control parameters are set as $s = [0.1, 0.1]^T$, $\phi = 0.03$ [rad] and $M_f = 1.0$ [kg-m²]. The walking pattern converges 1-periodic stable limit cycle, and the velocity of the virtual flywheel returns to the initial condition again after one cycle. It can be shown that $s_1 = s_2$ is a necessary condition for the augmented system to have a 1-periodic limit cycle. (The proof is omitted due to space limitation) Fig. 4 shows the simulated result of a 2-periodic steady limit cycle. We set the control parameter as $s = [0.2, 0.0]^T$ and change $s_1$ and $s_2$ every heel-strike. This condition is considered that the coupling control parameter $s_1$ and $s_2$ are connected with right and left legs, not stance and swing legs. Sometimes the augmented system exhibits period-doubling bifurcation and chaos according to tuning of $s$.

Fig. 6 shows the simulated results with external force. We set the control parameters as $\phi = 0.03$ [rad], $s = [0.01, 0.01]^T$, $M_f = 1.0$ [kg-m²] and $\alpha = 0.02$. The robot is pushed forward by a force 2.0 [N] at the hip position for 0.3 [sec](from 0.9 to 1.2 [sec]). From (d), (e) and (f), the total energy of the augmented system is the sum of virtual energy of the robot and kinetic energy of the flywheel, and changes only when the robot is pushed. From (b) and (c), it can be seen that the control inputs are almost constant after virtual slope changed. Form (a), The walking pattern slides and converges a new pattern gradually.
Figure 3: Simulated results of virtual coupling control

Figure 4: Simulated results of 2-periodic gait

Figure 5: Robust stability

Figure 6: Simulated results of variable gait
5 Robust Stability

The virtual passive walker is not robust against external forces. This nature is identical with that of original passive walker. In the last section, the virtual flywheel is introduced to cancel the impulsive energy changes. In this section we analyze the effect of coupling force for robust stability.

Fig. 5 shows the bounds that the robot can continue walking against the external forces. We set $\phi = 0.03$ [rad], and push the robot forward from 0.2 to 0.5 [sec] after heel-strike. From the results, the range that the walking can be continued becomes large as the parameter $s_1 = s_2$ become large. This implies that we are able to improve the robust stability by means of selecting parameter $s$. The more detail analysis is left in the future work.

6 Experiments

In order to check the validity of the proposed method, we constructed an experimental system. In Fig. 7 we show the experimental setup. The robot has straight legs of the same length and three DC motors with encoders at the hip position. The ankle joints are driven by tinning belts. Because of the symmetric structure the motion is constrained to sagittal plane driven by tinning belts. The ankle joints are identified off-line as:

- The dynamic parameters of the robot were identified off-line as: $m = 0.40$, $m_H = 3.0$ [kg], $a = 0.215$, $b = 0.465$ and $l = 0.68$ [m]. The ankle joint of the swing leg is controlled by PID controller in order to keep the foot posture horizontal.

Since the proposed methods are so called model matching control, the controls are not robust for uncertainty. In this research, we use model following control of the motion generated by VIM(Virtual Internal Model) which is a reference model driven by sensory information. By the use of VIM, the uncertainties of identification which is crucial factor in the case of model matching control can be compensated for. The dynamic equation of VIM with virtual control input is given by

$$
\dot{\theta} - C(\theta, \dot{\theta}) + g(\theta) = \ddot{\theta}(\theta, \phi).
$$

And the resultant control input for model following control of the real robot is given as

$$
\tau = \ddot{\theta} + C(\theta, \dot{\theta}) + g(\theta) + K_P(\theta_d - \theta) + K_I \int (\theta_d - \theta) dt.
$$

At $T_r$, and $T_e$ instances when the stance leg touches and detaches the ground. And then the effect of the reaction moment will disappear.

According to the switching control, the ZMP can be controlled forward till $t = T$ [sec] after transition where $T_r$ and $T_e$ are instances when the ZMP should be shifted forward just after the transition in order to cancel the reaction force from the floor. Based on the observation, we propose an easy control law for ZMP via switching $\phi$. The ZMP can be shifted forward the ankle joint by the effect of change of ankle torque's direction. In our method, this is identical with switching the signal of $\phi$. And we propose $\phi$ switching scheme as

$$
\phi = \begin{cases} 
\phi_1 < 0 & (T_r \leq t < T) \\
\phi_2 > 0 & (T \leq t < T_e).
\end{cases}
$$

According to the switching control, the ZMP is always at behind the ankle joint due to the reaction torque. Since our method provides one-directional ankle torque, the ZMP is always at behind. So at the transition, since the robot is strongly affected by the reaction moment from the floor, the ZMP will be shifted to the heel and walking cannot be continued. Thus the ZMP should be shifted forward just after the transition in order to cancel the reaction force from the floor. Based on the observation, we propose an easy control law for ZMP via switching $\phi$. The ZMP can be shifted forward the ankle joint by the effect of change of ankle torque's direction. In our method, this is identical with switching the signal of $\phi$. And we propose $\phi$ switching scheme as

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Fig. 8 shows the ankle's structure of the experimental machine. We can see that if the stance leg moves forward, the ZMP is at behind the ankle joint due to the reaction torque. Since our method provides one-directional ankle torque, the ZMP is always at behind. So at the transition, since the robot is strongly affected by the reaction moment from the floor, the ZMP will be shifted to the heel and walking cannot be continued. Thus the ZMP should be shifted forward just after the transition in order to cancel the reaction force from the floor. Based on the observation, we propose an easy control law for ZMP via switching $\phi$. The ZMP can be shifted forward the ankle joint by the effect of change of ankle torque's direction. In our method, this is identical with switching the signal of $\phi$. And we propose $\phi$ switching scheme as

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Fig. 9 shows the experimental results. In the experiment the control period was set to 2.0 [msec] and the angular velocity was measured through a filter whose transfer function is $70/(s + 70)$ for each actuator. To implement the control law, we used RTMATX[9]. The control parameters were set as $\phi_1 = -0.0056$ [rad], $\phi_2 = 0.014$ [rad], $T = T_r + 0.05$ [sec] and $s = [0.01, 0.01]^T$.
ity $\dot{\theta}_f(0) = 2.0$ [rad/sec].

From (a) and (b), it can be seen that the walking is realized under the condition that the virtual coupling force is exerted between walking robot and flywheel. From (c), although virtual energy is not kept constant after transition, constant energy is realized in other situation. Because the angular velocity is reset at heel-strike, virtual energy varies for a short time after transition.

In the previous works [3, 4], we reported that the experiment cannot be realized because the heel-strike is not inelastic in the real world, however, by the effect of $\phi$ control for ZMP we could solve the transition problem.

7 Conclusions

In this paper, we proposed a virtual coupling control to realize variable walking gait based on virtual passive dynamic walking and investigated the validity of proposed method by numerical simulations and experiments. We have already applied the proposed method for kneeed biped robot in numerical simulations. Now we are testing the validity of the method using an experimental machine which has knees and upper part of the body.

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References