Arbitrarily Shaped Formation Control of Multiple Electrically Driven Mobile Robots Using Backstepping Control

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Abstract

This paper presents a decentralized control scheme for the formation control of multiple electrically driven mobile robots based on artificial potential functions and backstepping control method. Backstepping control is used to extend the theoretical potential fields based formation control methods designed for mass-less kinematic agents to the case of fully actuated electrically driven mobile robots. The proposed method maintains the decentralized nature of the control method as well as its convergence and stability properties and improves the trajectories agents follow to the desired formation (transient system behavior). Asymptotic stability of the system is proved using Lyapunov analysis. Simulation results are presented to illustrate the effectiveness of this method.

Keywords: multi-agent systems, formation control, backstepping control, electrically driven WMRs

Introduction

During recent years, the control problem of multi-agent robotic systems has been the subject of considerable attention and extensive research [1]-[5]. The formation control problem of multi-agent systems is defined as the organization of a swarm of agents into a particular shape in a 2D or 3D space [1]. Formation control is useful in satellite configurations in the space or search and rescue robots propagation.

Many approaches have been developed to confront this problem, each with their own advantages and disadvantages. All these methods can be categorized in three different groups of centralized, decentralized and hybrid control. Compared to the conventional centralized control, distributed control of multi agent systems provides increased performance, efficiency and robustness. Some of the recently used control approaches in this area are potential fields [1], behavior-based [2], leader-following [3], graph-theoretic [4] and virtual structure [5] methods.

Most of the formation control methods cited before consider a group of mass-less agents with kinematic models. Although the results of these researches can serve as a basis for designing the desired behavior in multi-agent systems, the mass-less kinematic model does not represent the dynamics of realistic agents and falls short of achieving the desired behavior in engineering applications with real agent dynamics.

In this paper, we extend an artificial potential function based method of realizing the desired formation for mass-less kinematic agents, to the case of fully actuated electrically driven holonomic wheeled mobile robots using a backstepping control method. The main advantage of this novel control strategy is that it maintains the decentralized nature of the control method as well as its convergence and stability properties. It also introduces additional nonlinear terms to improve the transient performance [7]. The potential field based formation control method this work is based on [8], has the advantage of not being affected by the problem of local minima as opposed to the previous work where the existence of local minima in potential functions lead to guaranteeing only local convergence to the desired formation [9].

The rest of the paper is organized as follows: In section 2, the choice of potential function and the transformation strategy for obtaining arbitrarily shaped formations is explained briefly. In section 3, we consider the agents to be mobile robots with general vehicle dynamics and design a controller at the torque level using backstepping control method. Section 4 continues the backstepping design to obtain actual control inputs at actuator dynamics level. In section 5, simulation results concerning all the proposed controllers are presented to verify the theoretical results. Finally, conclusions are given in section 6.

The Potential Fields Method

In this section, the choice of potential function and the arbitrarily shaped formation control strategy for the kinematic model are explained briefly. As it was mentioned before, this potential field control method is based on the work in [8]. First, an artificial potential field is designed to obtain a formation with the shape of a regular polygon, then a bijective coordinate transformation is used to deform the polygonal formation into a completely arbitrarily shaped formation.

Let \( q_i = [x_i,y_i]^T \in \mathbb{R}^{2 \times 1} \) denote the position vector of agent \( i \), where \( q_1,\ldots,q_n \in \mathbb{R} \) are the agents Cartesian coordinates.

The polygonal formation potential function is considered of the form:

\[
V(q) = \sum_{i=1}^{n} V_{cf}(q_i) + \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} V_{at}(q_i, q_j) + \sum_{i=1}^{n} V_{pd}(q_i)
\]

(1)

Where \( \mathbf{q} = [q_1 \ldots q_n]^T \) is the state vector of the system and \( V_{ef}(q_i) \) is the potential between each agent and the center of the polygonal formation and of an attracting nature, while \( V_{at}(q_i, q_j) \) is the potential between agents in the group and of a repulsing nature and \( V_{pd}(q_i) \) is the orientation potential. The first term is defined as:

\[
V_{cf}(q_i) = \frac{1}{2} H_c(d_{ei} - R_i)^2
\]

(2)
Where $d_{ij}(t) = \|q_i(t) - q_j\|$ is the current distance between the $i$-th agent and the desired position for the center ($q_0$). $R$ is the radius of the polygon which can be obtained from the length of each side of the polygonal formation ($L$) using (3) and $K_d$ is a positive constant. The role of this term is to move each agent to a circumference with the center $q_0 \in \mathbb{R}^{2\times 1}$ and radius $R$.

$$R = L/[2 \sin(\pi/n)]$$  
(3)

The inter-agent potential is written as follows:

$$V_{ad}(q_i, q_j) = \begin{cases} \frac{1}{2}K_d(d_{ij} - L)^2 & \text{if } d_{ij} \leq L \\ 0 & \text{if } d_{ij} > L \end{cases}$$  
(4)

Where $d_{ij}(t) = \|q_i(t) - q_j(t)\|$ is the distance between the $i$-th and the $j$-th agents and $K_d$ is the positive constant. This term produces a repulsive force if two agents are too close, namely if $d_{ij} \leq L$, and is in charge of both collision avoidance and regulation of the relative distances among agents in the desired formation.

Finally, there is the orientation potential with the role of fixing the orientation of the formation on one of the infinite possible regular polygons with $n$ sides lying on the same circumcircle, this term is of the form:

$$V_{ad}(q_i) = \begin{cases} \frac{1}{2}K_o(d_{ad} - L)^2 & \text{if } q_i \in \mathcal{L} \\ 0 & \text{otherwise} \end{cases}$$  
(5)

Where $d_{ad}(t) = \|q_i(t) - q^\ast\|$ is the distance between the $i$-th agent and the point $q^\ast$ which is defined as the position to be occupied by one of the vertices of the polygon, and $K_o$ is a positive constant, and

$$\mathcal{L} = \{q \text{ s.t. } \|q - q^\ast\| < L\} \cup \{x_1\}$$  
(6)

Where $L' = L/[2 \cos(\arcsin(L/2R)/2)]$, and $x_1$ is one of the two points $q_1, q_2$ such that $\|q_1\| = \|q_2\| = R$ and $\|q_i - q_1\| = \|q_i - q_2\| = L'$.

In the next section, it will be explained how a control law is designed using this potential function to achieve a stable polygonal formation. The next step would be to find a transformation to transform a regular polygon to an arbitrary shape. The main idea is to consider two different reference frames, the real reference frame $(\dot{w}, \dot{z})$ and an auxiliary reference frame $(\dot{u}, \dot{v})$. Thus, the control strategy is as follows: first, each agent measures its own position and its neighbors' in the real reference frame $(\dot{w}, \dot{z})$, then it transforms these measurements to those of the auxiliary reference frame $(\dot{u}, \dot{v})$ using the transformation $T$ and calculates the control law for the polygonal formation in the auxiliary reference frame using the method that will be stated in the next section. After that, this control law is transformed into the real control law for the arbitrarily shaped formation using the inverse transformation $T^{-1}$ and applied to the agents.

Let $\tilde{q} = (q^T, 1)^T \in \mathbb{R}^2$ and $\tilde{q}' = (q'^T, 1)^T \in \mathbb{R}^2$ where $q^T = [v_{i1}', y_{i1}']^T$ is the image of $q_i$ in the auxiliary frame and assume that $\dot{q} = [\dot{q}_1^T \ldots \dot{q}_n^T] \in \mathbb{R}^{2n}$ and $\dot{q}' = [\dot{q}'_1^T \ldots \dot{q}'_n^T] \in \mathbb{R}^{2n}$.

The transformation matrix $T$, can be defined as a block diagonal, invertible matrix such that:

$$q' = T(q) \cdot q$$  
(7)

If we consider $q_D$ as the desired formation positions vector of the agents in the auxiliary reference frame (namely the vertices of a polygon) and if $q_D$ is the desired formation positions vector of the agents in the real reference frame, then using the transformation we can write

$$q_D = T(q_D) \cdot q_D$$  
(8)

Now, we know that realizing $q = q_D$ means that a regular polygon shaped formation is achieved in the auxiliary reference frame and we will prove in the next section that the control strategy is asymptotically stable or

$$\lim_{t \to \infty} q(t) = q_D$$  
(9)

Applying the coordinate transformation to (9), the following is obtained:

$$\lim_{t \to \infty} \tilde{q}(t) = \lim_{t \to \infty} T(q(t)) \cdot q(t) = T(q_D) \cdot q_D = q_D$$  
(10)

In other words, with this control strategy the desired formation is always created.

Backstepping Control Design for Agents with General Vehicle Dynamics

In this section, we consider that all the agents in the system have the same known dynamics which could be described by the following equation:

$$M(q_i) \ddot{q}_i + C_i(q_i, \dot{q}_i) \dot{q}_i + g_i(q_i) = \tau_i$$  
(11)

This can be written in the form:

$$\begin{cases} \dot{q}_i = v_i \\ \dot{v}_i = -M_i^{-1}(C_i(q_i, v_i) + g_i(q_i)) + M_i^{-1} \tau_i \end{cases}$$  
(12)

Where $q_i = (x_{i1}, y_{i1})^T$ is the position of the agent, $v_i = (v_{i1}, v_{i2})^T$ is the vector of velocities, $\tau_i = (\tau_{i1}, \tau_{i2})^T$ is the vector of torques applied to the wheels of the robot, $M_i(q_i)$ is a $2 \times 2$ positive definite inertia matrix, $C_i(q_i, \dot{q}_i)$ is the vector of centripetal and coriolis forces and $g_i(q_i)$ is the vector of gravitational forces.

This system can be viewed as a cascade connection of two components, the first component is (12) with $v_i$ as input and the second component is (13) with $\tau_i$ as input.

**Theorem 1:** The multi-agent system with agents having the dynamics in component (12) can be stabilized to the desired configuration by the following state feedback control law:

$$\dot{q}_i = v_i(q_i) = \Theta_{21}(q_i) = -\nabla V(q)$$  
(14)

Where $V \in \mathbb{R}$ is the potential function from the previous section.

**Proof 1:** let us choose the candidate Lyapunov function as the smooth positive definite potential function $V$. The time derivative of the $V$ is given by:

$$\dot{V}(q_i) = \sum_{j=1}^{n} q_{i1}^T \nabla q_{i1} + \sum_{j=1}^{n} q_{j1}^T \nabla q_{j1} + \sum_{j=1}^{n} q_{j1}^T \nabla q_{j1} + \sum_{j=1}^{n} q_{j1}^T \nabla q_{j1}$$  
(15)

Substituting the agent dynamics (12) and the control action (14) in (15) we obtain:
\[ \mathbf{V}(q) = -\sum_{i=1}^{n} \|\dot{q}_i\|^2 \leq 0 \]  \hspace{1cm} (16)

From (16), we have \( \dot{\mathbf{V}} \leq 0 \) and it can easily be seen that \( \dot{\mathbf{V}} = 0 \) if and only if \( \dot{q}_i = 0 \) \( \forall i,j = 1,...,n \). So, utilizing the LaSalle Yoshizawa theorem \([10]\), as \( \dot{\mathbf{V}} \to 0 \), the system converges to a constant configuration corresponding to the minimum of \( \mathbf{V} \).

From (2),(4),(5), it can be observed that \( \mathbf{V}(q) \) is the sum of three nonnegative terms, thus, to have \( \dot{\mathbf{V}} = 0 \), all three terms must be zero:

\[ V_{ct} = V_{at} = V_{st} = 0 \hspace{1cm} \forall i,j = 1,...,n \]  \hspace{1cm} (17)

This means that at \( \mathbf{V} = 0 \), every agent is placed on the circle \( (q_{ct},R) \), and the distance between each two agents is greater than or equal to \( L \), which considering the geometry means the formation is obtained. Also, the orientation of the formation is fixed at the desired point. Therefore the asymptotic stability of the configuration can be concluded and the proof is completed.

Hence, \( C_{at}(q_{st}) \) is the first virtual controller in the backstepping control design.

Now, consider the following input transformation:

\[ r_i = M_i(q_i)[\tau_{ct} + C_i(q_i,v_i)\dot{v}_i + g_i(q_i)] \]  \hspace{1cm} (18)

this, reduces (13) to the integrator \( \dot{v}_i = r_{at} \).

next, we use the change of variables:

\[ z_{st} = v_i - C_{st}(q_i) \]  \hspace{1cm} (19)

\[ \tau_{st} = \theta_{st} = w_i \]  \hspace{1cm} (20)

Therefore, the overall system in (12),(13) can be written as:

\[ \{ \dot{q}_i = C_{st}(q_i) + z_{st} \]  \hspace{1cm} (21)

**Theorem 2:** The multi-agent system with agents having the dynamics in (12),(13) can be stabilized to the desired configuration by the following state feedback control law:

\[ \theta_{2s} = \tau_i = M_i(q_i)[\frac{\partial C}{\partial q} T \dot{v}_i] - \frac{\partial g}{\partial q} - K_{1s}(v_i - \theta_{st}) + C_i(q_i,v_i)\dot{v}_i + g_i(q_i) \]  \hspace{1cm} (22)

Where

\[ \frac{\partial g}{\partial q} = \frac{\partial^{g^2}}{\partial q^2} \]  \hspace{1cm} (23)

\[ \frac{\partial^{2} g}{\partial q \partial v} = -\frac{\partial \dot{v}_i}{\partial q} (\dot{v}_i,\dot{v}_i) = \begin{bmatrix} \frac{\partial^2 g}{\partial q^2} & \frac{\partial^2 g}{\partial q \partial v} \\ \frac{\partial^2 g}{\partial q \partial v} & \frac{\partial^2 g}{\partial v^2} \end{bmatrix} \]  \hspace{1cm} (24)

Proof 2: Consider \( \mathbf{V}_1 \) as a Lyapunov function candidate:

\[ \mathbf{V}_1 = \mathbf{V}(q_1) + \frac{1}{2} z_{st}^2 \]  \hspace{1cm} (25)

The time derivative of the function \( \mathbf{V}_1 \) is given by:

\[ \dot{\mathbf{V}}_1 = \frac{\partial \mathbf{V}}{\partial q} T [\dot{q}_i(q_i)] + \frac{\partial \mathbf{V}}{\partial \dot{q}} T \dot{z}_{st} + \frac{1}{2} \dot{z}_{st} T \dot{w}_i \]  \hspace{1cm} (26)

According to (16), we can write the above equation as:

\[ \dot{\mathbf{V}}_1 \leq \frac{\partial g}{\partial q} \dot{z}_{st} + \frac{1}{2} \dot{z}_{st} T \dot{w}_i \]  \hspace{1cm} (27)

Let us choose the control law as:

\[ \dot{w}_i = -\frac{\partial \mathbf{V}}{\partial q} - K_{1s} z_{st}^2 , \hspace{1cm} K_{1s} > 0 \]  \hspace{1cm} (28)

It is straightforward to see the following inequality holds:

\[ \dot{\mathbf{V}}_1 \leq -K_{1s} z_{st}^2 \]  \hspace{1cm} (29)

Which shows that the desired configuration \( q_{st} = q_{di}, \dot{v}_i = 0 \) is asymptotically stable. Substituting for \( \dot{w}_i, \dot{z}_{st}, \dot{v}_i \), the control law in (22) can be derived and the proof is completed. Note that this term serves as the second virtual control law in the next section as we extend agent dynamics to actuator level.

**Backstepping Control Design for Mobile Robots at the Actuator Dynamics Level**

This section extends the work in previous sections to the case of \( n \) electrically driven mobile robots. The motor dynamics is as follows:

\[ \tau_{mi} = K_{mi} i_{at} \]  \hspace{1cm} (30)

Where \( \tau_{mi} = [\tau_{m1}, \tau_{m2}]^T \) is the torque generated by the dc motor on the \( i \)-th robot, \( i_{at} = [i_{at1}, i_{at2}]^T \) is the current of the dc motor on the \( i \)-th robot, \( \theta_{mi} = [\theta_{m1}, \theta_{m2}]^T \) is the angular velocity of the dc motor on the \( i \)-th robot, \( K_{mi} = \text{diag}[K_{mi1}, K_{mi2}] \) is the motor torque constant on the \( i \)-th robot, \( R_{at} = \text{diag}[R_{at1}, R_{at2}] \) is the resistance of the dc motor on the \( i \)-th robot, \( L_{at} = \text{diag}[L_{at1}, L_{at2}] \) is the inductance of the dc motor on the \( i \)-th robot and \( K_{e} = \text{diag}[K_{e1}, K_{e2}] \) is the back electromotive force coefficient of the dc motor on the \( i \)-th robot. Here, \text{diag}[ \cdot ] denotes the diagonal matrix.

The relationship between the \( i \)-th dc motor and the \( i \)-th mobile robot wheel can be described as:

\[ g_{ij} = \frac{\theta_{mij}}{v_{ij}} = \frac{R_{at} i_{atj} + K_{e} v_{ij}}{R_{at} i_{atj}} \]  \hspace{1cm} (31)

Where \( j = 1,2 \), \( g_{ij} \) is the gear ratio of the dc motor for each wheel on the \( i \)-th robot. By applying (31), the actuator dynamics can be written as:

\[ \tau_{js} = g_{ij} \dot{v}_{ij} + \frac{L_{at}}{R_{at} i_{atj}} + C_{ij} \dot{v}_{ij} \]  \hspace{1cm} (32)

Where \( g_{ij} = \text{diag}[g_{i1}, g_{i2}] \) is the gear ratio of the dc motor on the \( i \)-th robot.

**Theorem 3:** The multi-agent system with the agent dynamics in the following strict-feedback form:

\[ \dot{\mathbf{q}}_i = \dot{\mathbf{q}}_i + \sum_{j=1}^{n} \frac{\partial g_i}{\partial q_j} \mathbf{q}_j - g_i(q_i) \]  \hspace{1cm} (33)

\[ \dot{\mathbf{q}}_i = -M_i^{-1}[C_i(q_i,v_i)\dot{v}_i + g_i(q_i)] + M_i^{-1} K_i \tau_{i} \]  \hspace{1cm} (34)

\[ \dot{\mathbf{q}}_i = L_i^{-1} \dot{v}_i - R_{atj} i_{atj} - C_i \dot{v}_i \]  \hspace{1cm} (35)

Can be stabilized to the desired configuration by the following actual control input:

\[ u_i = L_i^{-1} \dot{v}_i - \frac{\partial g_i}{\partial q} T \dot{v}_i + \frac{\partial^2 g_i}{\partial q \partial v} T \dot{w}_i - \frac{\partial \dot{g}_i}{\partial v} T (M_i^{-1} - C_i) \dot{v}_i \]  \hspace{1cm} (36)

For some

\[ K_{2s} > 0 \]
Proof 3: Using the same backstepping control design technique as the one we used to obtain (22), let us choose the candidate Lyapunov function as:

\[ V_2 = V_1(q_i) + \frac{1}{2} ||u_{ai} - q_{ai}||^2 \geq 0 \]  

(37)

Using the change of variable \( z_{2i} = i_{ai} - q_{ai} \) and the proposed control input, it is straightforward to verify that the time derivative of the Lyapunov function satisfies the following inequality:

\[ \dot{V}_2 \leq -K_{2i}z_{2i}^2 - K_{2i}\dot{z}_{2i}^2 \]  

(38)

This proves the asymptotic stability of the configuration \( q_i = q_{ai}, \dot{q}_i = 0 \), using (36) as the actual control input or the voltage input to the dc motors on each robot’s wheels.

Simulation Results
To validate the effectiveness of the proposed method, we consider a group of 6 identical electrically driven mobile robots. A straight line formation is considered. The initial positions are chosen randomly in each case. The initial and final positions are indicated with squares and circles respectively. Each figure includes two sets of trajectories, the bold (black) ones show what happens in the real reference frame while the pale (yellow) ones show the trajectories in the auxiliary reference frame.

In Figure 1, kinematic agents are controlled using the control law in (14). The desired formation is realized but the paths travelled by the robots seem unnecessarily long and complex. In Figure 2, the case of general vehicle dynamics (11) and the control law (22) is presented. The nonlinear terms added by backstepping control have improved the trajectories significantly. Finally, Figure 3, demonstrates the simulation results for the case of electrically driven mobile robots in (30) and the control law (36). The superiority of the trajectories compared to the previous methods is quite obvious.

Conclusions
We presented a decentralized backstepping control strategy that drives a system of multiple electrically driven mobile robots to a desired formation by determining the voltage input on the dc motors of each robot’s wheels. A special case of potential functions was chosen to achieve any desired configuration as well as to avoid the local minima problem. The system’s asymptotic stability was proved using the Lyapunov stability theorem. All theoretical results were verified by simulations to demonstrate the effectiveness of the proposed control method.

References