Abstract—This paper describes a blind crosstalk channel identification method and its application to Discrete MultiTone (DMT)-based Digital Subscriber Line (DSL) systems. Cyclostationarity inherited in these systems is exploited to isolate effects of different sources, which allows separate selection and identification of crosstalk channels. Second-order statistics-based source separation is performed by controlling part of the cyclic-extension in the transmitted DMT block. The minimum changes required by this method facilitate its implementation in current DSL systems. All mentioned theoretical derivations are illustrated by numerical results obtained with measured DSL channels.

Index Terms—DSL, DMT, crosstalk, blind estimation.

I. INTRODUCTION

Crosstalk channel identification has known increased attention in high rate Digital Subscriber Line (DSL) systems. Utilizing vectored techniques in these systems facilitates crosstalk mitigation, which allows very high throughputs to be attained [1]. However, obtaining the theoretically achievable rates requires accurate identification of the channel state.

Usual MIMO channel estimation methods cannot be easily adapted to crosstalk channel identification in coordinated DSL systems. For instance, higher-order statistics-based source separation methods require for each source a different distribution nature, whereas in DSL systems, all sources have the same natural distribution due to Discrete MultiTone (DMT) modulation with a large number of subcarriers [2]. A least-square method in [3] performs crosstalk identification considering that transmitted crosstalk signals are provided at the receiver by a third party. In [4], crosstalk identification is carried out by an expectation-maximization algorithm. An alternative method in [5] identifies Near End CrossTalk (NEXT) by searching the best match between the crosstalk Power Spectral Density (PSD) and a base-set of predefined crosstalk PSDs. The search method in [5] is implemented by a matching pursuit algorithm in [6], and an MSE algorithm in [7]. The previous methods in [5], [6] and [7] are restricted to NEXT identification, and require a large base-set for accurate identification. In [8], Far End CrossTalk (FEXT) identification is proposed using training signals. Such approach requires sequential reinitialization of the lines each time a new line is activated. Based on source precoding, methods [2] and [12] allow channels identifications to be performed separately, where sources are uniquely precoded to have separate effects.

Our approach to carry out crosstalk identification is based on cyclostationarity of the signals in DMT-based DSL systems. In order to estimate a certain crosstalk channel, the effect of the related crosstalk source is isolated while the effects of the other sources are eliminated. This is possible by controlling the cyclic-extensions in the transmitted blocks of the corresponding crosstalk source.

Following this approach, multi-source channel identification is split into the identification of single-source channels. This makes possible the application of low complexity single-source channel identification methods, e.g. [13]. Additionally, this method enables the system to select only the crosstalk channels that have to be identified when a new line is activated. That can bring to a considerable save in computations as usually limited channels need identification.

Channel estimation is proposed in the time domain, from which the frequency response is available by FFT. Time domain estimation can be efficient in terms of the number of estimated variables [12] when the effective channel taps in the time domain are notably fewer than the number of subcarriers in the frequency domain. This is typically the case in DSL systems where the number of subcarriers can reach 4096.

The rest of the paper is arranged as follows. The system model is presented in Section II. In Section III, we derive the identification method for a general DMT system. Application to DSL systems is described in Section IV. Section V shows simulation results for crosstalk channel estimation. Finally, Section VI draws the conclusions of this contribution.

II. SYSTEM MODEL

In DMT-DSL systems, data are firstly converted from serial to parallel form. Bits allocated to each subchannel are grouped and mapped into symbols using multi-level constellation encoders. Symbols are then multi-carrier modulated through an IFFT. Cyclic-Extensions (CE), including Cyclic-Prefix (CP) and Cyclic-Suffix (CS), are appended to the beginning and end of the IFFT block respectively. The length of the DMT block is then \( P = M + L \), where \( M \) is the size of the IFFT and \( L \) is the CE length.

The following conditions are assumed to hold in our model:

A1) Encoded symbols are uncorrelated, with variance \( \sigma^2 \).
A2) Noise is wide sense stationary, either white or colored.
A3) Noise and source signals are statistically independent.

Cyclic extensions are implemented with the cyclic repetition of part of the IFFT output, as shown in Fig.1. Therefore periodicity is induced in the second-order statistics of the transmitted signals [9], [15]. In fact, the transmitted signal autocorrelation appears at delays \( \tau = \pm M \) as a pulse of length \( L \) and period \( P \), i.e., under assumption A1:

\[
R_x(n, \tau) = \mathbb{E} \{ x(n)x(n-\tau)^H \} = \sigma_x^2 [ \delta(t) + \delta(t-M) \sum_{l=0}^{L-1} \delta(n-l) + \delta(t+M) \sum_{l=-M}^{0} \delta(n-l) ]
\]

where \( x \) is the transmitted signal and superscript \( H \) stands for conjugate transpose. An example given in Fig.2 for \( M=12 \) and \( L=3 \) shows periodic pulse of length 3 at \( \tau = \pm 12 \).

As a periodic function, the time-varying autocorrelation admits a Fourier series representation known as cyclic autocorrelation. Each Fourier coefficient is defined at the \( k \)'th cyclic frequency as:

\[
\hat{R}_x(k, \tau) = \frac{1}{P} \sum_{n=0}^{P-1} R_x(n, \tau) e^{-j \frac{2\pi nk}{P}}
\]

Cyclic autocorrelations obtained for (1) exhibit the form of:

i) an impulse located at \( \tau=0 \) and \( k=0 \), i.e., the Fourier transform of the constant stationary process.

ii) a sinc function at \( \tau = \pm M \), with zeros located at \( k \) values that are integer multiples of the \( P/L \) ratio.

At the receiver, after filtering by a linear FIR channel \( h \), the received signal autocorrelation can be expressed as:

\[
R_r(n, \tau) = \sum_{u,l} h(u) R_x(n-l,u) e^{-j \frac{2\pi nk}{P}} + R_w(n, \tau)
\]

where \( w \) represents the noise. Linear time-invariant filtering is known to conserve inputs cyclostationarity [9]. This appears in Fig.3 where the signal from the previous example has been filtered by \( h=[1 -0.5 \ 0.2]^T \). Cyclic autocorrelation zeros are indeed located at integer multiples of the \( P/L \) ratio.

Fig.1. Autocorrelation of a DMT block at a delay equivalent to one IFFT block. Cyclic extensions and their data copies are also represented.

Fig.2. Example illustrating periodicity in transmitted signals autocorrelation.

III. IDENTIFICATION METHOD

A. Source separation condition

In this subsection, the previous cyclostationary behavior is employed to propose a general source separation method for a DMT system with \( M_T \) sources. Some specific aspects of DSL vectored transmission will be discussed in the next section.

For source \( i \), where \( i \in \{ 0, \cdots, M_T-1 \} \), consider \( P \) as the length of the DMT block, and \( L_i \) as the length of the autocorrelation pulse as depicted in Fig.1. The received signal contains contributions from all the sources. Nevertheless, at a given cyclic frequency \( k \), the cyclic autocorrelation matrix of the received signal includes only the effect of source 1 if the following condition is met for every integer \( c_1 \) and any integer \( c_2 \):

\[
L_i \times k \neq c_1 \times P \quad i = i_1 \quad (4.1)
\]

\[
L_i \times k = c_2 \times P \quad \forall i \neq i_1 \quad (4.2)
\]

This can be illustrated by an example of one receiver and two sources with \( P=15 \), \( L_1 = 3 \) and \( L_2 = 5 \). Received cyclic autocorrelations of the first source are depicted in Fig.3. A cyclic autocorrelation matrix computed at \( k=3 \) (\( k=5 \)) includes only effect of the first (second) source, since the second (first) source has zero contribution at this value of \( k \).

B. Channel estimation

In this subsection, source separation based on cyclic autocorrelations is applied to identify a selected channel. Consider a linear time invariant MIMO channel represented by the \( (M_T \times M_T) \) channel matrix \( h(l) = \{ h_{ji}(l) \} \), where \( h_{ji}(l) \) denotes the \( l \)th tap of the Channel Impulse Response (CIR) between input \( i \) and output \( j \). The CIR is represented by a number of taps equal to \( L_k \). The relationship between the vector of system inputs \( x(n) \triangleq [x_0(n) \cdots x_{M_T-1}(n)]^T \) and the
vector of system outputs \( r(n) \) can be modeled as:
\[
r(n) = \sum_{l=0}^{L-1} h(l)x(n-l) + w(n)
\]
where \( w(n) \) is a white noise process. Without loss of generality, the received signal at terminal \( j \) is expressed by:
\[
r_j(n) = \sum_{l=0}^{L-1} y_{j,l}(n) + w_j(n)
\]
where \( y_{j,l}(n) = \sum_{l=0}^{L-1} h_{j,l}(l)x_j(n-l) \). Then, under assumptions A1-3 the autocorrelation of (6) is given by:
\[
R_{r_j}(n,\tau) = \sum_{l=0}^{M-1} R_{y_{j,l}}(n,\tau) + R_w(n,\tau)
\]
from which the cyclic autocorrelation is:
\[
\tilde{R}_{r_j}(k,\tau) = \frac{1}{P} \sum_{l=0}^{P-1} \sum_{n=0}^{M-1} R_{y_{j,l}}(n,\tau) e^{-j\frac{2\pi kn}{P}} + \tilde{R}_w(k,\tau)
\]
By satisfying (4) with \( i_i \) as the desired source, (8) can be rewritten as:
\[
\tilde{R}_{r_j}(k,\tau) = \tilde{R}_{y_{j,i_i}}(k,\tau) + \tilde{R}_w(k,\tau)
\]
Limiting the crosstalk to the effect of \( i_i \) in (9) enables the estimation of \( h_{j,i_i} \). Hence, applying (3) in (9) and taking \( k \neq 0 \):
\[
\tilde{R}_{r_j}(k,\tau) = \frac{1}{P} \sum_{l=0}^{P-1} \sum_{n=0}^{M-1} h_{j,i_i}(l) R_j(n,u) h_{j,i_i}(u-\tau+l) e^{-j\frac{2\pi kn}{P}} e^{-j\frac{2\pi kl}{P}}
\]
\[
= \sum_l h_{j,i_i}(l) e^{-j\frac{2\pi kl}{P}} \left( \frac{1}{P} \sum_{u=0}^{P-1} R_j(n,u) e^{-j\frac{2\pi ku}{P}} h_{j,i_i}(u-\tau+l) \right)
\]
\[
= \sum_l h_{j,i_i}(l) e^{-j\frac{2\pi kl}{P}} \left( \tilde{R}_j(k,\tau-l) * h_{j,i_i}(l-\tau) \right)
\]
where \(*\) stands for the convolution operation. Noise has not been taken into account in the formulation of (10), since it has no effect at cyclic frequency \( k \neq 0 \) w.r.t. assumption A2.

According to equation (10), the stated identification problem is therefore equivalent to a single-source channel identification problem. Following the methodology described in [15] for a similar case, (10) can be solved using two cyclic frequency values. Taking these values equal to \( \pm \frac{2\pi}{k_0} \), substitution into (10) leads after some mathematical operations to:
\[
\sum_{l=0}^{L-1} \left( \tilde{R}_j(k,\tau-l) - \tilde{R}_j(-k,\tau-l) e^{j\frac{2\pi kl}{P}} \right) h_{j,i_i}(l) = 0
\]
where \( \tilde{R}_j(k,\tau) = \tilde{R}_j(k,\tau) * \tilde{R}_j(-k,\tau) \). By evaluating (11) at different values of \( \tau \), one can set a system of equations with unknowns \( h_{j,i_i} = [h_{j,i_i}(0) \ldots h_{j,i_i}(L_{h}-1)]^T \) as:
\[
Ah_{j,i_i} = 0
\]
where \( A \) is a \((4P+3L_h-6\times L_h)\) Toeplitz matrix with first row:
\[
[\tilde{R}(k,0) - \tilde{R}(-k,0) \quad \vdots \quad \tilde{R}(k,B)e^{j\frac{2\pi k}{P}} - \tilde{R}(-k,B)e^{-j\frac{2\pi k}{P}} ]
\]
and first column:
\[
[\tilde{R}(k,0) \tilde{R}(-k,0) \vdots \tilde{R}(k,B)e^{j\frac{2\pi k}{P}} - \tilde{R}(-k,B)e^{-j\frac{2\pi k}{P}} ]
\]
where \( B = 4P + 2L_h - 5 \). Applying (12) with an estimation \( \hat{A} \) of \( A \) leads to an error \( e \) defined by:
\[
\hat{A}h_{j,i_i} = e
\]
From (13), \( h_{j,i_i} \) is the solution that minimizes the error energy:
\[
\hat{h}_{j,i_i}^m = \min_{\text{A}} \| \hat{A}h_{j,i_i} \|^2
\]

In the simulation part, a gradient algorithm is employed to perform the above minimization, i.e.
\[
\hat{h}_{j,i_i}^m = \hat{h}_{j,i_i} - \mu m \nabla g^m
\]
where \( \hat{h}_{j,i_i}^m \) denotes the channel updated estimate at the \( m \)th iteration, \( \mu m \) is the step size, and \( \nabla g^m \) is the gradient of the constrained cost function (14):
\[
\nabla g^m = (I - \hat{h}_{j,i_i}^m \hat{h}_{j,i_i}^m H)(\hat{A}^mH \hat{A}^m)\hat{h}_{j,i_i}^m
\]

IV. APPLICATION TO DMT-BASED DSL SYSTEMS

A. Estimation scenario

Consider a DSL system where transceivers located at the Central Office (CO) communicate with transceivers at remote locations using copper pairs in the same binder. DMT blocks of length \( P \) and cyclic-extensions of length \( L \) are used by all the transceivers. Activating a new line initiates a channel identification process involving the newly activated line and the already active ones. The following scenario describes part of this process taking place between the newly activated transceiver at the CO side and the already active transceivers at the remote locations side.

- **Upstream** (transmitters located at the remote locations):
  1. The cyclic frequency \( k \) is selected at zero cyclic autocorrelation to satisfy (4.2) at given \( P \) and \( L \).
  2. For an upstream transmitter, referred to as \( i_i \) in (4.1), part of the CE is changed (as described in the next subsection) in order to satisfy (4.1) at given \( P \) and selected \( k \).
  3. For the newly activated transceiver at the CO side, the received signal is used to compute a cyclic autocorrelation matrix at \( k \). With null contributions from all transmitters except \( i_i \), this matrix allows identifying the crosstalk channel between the new activated CO transceiver and the transmitter \( i_i \) at the remote location side.
  4. Steps 2 and 3 are repeated for another transmitter in Step 2.
- **Downstream** (receivers located at the remote locations):
  1. The cyclic frequency $k$ is selected at zero cyclic autocorrelation to satisfy (4.2) at given $P$ and $L$.
  2. For the newly activated CO transceiver, part of the CE is changed to satisfy (4.1) at given $P$ and selected $k$. All the other transceivers continue to transmit normally.
  3. A cyclic autocorrelation matrix is computed at $k$ simultaneously for all receivers. This matrix includes only effects from the newly activated transmitter at the CO location. Therefore, it allows the crosstalk channel between this transmitter and each receiver to be identified.

B. **Practical considerations in changing the CE**

For vectored DSL transmission, all the lines must be synchronous and data blocks must be aligned in order to achieve crosstalk mitigation. It is also necessary to have a sufficient CP to avoid interblock interference and maintain subcarriers orthogonality. Block alignment is preserved for a given transceiver, i.e. $i_j$, during its channel estimation by keeping the IFFT block and the CE the same lengths as of the other transceivers, while replacing part of the CS with data uncorrelated to the IFFT block. At the same time, the transmission of this transceiver is delayed for a duration $\Delta$ equivalent to the length of the replaced CS part.

Replacing part of the CS with uncorrelated data reduces the length of the autocorrelation pulse, i.e. $L$ as illustrated by Fig.1, which allows changing the position of the cyclic autocorrelation zero to satisfy (4.1) at given values of $P$ and $k$. Actually, block alignment in coordinated transmission is required in a time window during which valid data from all the lines are extracted. This window is indicated as the valid window in the example of Fig.4. Alignment is preserved in this window by delaying the transmission of transceiver $i_j$ in order to shift the changed CS part out from the valid window. With such delay, original valid data in the changed part of the CS are replaced into the valid window by their copies in the IFFT block. A margin for such small transmission delay exists even in very high rate synchronized systems, where the minimum CS length in these systems is selected relatively to the delay spread of the longest line.

V. **Simulation Results**

Simulations were carried out with the aim of assessing the performance of the proposed method. Channel measurements were obtained through Orange Labs over a DSL bundle of 28 cooper pairs. Table 1 shows experimental and simulation settings. A CE of length 640 is used with transmission over 4096 subcarriers ($M = 8192$). During channel estimation, (4.2) is satisfied at the used transmission values by selecting an appropriate value of $k$ (e.g. $k = 69$). For transceiver $i_j$, and according to the scenario in Section IV, an appropriate length of the autocorrelation pulse (e.g. $L_i = 600$) is used to satisfy (4.1), i.e. $L_i \neq (c_1 \times (2 \times 4096 + 640) + 69 = 128c_1)$.

<table>
<thead>
<tr>
<th>Table I</th>
<th>Experimental and simulation settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmitted signal PSD ($\sigma^2_i$)</td>
<td>-60 dBm/Hz</td>
</tr>
<tr>
<td>Noise PSD ($\sigma^2$)</td>
<td>-140 dBm/Hz</td>
</tr>
<tr>
<td>Lines lengths</td>
<td>75, 150, and 300 m</td>
</tr>
<tr>
<td>Gap ($\Gamma$)</td>
<td>9.78 dB</td>
</tr>
<tr>
<td>Frequency band</td>
<td>10 kHz-30 MHz</td>
</tr>
<tr>
<td>Subcarrier spacing ($\Delta f$)</td>
<td>8 kHz</td>
</tr>
<tr>
<td>Number of subcarriers (N_s)</td>
<td>4096</td>
</tr>
</tbody>
</table>

Performance is measured in terms of achievable throughput after crosstalk channel estimation and crosstalk mitigation in a system applying linear precompensation [10]. Achievable throughput of the $j^{th}$ line is expressed by:

$$D_j = \Delta f \sum_{f_c=1}^{N_s} \log_2 \left( 1 + \frac{SNIR_j^m(f_c)}{\Gamma} \right)$$

where $f_c$ denotes the subcarrier index, and $m$ gives the number of iterations. Each iteration is equivalent to a DMT symbol duration of 125 $\mu$s. $SNIR_j^m(f_c)$ is the signal to noise plus residual interference ratio after crosstalk mitigation [14]:

$$SNIR_j^m(f_c) = \frac{\left| h_j^m(f_c) \right|^2 \sigma_s^2}{\sum_{m=1}^{N_s} \left| h_j^m(f_c) \right|^2}$$

where:

$$e_{f_c}^{m} = \hat{h}_j^m(f_c) - h_j^m(f_c)$$

is the error in estimating the crosstalk channel between pairs $j$ and $i$, and $N$ is the number of active lines.

The impact of direct path signal on crosstalk channel estimation is examined first. Owing to the cable topology, direct path signals are relatively strong compared to crosstalk. The proposed method is used to cancel the direct path effect and perform crosstalk channel estimation while the considered transceiver pair continues to transmit. The direct path effect and the maximum achievable throughput vary according to the line length. That appears in Fig.5 when the channel estimation is performed with three different line lengths. Perfect cancellation of the direct path effect improves...
the estimation accuracy, which maximizes the throughput gain. For example, a maximum throughput of 620 Mbps is achieved for a 75 m line, which represents about 47% increase compared to the initial throughput for which no estimation is performed and crosstalk is regarded as noise.

Fig. 5 shows comparable convergence delays for different line lengths. Although the estimation of fewer channel taps is needed for the shortest lines, reaching convergence in these lines requires eliminating the effect of a stronger direct path.

Results are extended in Fig. 6 to include interfering effects of crosstalk sources. The proposed method is applied to estimate the crosstalk channel of a desired source after cancellation of the other sources effects. The obtained throughputs are averaged over 10 realizations of the estimation of a randomly chosen desired source. When the number of interferers increases, non-perfect estimation of the cyclic autocorrelation matrix generates residual errors in the crosstalk channel estimation. This appears at the first iterations as differences between achieved throughputs for the different configurations in Fig. 6. However, improvement in estimation at the later iterations reveals good and comparable performance between the curves.

VI. CONCLUSION

A new method has been investigated for blind crosstalk channel identification in DMT-based DSL systems. Controlled by CE in DMT blocks, source separation helps performing separate and low complexity identification of desired crosstalk channels. The identification process takes place without interrupting the vectored transmission in the coordinated system. The new approach does not impose any restriction on the channel zeros, and is applicable to any stationary white or colored noise. Simulation results demonstrate the good performance of the proposed method where significant increase in throughput has been achieved as a result of accurate estimation. Interfering effects, either from direct path sources or from crosstalk sources in the system, are shown almost perfectly eliminated.

REFERENCES