An Agent Based Model of the E-Mini S&P 500 Applied to Flash Crash Analysis

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Abstract—We propose a zero-intelligence agent-based model of the E-Mini S&P 500 futures market, which allows for a close examination of the market microstructure. Several classes of agents are characterized by their order speed and order placement within the limit order book. These agents’ orders populate the simulated market in a way consistent with real world participation rates. By modeling separate trading classes the simulation is able to capture interactions between classes, which are essential to recreating market phenomenon. The simulated market is validated against empirically observed characteristics of price returns and volatility. We therefore conclude that our agent based simulation model can accurately capture the key characteristics of the nearest months E-Mini S&P 500 futures market. Additionally, to illustrate the applicability of the simulation, experiments were run, which confirm the leading hypothesis for the cause of the May 6th 2010 Flash Crash.

I. INTRODUCTION

With the advent of electronic financial markets for the exchange of securities and derivatives, the electronic centralized order book has become the standard market mechanism for price discovery in today’s financial markets. Through this form of exchange market participants have been offered a more liquid market system that has kept bid-ask price spreads small, increased market depth, and decreased executions transaction times [1].

Many market participants now employ algorithmic trading, commonly defined as the use of computer algorithms to automatically make trading decisions, submit orders, and manage those orders after submission [2]. Algorithmic trading is so prevalent that the speed of order submission has emerged as a principal characteristic for distinguishing trading agents. Market participants known as high-frequency traders (HFTs), can trade hundred times of a second, using fast (often deterministic) algorithms, as well as, specialized network connections and operation rules with the trading exchanges [3]. HFTs are often orders of magnitude faster in order submission than other trading agents, and even other algorithms [3].

The rise of algorithmic trading has had broad and direct impacts on the financial markets. Concerns about the effects of high-frequency trading on the price discovery process and market price stability were first widely acknowledged after the events of May 6, 2010. During which the world markets were perturbed by one market participant’s algorithm that caused a sharp price drop, known as a flash crash, in the E-Mini S&P futures market. The action in the E-Mini market spread to other futures markets and equities. Markets fell nearly 6% in minutes, after which they recovered nearly as quickly. Individual securities experienced similar incidents falling as much as 15%, before rebounding close to their original price [4]. At the time of the May 6, 2010 flash crash, algorithmic trading was thought to be responsible for more than 70% of trading volume in the U.S. Kirilenko et al. [4] have shown that the key events in the Flash Crash have a clear interpretation in terms of algorithmic trading.

Since this event, a number of different rules have been proposed to help create “circuit breakers”, mechanisms to allow markets to recover from sharp price movements [5]. However, the vulnerability of these markets is still in question. Therefore, a study is needed to examine the impact of low latency traders on rapid price movements.

We assert that price discovery and the dynamics of the order book are the result of a complex system containing large number of agents (i.e., market participants), which interact with one another in a manner that cannot be stochastically determined and therefore cannot easily be empirically studied. This agent-based simulation can be used to study potential regulatory actions and the extreme dynamics such as flash crashes. The simulation is parameterized to replicate market for the lead month E-mini S&P 500 futures contract. The simulation contains classes of agents configured to closely reflect real participation rates in the E-mini market. To illustrate the usefulness of this type of simulation an experiment was run to examine the cause of the May 6th 2010 Flash Crash.

II. BACKGROUND

A. Literature Review of Financial Simulations

A simplified conception of a financial market includes a set of market participants, a trading mechanism, and a set of securities. An agent-based model (ABM) has a similar structure and includes a set of agents, a topology and an environment [6]. In a typical ABM of a financial market, the market participants are agents, the market mechanism is the
simulation is about 1% in 9 seconds and deduced. Figure d through the model they postulated. The mass of the hypothesis deduced algorithm which sold 45,000 contracts, valued at $1.9 billion, in 20 minutes. The algorithm examined the previous minute trading volume and placed an order for 9% of that volume [9]. The initial trades were absorbed by high frequency traders, fundamental buyers, and market makers. However, once the high frequency traders hit their long position limit they also began selling, further driving the price down. The execution algorithm, analyzing only volume, attributed this to buy side interest and began placing larger orders. Fundamental traders and market makers began leaving the market, which decreased the buy side liquidity and forced high frequency traders to trade amongst themselves, further driving the price down. Within 20 minutes the E-mini fell in price by more than 5% [4]. Due to the interconnectedness of markets this event propagated through both the commodities and the equities market [4].

Flash crashes and flare ups have gotten more focus after the infamous May 6th incident. However, this is largely because the flash crash on May 6th propagated across multiple markets. In fact, flash crashes and flare ups happen regularly in both the commodities markets and the equity markets [10]. Natural gas experienced a flash crash on June 8th, 2011. The price fell about 1% in 9 seconds and recovered over the next several minutes [10]. Figure 1 illustrates the number of reported flash crashes and flare ups, in just the equity markets over the last 5 years, as reported by [10].

Figure 1: Number of Flash Crashes & Flare ups
The May 6th flash crash is not an isolated event. However, there has been a dramatic decline in these events since 2008. This maybe because algorithm programing has improved or new regulations, such as circuit breakers were created for stocks. Circuit breakers pause trading if a stock moves more than 10% in a 5 minute period [5].

There has been a large amount of press and regulations recently focusing on flash and flare events. However, are these events actually harmful to the market? There are people who argue that flash and flare events are detrimental to the market. They shake market confidence and when a trade is executed at an extreme price there is no guarantee that it will be an algorithmic trader getting hurt. In fact, small investors whose stop-loss orders were triggered sold their stocks at unreasonably low prices. Although some of these trades were cancelled, many were not, which resulted in loss of money and higher taxes for some traders [11]. Additionally, there is no regulation on when and how these trades are canceled, which has led to accusation of preferential treatment for certain traders. In the end it is in the best interest of the exchanges, algorithmic traders, and traditional traders to have a stable market.

III. SIMULATION DESIGN

A. Agent Design

Current literature on the events of May 6, 2010 and other less significant flash crashes, suggest that the markets are divided into subcategories of traders and the combination of trading styles are responsible for these events. These combinatorial aspects lead us to believe that it was necessary to have multiple categories of trading agents in the simulation. However, as to not presume specific trading strategies for agents, it was necessary to design the agents to be zero intelligent. The agents were design with constraints on their behaviors, such as position limits. From [4] and [9], we were able to use their characterizations of the participants of the E-Mini S&P Futures market by placing them into one of six categories of trader types:

Fundamental Buyers and Sellers – take long or short positions on the asset during the entire duration the markets exist and trade with a low frequency.

Market Makers – take the position of straddling both sides of the market by taking long and short positions on an asset. Intermediaries’ trades are meant to give the market liquidity.

Opportunistic – take a long or short positions on the asset during the duration of the market day like a fundamental trader. However, they implement trading strategies that make them resemble Intermediaries because they do not take a large position.

High Frequency - take the position of straddling both sides of the market by taking long and short positions on an asset for short periods and trading with high frequency near the best-ask and best-bid sides of the book. They allow themselves to take large positions for short periods of time but will try to be neutral by the end of day.

Small Traders – take either a long or short positions on the asset during the entire duration of the markets exist and trade with a very low frequency.

From the work done by Kirilenko, et al [4] the following set of trader characteristic data was taken and used to derive the following basic market, which were then use to construct our agents.

Table 1: S&P 500 Market Participation Description

<table>
<thead>
<tr>
<th>Trader Type</th>
<th># of Traders</th>
<th>Trade Speed</th>
<th>Position Limits</th>
<th>Market Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>6880</td>
<td>2 hr</td>
<td>-30 – 30</td>
<td>1%</td>
</tr>
<tr>
<td>Fundamental Buyers</td>
<td>1268</td>
<td>1 min</td>
<td>-∞ - ∞</td>
<td>9%</td>
</tr>
<tr>
<td>Fundamental Sellers</td>
<td>1276</td>
<td>1 min</td>
<td>-120 – 120</td>
<td>9%</td>
</tr>
<tr>
<td>Market Makers</td>
<td>176</td>
<td>20 sec</td>
<td>-120 – 120</td>
<td>10%</td>
</tr>
<tr>
<td>Opportunistic</td>
<td>5808</td>
<td>2 min</td>
<td>-120 – 120</td>
<td>33%</td>
</tr>
<tr>
<td>High Frequency</td>
<td>16</td>
<td>0.35 sec</td>
<td>-3000-3000</td>
<td>38%</td>
</tr>
</tbody>
</table>

This classification of traders was determined using the following two variables to determine which category a trader fell under.

- Trade Speed - Average amount of time taken between order placements or cancelations.
- Position Limit - Number of contracts allowed to be held.

Using order book data from the E-Mini S&P 500 contract provided by the CFTC and the classification process, we described empirically the style in which the agents placed orders into the order book. This was done by determining each trading class order size and order price distribution.

- Order Size - Distribution of order quantity size.
- Order Price Selection – Distribution describing the number of ticks a order’s price was in reference to the last trade price (see Figure 2).
Using these four variables in unison we were able to create the six classes of agent for our market. These characteristics keep the traders in the realm of zero intelligence through their lack of interaction with market data to determine their actions.

Figure 2: Order Price selection for Market Makers

Figure 3: Components of an Agent Class

B. Topography

The design of the simulated market exchange systems follows a traditional ‘price then time’ order book market using a set of agents to input transactional messages to the order book. This topography allows for asset price creation on the part of the market participants through their individual actions (order, cancel) and the market matching engine connecting them (execute).

C. Model Validation

In the creation of a properly functioning market simulation it becomes a necessary to strike a balance between characterizing the market structure and participants perfectly and making the market work in a manner that behaves similar to the “stylized facts” [12]. “Stylized facts” are statistical financial time series phenomena that are typically found in market data. We demonstrate the accomplishment of aforementioned tasks by achieving the following set of criteria for validation:

- Market Structure Design
  - Trader Order and Execution Rates
  - Order Book Construction
- Stylized Price Characteristics
  - Distribution of Price Returns
  - Volatility Clustering
  - Absence of Autocorrelation of Returns
  - Aggregation of Returns

1. Market Structure Design

   Trader Order and Execution Rates

   To demonstrate that the model appears to have the same physical characteristics in trade volume and cancelation rates, a simple comparison between the real and simulated S&P 500 E-Mini markets is provided (Table 1)

   Table 2: Parameter Comparisons

<table>
<thead>
<tr>
<th>Trader Type</th>
<th>Simulated Volume</th>
<th>Actual Volume</th>
<th>Simulated Cancelation Rate</th>
<th>Actual Cancelation Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>1%</td>
<td>1%</td>
<td>40%</td>
<td>20-40%</td>
</tr>
<tr>
<td>Fundamental Buyers</td>
<td>10%</td>
<td>9%</td>
<td>44%</td>
<td>20-40%</td>
</tr>
<tr>
<td>Fundamental Sellers</td>
<td>10%</td>
<td>9%</td>
<td>44%</td>
<td>20-40%</td>
</tr>
<tr>
<td>Market Makers</td>
<td>10%</td>
<td>10%</td>
<td>35%</td>
<td>20-40%</td>
</tr>
<tr>
<td>Opportunistic</td>
<td>31%</td>
<td>33%</td>
<td>50%</td>
<td>40-60%</td>
</tr>
<tr>
<td>High Frequency</td>
<td>38%</td>
<td>38%</td>
<td>77%</td>
<td>70-80%</td>
</tr>
</tbody>
</table>

Order Book Construction

The shape that the limit order book takes an equilibrium state that resembles ‘V’ formation when looking at the queue of orders away from the best bid/ask. This was first observed and described by looking at the Paris Bourse order book [13].

This structure has been an interest for econ-physicists who have tried to determine simple models that attempt to recreate this shape. One such model accomplished this by slowly modifying order prices closer to the current best bid/ask over a time given horizon [7]. More advanced models use an advanced calibration on the market data to construct an order placement schema and cancellation methods [13].

The arrival and cancellation process has been the other mechanism used to recreate the limit order book. Zovko and Farmer [14] looked at the creation of the limit order book from the perspective of the average size of orders placed at different price points. They suggested that this order process could be roughly estimated through a stochastic process of arrivals and cancelations using a Poisson distribution.
The model presented in this paper uses multiple agents and order types that are drawn from their specific probability distribution to determine order prices selection. In combination with a Poisson arrival and cancelation process with a mean that is relative to the speed that a trader class is allowed to place and cancel orders. This process allows us to capture a 'V' structure without having to compromise our model by having a single agent class (see Figure 4).

2. Stylized Price Characteristics

Distribution of Price Returns

It has been largely observed that the empirical distributions of financial returns and log returns are fat-tailed. Mandelbrot [16] observed that the tails of a distribution of prices changes are extraordinary long and the sample second moment typically vary in an erratic fashion. This has caused various suggestions regarding the form of the distribution, ranging from the Student-t, hyperbolic, normal inverse Gaussian, and others, but no general consensus exists for the form of the tails for all markets.

Volatility Clustering

The characteristic of volatility clustering is seen in the absolute price returns for securities that have slowly decaying autocorrelation in variance, meaning that price changes tend to follow other price changes of the same size. This was first noted by Mandelbrot [16] and was finally translated into a general based models by Kirman and Teyssiere [17] when they discovered a model would exhibit autocorrelation patterns in the absolute returns if a variable was herded by the positive or negative opinion of an asset:

\[
\text{Herd}_t = \text{Herd}_{t-1} + \text{UniformDistribution}(\text{between} -N\% \& N%)\]

In the observed data from the S&P 500 E-mini and the simulation we can observe this phenomenon by testing for the normality of the distribution of price returns. Figures 5 and 6 illustrate that both the real and simulated date diverges from normality at the tails.
Figure 7 & 8: Volatility clustering of absolute price returns for S&P 500 E-mini and simulation

The simulation implements the same herding variable to influence the decision of opportunistic traders in the model (see Figures 7 and 8). This creates a similar autocorrelation pattern in the absolute returns that is seen in the S&P 500 E-Mini Futures Contract’s minute price returns.

Absence of Autocorrelation of Returns

In proving that markets are efficient, it has been common practice to show that there is no predictability between price movements. In demonstrating this we look at the autocorrelation or returns to show that there is no predictability of markets. Figures 9 and 10 illustrate that this property exists in both the real and simulated market data.

Figure 9 & 10: Absence of autocorrelation for price returns for S&P 500 E-mini and simulation

Aggregation of Returns

The final stylized fact was that as one increases the time scale over which the returns are calculated, the distribution approaches the Gaussian form. This cross-over phenomenon was noted by Kullmann et al [18], where the evolution of the Pareto exponent of the distribution with the time scale is studied.

Figure 11 & 12: Aggregation of price returns for S&P 500 E-mini and simulation

Figures 11 and 12 illustrate the standardized distributions of returns for S&P 500 index for the month of June 2011 and the simulation. It is clear that the larger the time scale
increases, the more Gaussian both sets of distributions become. The next section illustrates how the agent based model described in this paper can be used to examine market phenomenon.

IV. SIMULATION APPLICATION

The agent based model was used to examine the flash crash of May 6th, 2010. An agent is introduced to the model, which tries to sell a large number of contracts. The agent examines the previous minute of trading and executes an aggressive sell order for 9% of the trading volume. Market makers and HFT are constrained by a rule, which forces them to lower their position level if they reach their position limit. Additionally, market makers were calibrated to withdraw from the market if the price falls 24 ticks below the moving average. Furthermore, Fundamental traders withdraw from the market and stop loss orders are triggered if the price drops 70 ticks below the starting price. Lastly, a market pause is initiated when the price drops more than 1.3% in a second. The simulation was allowed to run with no other outside influences. The graph of the price and the moving average volume is displayed below. Additionally the graph of the actual E-mini S&P 500 flash crash is displayed for comparison in Figures 13 and 14.

Since the simulated market is 1/32 of the actual of the E-mini market, there are some scaling issues that arise. Most noticeably the prominent spike in volume is not as defined in the simulated market. This is because there is not as much liquidity in the market, thus it does not take as much trade volume to cause a flash crash. However, the simulation accurately represents a spike in volume corresponding to the flash crash. However, the simulation accurately replicated the defining characteristics of a flash crash.

Once the large agent begins selling shares, the first noticeable impact in the market was the Bid depth began to decay at a fast pace. After which fundamental traders begin pulling out of the market and the order book depth on both sides of the market crashes, which is illustrated in Figure 15.

The crash in bid depth occurs at the same time the price begins an exponential decline (i.e., flash crash). This relationship between rapid liquidity loss and rapid price drops is not a ground breaking finding. However, this leads to the question: “Is it possible to prevent the rapid loss of liquidity by changing parameters in the market?”

In recent years High Frequency Traders have been gaining an increasing amount of press. Their rapid speed allows them to trade on millisecond price fluctuations. A major benefit of high frequency traders is they provide liquidity to the market. However, questions have arisen about whether high frequency traders contributed to the flash crash. In the CFTC and SEC Report [9] it was shown that High Frequency Traders consumed a large amount of the initial large sell order. Once these algorithms reached their contract holding constraints they began aggressively selling in front of a large trader. This helped to drive the priced down but was it the reason for the flash crash? An experiment was run to determine if high frequency trader enabled the flash crash. The size of the large sell orders were adjusted as well as the number of high frequency traders to see how these affected the minimum price of the simulation (i.e., measure of how bad the flash crash is). A total number of 40 simulations were run for each variable pair and the minimum price in each simulation was recorded. The median price for each variable pair is illustrated in the Figure 16.
As the number of high frequency traders were reduced to zero the minimum price of the simulation increased. This illustrates that HFTs are necessary for the events on May 6th to occur. It has been hypothesized that HFT traded amongst themselves, as liquidity dried up. This rapid trading was termed “Hot Potato”. The Staff of the CFTC & SEC [9] discovered that the rapid trading increased the trade volume, which further increased the large execution algorithm sell orders. This self-perpetuating cycle was concluded to be the cause of the flash crash. In future work we will examine, if in fact “hot potato” is occurring in the simulation.

If the execution algorithm is parameterized to execute smaller trade sizes, the severity of the price drop is greatly decreased. Although, the aforementioned cycle is occurring the market is able to absorb it because it increases at a slower rate. This leads to the conclusion that the Flash crash would not have occurred without HFT and large sell orders from the execution algorithm.

V. CONCLUSION

The simulation was design to be flexible enough to accurately model any electronic financial market. The simulation has been calibrated to reflect the E-MINI S&P 500 Futures market. Using both market structure and stylized price characteristics the model is shown to be an acceptable representation of the E-MINI Market.

To illustrate the significance of this type of simulation an analysis of the May 6th, flash crash was presented. It was found that HFTs were needed to generate the May 6th flash crash. However, it was also determined that if the sell execution algorithm used a smaller percentage of the previous minute’s trade volume, to determine its order size, or taken into account price movement, the flash crash would not have occurred.

This simulation can be used to help regulators determine important factors that contribute to market phenomenon. Additionally, it can be used to examine how new regulations may affect the current market dynamics.

Future work will seek to use the simulation to examine possible indicators of the flash crash. Additionally, the simulation will be expanded to allow for multiple markets and more granular trading strategies.

ACKNOWLEDGMENT

We would like to thank the members of the Commodity and Future Trade Commission, and Andrei Kirilenko, the Chief Economist of the Commodity and Future Trade Commission.

REFERENCES


