Synchronization of noise-induced oscillations by time-delayed feedback

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Abstract. We consider two mutually coupled neural populations represented by two FitzHugh-Nagumo systems prepared at parameter values at which no autonomous oscillations occur. Each system is driven by its own source of random fluctuations realized by Gaussian white noise. We show that an extended time-delayed feedback scheme is able to influence the global cooperative dynamics of the ensemble of neural populations by local application of the stimulus to a single system. We discuss effects on the stochastic synchronization of coupled neural oscillators in dependence upon the control parameters given by the time delay, feedback strength, and memory parameter. We show that increasing the memory parameter enhances the cooperative dynamics of the two subsystems.

Keywords: Synchronization, noise, coupling, time-delayed feedback

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The network of neurons in the brain exhibits a subtle balance of dynamic chaos and selforganized order. A number of neurological diseases like Parkinson or epilepsy are characterized by a disturbance of this balance, e.g. synchronized firing of electrical pulses of the neurons [1, 2]. Modern concepts of time-delayed feedback control have recently been applied to suppress this undesired synchrony [3, 4].

Various models have been introduced to describe the neural systems. Here, we consider two mutually coupled FitzHugh-Nagumo systems which are subject to independent random fluctuations [See Fig. 1(a)]:

\[
\begin{align*}
\dot{x}_1(t) &= x_1(t) - \frac{x_1^3(t)}{3} - y_1(t) + C[x_2(t) - x_1(t)] \\
\dot{y}_1(t) &= x_1(t) + a + K \sum_{n=0}^{\infty} R^n [y_1(t - (n+1)\tau) - y_1(t - n\tau)] + D_1 \xi_1(t) \\
\dot{x}_2(t) &= x_2(t) - \frac{x_2^3(t)}{3} - y_2(t) + C[x_1(t) - x_2(t)] \\
\dot{y}_2(t) &= x_2(t) + a + D_2 \xi_2(t),
\end{align*}
\]

where \(x_1, y_1\) and \(x_2, y_2\) represent two different neurons or neuron populations, which are diffusively coupled with coupling strengths \(C\). The variables \(x_1, x_2\) are related to the transmembrane voltage and \(y_1, y_2\) correspond to various quantities connected to the electrical conductance of the relevant ion currents. Throughout this paper, we fix the system’s parameters as \(a = 1.05\), \(\varepsilon_1 = 0.005\), and \(\varepsilon_2 = 0.01\). Each neuron is driven by Gaussian white noise \(\xi_i(t)(i=1,2)\) with zero mean and unity variance. The noise intensities are denoted by parameters \(D_1\) and \(D_2\), respectively. We keep \(D_2\) fixed at \(D_2 = 0.09\)
FIGURE 1. Panel (a): Dynamics of a single FitzHugh-Nagumo system without control according to Eq. (1a) for a noise intensity $D = 0.6$. Panel (b): Time series of two coupled FitzHugh-Nagumo systems, $x_1, x_2$, and their sum $x_\Sigma = x_1 + x_2$ for different noise intensities $D_1 = 0, 0.05$, and $1$ in the absence of control, i.e. $K = 0$. Other parameters: $C = 0.07$ and $D_2 = 0.09$. The red dashed and dash-dotted lines denote the null-isoclines.

in the following.

The parameters of the time-delayed feedback scheme are the feedback gain $K$, the time delay $\tau$, and the memory parameter $R$. With this method, a control force is constructed from the differences of the states of the system which are one time unit $\tau$ apart. The memory parameter $R \in (-1, 1)$ can be seen as a weight of states that are further in the past. One could also consider application of the feedback scheme to both subsystems and effects of different values of the control parameters for each subsystem, but these investigations are out of the scope of this work.

The case $R = 0$, also known as Pyragas control [5], was investigated in Ref. [6]. Thus, we extend the work of Hauschildt et al. [6] by application of a different external stimulation with multiple time delays. This method, also known as extended time-delayed feedback, was initially proposed by Socolar et al. in order to stabilize unstable periodic orbits [7]. It generalizes the Pyragas scheme by introducing an additional memory parameter. Previously, time-delayed feedback has also been used to influence noise-induced oscillations of a single excitable system [8, 9]. The effect of extended time-delayed feedback control upon noise-induced oscillations below a Hopf bifurcation has been studied in [10]. For further application of this control scheme see Ref. [11] and the references therein.

In the absence of control ($K = 0$) and for proper choices of the noise intensities and coupling strength, the two subsystems exhibit cooperative dynamics as can be seen from the time series in Fig. 1(b). There are various measures of the synchronization of coupled systems. For instance, one can consider the average interspike intervals of each subsystem, i.e. $\langle T_1 \rangle$ and $\langle T_2 \rangle$, calculated from the $x$ variable of the respective subsystem and their ratio $\langle T_1 \rangle / \langle T_2 \rangle$ as depicted in Fig. 2. The bright regions in Fig. 2(c) indicate a strongly synchronized behaviour of the two subsystems.

It was shown in Ref. [6] that application of time-delayed feedback of Pyragas type is able to change coherence, time-scales, and synchronization of noise-induced oscillations. In the following, we consider effects of the additional control parameter $R$ for the cases of moderately, weakly, and strongly synchronized realizations of the neural system by choosing parameters as (i) $C = 0.2, D_1 = 0.6$, (ii) $C = 0.1, D_1 = 0.6$, and (iii)
FIGURE 2. Absence of control \((K = 0)\): Average interspike intervals of each subsystem \((\langle T_1 \rangle\) as black solid curve and \((\langle T_2 \rangle\) as red dashed curve) and their ratio \((\langle T_1 \rangle/\langle T_2 \rangle\) as green dotted curve) on dependence on the noise intensity \(D_1\), the coupling strength \(C\) in panels (a) and (b), respectively. Panel (c): Ratio of the average interspike intervals in the \((D_1,C)\) plane.

\[C = 0.2, D_1 = 0.15, \text{ respectively.}\]

The results are shown in Fig. 3. The ratio of the average interspike intervals is depicted in the plane parameterized by the time delay \(\tau\) and feedback gain \(K\), where the left, middle, and right plot corresponds to the case of moderately, weakly, and strongly synchronized systems, respectively. The memory parameter is chosen as \(R = 0, 0.35, 0.7,\) and 0.9 in panel (a), (b), (c), and (d) of each subfigure, respectively. In all three cases, one can see that an increase of the memory parameter \(R\) yields a more uniform behaviour over a large range of control parameters in comparison to the case of \(R = 0\). For instance, the minima at \(\tau = 2\) are less pronounced. The extended time-delayed feedback method is less likely to desynchronize the two subsystems. Therefore, this control method can be considered more robust in terms of a variation of the feedback gain \(K\) and time delay \(\tau\). However, not only the cases of strong desynchronization are weakened, but the realizations of enhanced cooperative dynamics loose some synchronization as well. This can be seen, for instance, in the right plot of Fig. 3, where the bright regions become darker.

These effects can also be seen in Fig. 4, which shows the ratio of the average interspike intervals in dependence on the time delay \(\tau\). The feedback gain is fixed at \(K = 1.5\) in all plots and the memory parameter \(R\) is chosen as \(R = 0\) and 0.9 in panels (a) and
FIGURE 4. Ratio of the average interspike intervals of moderately, weakly, and strongly synchronized systems in the left, middle, and right plot, respectively. In each subfigure, panel (a) and (b) corresponds to a memory parameter of $R = 0$ and 0.9, respectively. The color code is as in Fig. 2(a) The feedback gain $K$ is fixed at $K = 1.5$.

(b) of each subfigure, respectively. Thus, Fig. 4 can be understood as a horizontal cut through Fig. 3. For a larger memory parameter, both the maxima and minima are less pronounced.

In conclusion, we have investigated the effect of extended time-delayed feedback on the cooperative dynamics of two coupled excitable neural systems in the presence of noise. We have shown that incorporating states into the control force, which are further in the past, the feedback scheme is able to enhance the synchronization of the two subsystems. An increase of the additional memory parameter leads to an enhanced synchronization of the two subsystems.

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