Spatial Data Multiplexing over OFDM/OQAM modulations

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Abstract — Multi-carrier modulations (MCM) and especially orthogonal schemes such as orthogonal frequency division multiplexing (OFDM) are currently used in many radio transmission standards. Among these modulations, OFDM/OQAM (also referred as OFDM/Offset QAM) is an interesting alternative to classical OFDM modulation, as it does not require the use of guard interval. This characteristic makes its spectral efficiency optimal. In this paper we investigate the combination of OFDM/OQAM and multiple-input multiple-output (MIMO) over radio channels associated to non iterative and iterative receivers. We focus on spatial data multiplexing (SDM) which is a well known MIMO technique designed to improve the capacity.

Index Terms — OFDM, OQAM, MIMO, SDM, iterative receiver

I. INTRODUCTION

The use of radio communication systems with multiple transmit and receive antennas, also referred as a multiple-input multiple-output (MIMO) system, can be used to increase capacity. Because of the time-dispersion that occurs in radio mobile communications, the MIMO channel is frequency-selective. Orthogonal frequency division multiplexing (OFDM) presents the property to convert such a frequency-selective MIMO channel into a set of parallel frequency-flat MIMO channels. This is of course valid only if the cyclic-prefix CP is longer than the maximum delay introduced by the channel. Consequently, as long as this condition is verified, it is possible to equalize independently each sub-carrier with simple single-tap equalizer. This makes cyclic-prefix OFDM (CP-OFDM) a suitable scheme to be associated with MIMO. Nevertheless, there exists other OFDM based modulations such as OFDM/OQAM [1–3] for which no prefix cyclic is inserted. These modulations are robust to both frequency and time selectivity. In section II we describe the OFDM/OQAM modulation, and we show that the introduction of appropriate pulse shaping can efficiently combat time and frequency distortions caused by the channel. Then, we discuss the problem of ideal channel estimation for OFDM/OQAM systems. In section III we present the specific issues related to the combination of OFDM/OQAM with multiple transmit antennas. We propose in section IV solutions using a 2x2 spatial data multiplexing (SDM) configuration. In section V we provide simulation results, and in section VI we give the general conclusions and perspectives.

II. THE OFDM/OQAM MODULATION

In classical CP-OFDM modulation, we transmit complex symbols \( s_{m,n} \) in parallel over the sub-carriers. The complex orthogonality between these sub-carriers guarantees simple reception. Although robust to frequency-selectivity, CP-OFDM presents a loss in spectral efficiency and suffers from a bad localization in the frequency domain due to the rectangular prototype function. In the following we describe OFDM/OQAM which is an alternative to classical CP-OFDM.

A. OFDM/OQAM Theory

Contrary to CP-OFDM, OFDM/OQAM modulation does not require the use of a guard interval, which leads to a gain in spectral efficiency. Without adding the guard interval redundancy, similar performance can be reached by modulating each sub-carrier by a prototype function. To obtain a sufficient robustness to the channel variations, this prototype function must be very well localized in both time and frequency domains. The localization in time aims at limiting inter-symbol interference ISI and the localization in frequency aims at limiting inter-carrier interference ICI due for instance to Doppler effects.

The orthogonality between the sub-carriers must also be maintained after the modulation. Near optimally localized functions having these properties exist but only guarantee orthogonality on real values. An OFDM modulation using these functions is denoted OFDM/OQAM. We note that in OFDM/OQAM, each sub-carrier carries a real-valued symbol \( a_{m,n} \), which corresponds to either the real part or the imaginary part of a complex OFDM symbol \( s_{m,n} \), where \( m \) is the frequency index, and \( n \) is the time index. In the following we note \( T_0 \) the duration of the complex OFDM symbol \( s_{m,n} \) with zero-length cyclic prefix, then \( T_0 = T/2 \) is the duration of the real OFDM/OQAM symbol \( a_{m,n} \). By denoting \( AF \) the inter-carrier spacing we have \( T_0 AF = 1/2 \). This means that the density of the sub-carriers in the time-frequency plane is twice greater in OFDM/OQAM than in classical OFDM, with a zero-length cyclic prefix.
TABLE I

<table>
<thead>
<tr>
<th>Symbol</th>
<th>(2m-1)ΔF</th>
<th>(2m)ΔF</th>
<th>(2m+1)ΔF</th>
</tr>
</thead>
<tbody>
<tr>
<td>nT₀ - T₀/2</td>
<td>s² 2m-1,n-1</td>
<td>j s² 2m,n-1</td>
<td>s² 2m+1,n-1</td>
</tr>
<tr>
<td>nT₀</td>
<td>j s² 2m,n-1</td>
<td>s² 2m,n</td>
<td>j s² 2m+1,n</td>
</tr>
<tr>
<td>nT₀ + T₀/2</td>
<td>s² 2m-1,n-1</td>
<td>j s² 2m,n</td>
<td>s² 2m+1,n-1</td>
</tr>
</tbody>
</table>

As the information carried by two real-valued data correspond to the one carried by one complex-valued data, OFDM/OQAM has the same spectral efficiency as classical OFDM with no guard interval. The OFDM/OQAM transmitted signal can be expressed as follows

\[ s(t) = \sum_{n=-\infty}^{\infty} \sum_{m=0}^{M-1} a_{m,n} f_{m,n}(t), \]

where \( M \) is the number of sub-carriers, \( a_{m,n} \) is real-valued symbol transmitted on the \( m \)th sub-carrier at the \( n \)th symbol; \( f(t) \) denotes the real-valued prototype function. Equation (1) can be rewritten in a simpler manner

\[ s(t) = \sum_{n=-\infty}^{\infty} \sum_{m=0}^{M-1} a_{m,n} f_{m,n}(t), \]

where \( f_{m,n}(t) \) are the shifted versions of \( f(t) \) in time and frequency. Therefore the orthogonality condition between the sub-carriers is

\[ \text{Re}\left( \int f_{m,n}(t) f_{m',n'}^*(t) dt \right) = \delta_{m,m'} \delta_{n,n'}, \]

(3)

In case of no channel, the demodulated symbol over the \( m \)th sub-carrier at the \( n \)th instant is

\[ r_{m,n} = \int s(t) f_{m,n}^*(t) dt = a_{m,n} + \sum_{(m',n') \neq (m,n)} a_{m',n'} \int f_{m',n'} f_{m,n}^* dt + n_{m,n}, \]

(4)

As the OFDM/OQAM prototype function \( f(t) \) is chosen to be well localized both in time and frequency domains, the intrinsic interference term from (6) only depends on a restricted set of time-frequency positions \((m',n')\) around the considered symbol. Assuming that \( h_{m,n} \) is constant over the summation zone, we can write

\[ r_{m,n} = \int s(t) f_{m,n}^*(t) dt = h_{m,n} \left\{ a_{m,n} + \sum_{(m',n') \neq (m,n)} a_{m',n'} \int f_{m',n'} f_{m,n}^* dt \right\} + n_{m,n}, \]

(7)

with a good approximation, after equalization, a simple recuperation of the real part is sufficient.

A particular prototype function called IOTA (Isotropic Orthogonal Transform Algorithm) satisfying equation (3) is evaluated in this paper. To simplify the notations, in the following we call OFDM/IOTA an OFDM/OQAM system using the IOTA function. With IOTA prototype function, it is possible to assume as in [4] that the intrinsic interference \( I_{m,n} \) depends on the 8 nearest neighbors of \( a_{m,n} \) shown at Fig. 1.

![Fig. 1. First-order neighbors in time-frequency representation for OFDM/IOTA.](image)

B. Ideal Channel Estimation

In classical OFDM, ideal channel estimation assumes that the channel time response is given, which means, the coefficients multiplying the time samples of the transmitted signal are known. Because transmission and reception matrices are Fourier matrices, the relation between channel time response and the channel coefficients \( h_{m,n} \) is very simple. Thus, in CP-OFDM, perfect knowledge of the time response implies perfect estimation of \( h_{m,n} \).

In OFDM/OQAM modulation, because of the time and frequency overlap of the functions carrying the symbols, the relation between channel time response and the \( h_{m,n} \) is not straightforward. Using the IOTA function, a simple solution is proposed on Fig. 2.
Fig. 2. Ideal channel estimation for OFDM/IOTA.

In our simulations, to obtain ideal channel coefficients, we duplicate the transmission chain once with the Single Input Single Output (SISO) channel but without noise and once without any channel. To compute $h_{m,n}$, we assume that for each time-frequency position, the channel is constant over the first-order neighbors,

$$y_{m,n} = r_{m,n} - n_{m,n} = h_{m,n}a_{m,n} + I_{m,n} = h_{m,n} (a_{m,n} + i_{m,n}), \quad (8)$$

$$y_{m,n}' = a_{m,n} + i_{m,n}, \quad (9)$$

$I_{m,n}$ and $I_{m,n}$ are given in equations (4) and (6). We obtain $h_{m,n}$ simply from the ratio of the two received signals,

$$h_{m,n} = y_{m,n}/y_{m,n}', \quad (10)$$

III. OFDM/QAM USING MULTIPLE ANTENNAS

While being an OFDM based modulation, OFDM/QAM inherits from many properties related to classical OFDM systems. For instance, the OFDM/QAM system is supposed to be designed to cause negligible inter-symbol interference just like the cyclic prefix in CP-OFDM is assumed to be chosen longer than the delay spread. This property allows in both cases the use of simple equalizers.

In the following we focus on the differences between OFDM/QAM and CP-OFDM and we propose a 2x2 OFDM/QAM system based on Spatial Data Multiplexing (SDM).

A. Transmission Scheme

After channel coding, bit-interleaving, mapping over complex symbols, data are demultiplexed onto two antennas. Then, over each branch $i$ ($i=1,2$), these complex symbols $s_{i,m,n}$ either constitute directly the input of the CP-OFDM modulator, or are converted to real symbols $a_{i,m,n}$ as described in Table I, before being sent to the OFDM/QAM modulator.

B. Reception Scheme

In the CP-OFDM case, we call $s_{1,m,n}$ and $s_{2,m,n}$ two complex symbols transmitted at a given time-frequency position. After passing through a radio channel, at receive antenna $i$ we demodulate ($i=1,2$)

$$r_{i,m,n} = h_{i,m,n}s_{i,m,n} + h_{i2,m,n}s_{2,m,n} + n_{i,m,n}, \quad (11)$$

In the OFDM/QAM case, we transmit real symbols $a_{1,m,n}$ and $a_{2,m,n}$ obtained from complex symbols. After channel and demodulation,

$$r_{i,m,n} = h_{i1,m,n}a_{1,m,n} + h_{i2,m,n}a_{2,m,n} + I_{i2,m,n} + n_{i,m,n}, \quad (12)$$

where $I_{i,j,m,n}$ is the intrinsic interference due to $a_{i,j,m,n}$ transmitted over the $j^{	ext{th}}$ antenna and received at the $i^{	ext{th}}$ antenna.

$$I_{i,j,m,n} = \sum_{m',n'} h_{i,j,m',n'} a_{i,m',n'} \int f_{m',n'}(t) f_{m,n}'(t) dt , \quad (13)$$

Using IOTA function, and assuming the channel constant over the first-order neighbors; equations (7) and (12) lead to

$$r_{i,m,n} = h_{11,m,n}(a_{i,m,n} + \tilde{I}_{1,m,n}) + h_{12,m,n}(a_{2,m,n} + \tilde{I}_{2,m,n}) + n_{i,m,n}, \quad (14)$$

We define the virtually transmitted symbols as

$$c_{i,m,n} = a_{i,m,n} + \tilde{I}_{i,m,n}, \quad (15)$$

We will show that by considering these virtually transmitted symbols instead of the effective ones, signal equalization in the MIMO case can be performed either with a simple equalizer built on the Minimum Mean Square Error (MMSE) or the Zero Forcing (ZF) criteria, or with an iterative receiver.

IV. PROPOSED CONFIGURATION

In this section, we propose three receivers adapted to OFDM/IOTA in a 2x2 SDM scheme.

The first two configurations are based on equation (14) and on the notion of virtually transmitted symbols $c_{i,m,n}$. Here, we have to suppose twice that the channel is constant over the first-order neighbors, in the receiver equations and in the ideal channel estimation. In a third configuration, this assumption will be only used for the recovery of channel coefficients.

A. Ideal Channel Estimation

With the knowledge of the channel time response, the goal is to recover the $h_{i,j,m,n}$ ($i=1,2; j=1,2$). A 2x2 MIMO channel can be seen as a combination of four SISO sub-channels. To estimate a SISO sub-channel, we use the same methodology as explained in II.B. As a result, the assumption of a quasi constant channel over the first-order neighbors needs to be applied four times more than for the SISO case.

B. Non Iterative MMSE Receiver

After OFDM/IOTA demodulation over each antenna, $r_{i,m,n}$ represent the inputs of the MMSE equalizer having the equalized virtually transmitted symbols $\tilde{c}_{i,m,n}$ as outputs. Then, a retrieval of the real parts yields the real equalized symbols $\tilde{a}_{i,m,n}$ to be re-transformed to the complex equalized symbols $\tilde{c}_{i,m,n}$ as stated in Table I.

These symbols are multiplexed one by one inversing exactly the transmission operation, before being demodulated with a soft demodulation giving soft bits; Fig. 3 describes this non iterative receiver.
C. First Iterative MMSE Receiver

In this section, we present an iterative MMSE receiver \textbf{IOTA-SDM-IC1} designed to improve the performance of OFDM/IOTA spatial data multiplexing systems by iterative interference cancellation. This receiver depicted on Fig. 4 also treats the virtually transmitted symbols as the non-iterative receiver.

A soft demapping serves to calculate reliability information for each bit in the form of Log-Likelihood Ratios (LLR). In OFDM/IOTA case, we treat real symbols; if \( s_{i,m,n} \) are 4-QAM symbols then the soft demapper simply considers twice 1-bit symbols. The following multiplexer has a size=2, we get two real symbols together before multiplexing to be compatible with the demultiplexing operation at the transmission holding complex symbols \( s_{i,m,n} \).

An interference canceller, giving equalized virtually transmitted symbols, takes place directly after OFDM/IOTA demodulation. It is made of two finite length filters where \( \tilde{p}_{i,m,n} \in \mathbb{C}^{2^n} \) and \( \tilde{q}_{i,m,n} \in \mathbb{C}^{2^n} \) represent the vectors of these two filters. These vectors correspond to the virtually transmitted symbol \( c_{i,m,n} \) the output is:

\[
\tilde{c}_{i,m,n} = \tilde{p}_{i,m,n}^H \tilde{p}_{i,m,n} - \tilde{q}_{i,m,n}^H \tilde{c}_{i,m,n},
\]

where \( \tilde{c}_{i,m,n} \in \mathbb{C}^{2^n} \) is the vectors family defined by

\[
\tilde{c}_{i,m,n} = \begin{bmatrix} 0 \\ \tilde{c}_{2,m,n} \end{bmatrix} \quad \text{and} \quad \tilde{c}_{2,m,n} = \begin{bmatrix} \tilde{c}_{1,m,n} \\ 0 \end{bmatrix},
\]

where \( \tilde{c}_{i,m,n} \) is the estimate of the virtually transmitted symbol \( c_{i,m,n} \), this estimate is calculated via the iterative loop.

In a typical iterative receiver, the role of the soft mapping is the opposite of the soft demapping by reconstituting symbols from there LLRs [5,6,8]. In the proposed OFDM/IOTA receiver, we need to estimate \( c_{i,m,n} \); for that, we adopt a complex soft mapping having estimates of complex transmitted symbols \( s_{i,m,n} \) as outputs. After that, to generate the imaginary part of the virtually transmitted symbols, we apply an OFDM/IOTA modulator directly followed by an OFDM/IOTA demodulator in a 2x2 SDM configuration. At the output, we have available the estimates of the virtually transmitted symbols \( c_{i,m,n} \).

D. Second Iterative MMSE Receiver

Also based on the MMSE criteria and iterative interference cancellation, the second proposed iterative receiver called \textbf{IOTA-SDM-IC2} does not treat the virtually transmitted symbols. Contrary to \textbf{IOTA-SDM-IC1}, in the receiver expression, we do not assume that the channel is quasi constant over the first-order neighbors.

Starting from the exact expression, as in equation (12), the first iteration is similar to that of \textbf{IOTA-SDM-IC1}. Then, the idea is to cancel iteratively the interferences in two stages:

1) First stage: from equation (13), estimate the intrinsic interference terms \( \hat{i}_{i,m,n} \), and subtract these estimates from \( y_{i,m,n} \):

\[
y_{i,m,n} = r_{i,m,n} - \left( \hat{i}_{i,m,n} + \hat{i}_{2,m,n} \right), \quad i=1,2
\]

2) Second stage: the vector \( (y_{1,m,n}, y_{2,m,n}) \) is the input of an interference canceller treating traditional co-antenna interference exactly like in the case of classical OFDM. Although not used explicitly in the receiver expression, it is important to note that the assumption of a quasi constant channel over the first-order neighbors is still used in the calculation of the coefficients \( h_{i,m,n} \) from ideal channel estimation.

V. SIMULATION RESULTS

In this section, we compare OFDM/IOTA to CP-OFDM in SISO and 2x2 SDM over different kinds of channels.

A. Simulation Parameters

While choosing channels to simulate, our objective is to test OFDM/IOTA over two distinct channels, the first one having a high time selectivity and low frequency selectivity, and the second one with lower time selectivity but higher frequency selectivity. For that purpose, we have chosen a Pedestrian A channel with a velocity of 120 km/h and a Vehicular A channel at 60 km/h. For MIMO simulations, the 2x2 SDM channel made of four SISO sub-channels is spatially non-correlated.

Table II lists the principal common parameters used for OFDM/IOTA and CP-OFDM simulations over Pedestrian A and Vehicular A channels.

<table>
<thead>
<tr>
<th>TABLE II SIMULATION PARAMETERS</th>
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<tbody>
<tr>
<td>Pedestrian A</td>
</tr>
<tr>
<td>Complex modulation</td>
</tr>
<tr>
<td>FFT size</td>
</tr>
<tr>
<td>( \Delta ) size in CP-OFDM</td>
</tr>
<tr>
<td>Convolutional coding</td>
</tr>
<tr>
<td>Carrier frequency</td>
</tr>
<tr>
<td>Sampling frequency</td>
</tr>
<tr>
<td>Velocity</td>
</tr>
</tbody>
</table>

In CP-OFDM, we define \( \frac{E_{in}}{N_0} / \text{SNR} \) by:

\[
\frac{E_{in}}{N_0} = N_c N_r \frac{T_0 + \Delta}{T_0} R R_c \log_2 M, \quad \text{SNR},
\]

(19)

where: \( N_c \) and \( N_r \) are the transmission and reception antennas numbers respectively, \( T_0 \) is the useful CP-OFDM symbol duration, \( \Delta \) is the cyclic prefix duration, \( R_c \) is the channel coding rate, \( R \) is the space-time coding rate (in 2x2 SDM, \( R=2 \)), and \( \text{SNR} \) is the ratio between complex symbols variance and noise variance for a normalized channel. For OFDM/IOTA, equation (19) becomes:

\[
\frac{E_{in}}{N_0} = N_c N_r \frac{1}{R R_c \log_2 M} \text{SNR},
\]

(20)
For an iterative receiver, we define the Matched Filter Bound (MFB) as the asymptotical performance obtained when the estimated symbols are identical to the transmitted ones [6].

**B. Simulation Results**

In this section, we present OFDM/IOTA and CP-OFDM performance for SISO with MMSE equalizer and in a 2x2 SDM MIMO configuration with all the receivers described in section IV. SISO performance of both OFDM/IOTA and CP-OFDM are represented on Fig. 5 with a MMSE receiver for both modulations. Over Pedestrian A (120 km/h) channel, a 1dB gain results from both the removal of the cyclic prefix and from the better resistance of OFDM/IOTA to Doppler effects. Over Vehicular A channel (60 km/h), frequency dispersion is smaller and time dispersion is larger than for the Pedestrian A channel. As OFDM/IOTA does not have any cyclic prefix, a small degradation of the performance due to inter-symbol interference compensates the gain corresponding to the removal of the cyclic prefix.

For the MIMO case, we first compare the non iterative MMSE receiver performance on Fig. 6 over Pedestrian A and Vehicular A channels. Depending on the propagation channel, performance of OFDM/IOTA is either the same or worse than CP-OFDM. For OFDM/IOTA, additional degradation occurring in MIMO comes from channel estimation, even if supposed to be ideal. This degradation is due to the assumption of a constant channel over the symbols plus their first-order neighbors as described in equation (14).

Fig. 7 captures the performance of OFDM/IOTA and CP-OFDM over AWGN channel in a 2x2 SDM configuration. Both modulations use iterative receivers, here we show the
strictly constant over the whole time-frequency plan, so no bias is introduced in the channel estimation process. Thus we observe that once performance has converged, OFDM/IOTA outperforms CP-OFDM of about 0.3 dB.

Fig. 8 and Fig. 9 show the performance of CP-OFDM and OFDM/IOTA with iterative receivers over Pedestrian A and Vehicular A channels respectively. As for non iterative schemes, OFDM/IOTA performance after 5 iterations is similar to CP-OFDM over Pedestrian A channel and suffers from a 2 dB degradation over Vehicular A channel. Nevertheless it can be observed that for OFDM/IOTA there is still a gap of almost 1 dB with the MFB even after 5 iterations. In the Pedestrian A case, the MFB of OFDM/IOTA outperforms that of CP-OFDM by 0.5 dB. This remark means that further investigations on both channel estimation and on the iterative receiver itself can lead to improved performance.

VI. CONCLUSION

In this paper we described OFDM/IOTA transceivers adapted for MIMO transmissions. Non-iterative and iterative receiver structures were investigated and evaluated. Simulations over radio channels show that OFDM/IOTA outperforms CP-OFDM in the SISO case but when going to MIMO, degradation due to the assumption of a flat channel over a time-frequency zone may take place. However, if the channel impulse response is relatively short, no significant degradation occurs even for high velocities and OFDM/IOTA asymptotical performance is better than that of CP-OFDM. In contrast, with larger delay spreads, we observed that this degradation with respect to CP-OFDM increases. This degradation only exists for OFDM/IOTA with ideal channel estimation, and not for CP-OFDM. Based on these promised preliminary results, further studies with realistic channel estimation would provide a better picture of the performance of MIMO-OFDM/IOTA over radio channels.

REFERENCES