A New Multi-Dimensional Graphical Approach for Economics
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Abstract
This research will present a new multi-dimensional graphical approach for economics. The new multi-dimensional graphical approach is based on the design, formulation and application of multi-dimensional coordinate spaces. All these multi-dimensional coordinate spaces can support the graphical visualization of multi-variable economic data. This paper is divided into three parts. The first part reviews the literature about the evolution of graphical methods in economics. The second part will explain the differences between Cartesian coordinate plane and coordinate space. The third part of this paper shows a new set of multidimensional coordinate spaces that can be applied in the study of economics.

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References

1. The Evolution of Graphical Methods in Economics
   For long time, many economists, academics and policy makers they are using different mathematical and statistical methods and models in the study of economics. It is possible to be observed in different academic journals, text books in economics and economic reports. Hence, the study of economics highly depended on the development of new mathematics and statistics methods.

   Moreover, it is necessary to mention about the role of econometrics in the study of economics. The econometrics can be considered as the unified research field that joins mathematics, statistics and economics together. The uses of econometrics can be considered the essential quantitative analytical tool for many economists, academics and policy makers for the study of economics. It is based on the uses of economic data to verify the cause and effect of any economic phenomenon.
In fact, the application of mathematical and statistical methods and models in the study of economics also involves the uses of graphs. Therefore, this paper shows a strong link between the introduction of graphical methods in economics and the development of theories, methods and techniques in mathematics and statistics. In the 18th century, for example, several new graphical methods were developed as a result of enhanced mathematics and statistics research. These graphical methods include line graphs of time series data (since 1724), curve-fitting and interpolation (1760), measurement of error as a deviation from a graphed line (1765), graphical analysis of periodic variation (1779), statistical mapping (1782), bar charts (1756) and printed coordinate paper (1794) (See Beniger and Robyn, 1978).

In the 18th century, new graphical methods for economic analysis were also introduced by renowned economists like William Playfair, Francis Ysidro Edgeworth and William Stanley Jevons. According to Harro Maas (2005), William Playfair (1801) constructed a wonderful collection of plates and graphs at the end of the eighteenth century. In his book entitled Commercial and Political Atlas, Playfair focused on the study of trade cycles. This placed him far ahead of other economists at the time in terms of visualizing socio-economic data.

The development of the usage of graphical methods in economics can be classified into two distinct phases. The first phase is the “descriptive graphical method”. The descriptive graphical method is supported by simple tables, histograms, line graphs and scatter-plots. All these types of graphs are based on the visualization of a single economic variable (vertical axis) through a specific period of time (horizontal axis). The main objective of the descriptive graphical method in economics is to study the behavior of a single economic variable (e.g. exports, imports, unemployment, G.D.P., inflation rate etc…) within a time frame (per decade, annually, monthly, weekly or daily) based on time-series. In fact, William Playfair may be considered the pioneer and promoter of the descriptive graphical method.

The second phase in the development of graphical methods in economics is called the analytical graphical method. The analytical graphical method in economics features two 2-Dimensional Cartesian coordinate plane and 3-Dimensional coordinate space. According to Harro Maas, it was William Stanley Jevons who first explored the merits of the graphical method for the political economy. Jevons did this by introducing the function called “King-Devenant Law of Demand”. This is a case in which the analytical graph system is used in economics, where the form of the graph gives an idea of the possible class of the functions describing the relationship between \(X\) and \(Y\) variables. However, Jevons termed \(X\) and \(Y\) variables as variable and variant respectively, terminologies that suggest a causal interpretation of the relationship between \(X\) and \(Y\).

It is perhaps fitting to mention here that the formal graphical method based on the 2-Dimensional coordinate system \((X,Y)\) was introduced in 1637 by René Descartes, whose contributions to different scientific disciplines, of which economics was one, were substantial. The Cartesian coordinate system opened a new era in economic analysis by providing for the analysis of a single economic phenomenon based on the relationship between two variables.

As far as the application of the analytical graphical method in economics is concerned, it is necessary to mention the major contribution of Antoine Augustin Cournot. Cournot (1838) derived the first formula for the rule of supply and demand as a function of price. He was also the first economist to draw the supply and demand curves on a graph. Cournot believed that economists should utilize graphs only to establish probable limits and express less stable facts in more absolute terms. He further held that the practical use of mathematics in economics involves not only strict numerical precision, but also graphical visualization. Besides Cournot and Jevons,
other innovative economists who contributed to the analytical graph system in economics over time were Leon Walras, Vilfredo Pareto, Alfred Marshall and Francis Ysidro Edgeworth (McClelland, 1975).

In the 20th century the use and application of graphs among economists were often based on sophisticated mathematical techniques introduced during the development of new economic models. In particular, calculus, trigonometry, geometry and statistical and forecasting methods started to be employed by economists, academics and policy makers in constructing their graphs during that time. It is based on the uses of 2-Dimensional Cartesian coordinate plane and the 3-Dimensional coordinate space were also a part of complex economics research (Avondo-Bodino, 1963). The rapid development of the analytical graphical method has been facilitated by modern technology and sophisticated analysis instruments such as the electronic calculator and the computer. The development of analysis instruments in economics took place in two stages. The first stage involved “basic computational instruments”, where electronic calculators were used to compute basic mathematical expressions (e.g. long arithmetic operations, logarithms, exponents and squares). This took place between the 1950’s and 1960’s.

The second stage, involving “advanced computational instruments”, took place in the mid-1980’s when high speed and storage-capacity computers using sophisticated software were first introduced. The use of sophisticated software enabled efficient information management. This facilitated the application of difficult simulations as well as the creation of high resolution graphs under the 3-Dimensional coordinate space. Breakthroughs and new developments in designing and manufacturing cutting-edge analysis instruments undoubtedly contributed substantially to development and research in economics.

2. Differences between the Cartesian coordinate plane and coordinate space

Firstly, the idea to use coordinate systems in economics is to represent graphically the behavior of economic data in certain periods of time and space. Hence, we have two coordinate systems: the 2D Cartesian coordinate plane and 3D coordinate space (see Figure 1). In the case of 2D Cartesian coordinate plane shows the relationship between one dependent variable \( Y \) and one independent variable \( X \) given by the function \( Y=f(X) \). And the 3D coordinate space \((X, Y, Z)\) shows only the relationship that exist between two independent variables \((X,Y)\) and one dependent variable \( Z \).

Basically, the difference between the 2D Cartesian coordinate plane and 3D coordinate space is originated by the number of axes. In our case each axis represents a dimension in the space and time. In the case of the 2D Cartesian coordinate plane is based on the uses of two axes and the 3D coordinate space is based on the uses of three axes. Therefore, these two coordinate systems are available to show graphically any historical economic data, economic modeling and economic simulation through the uses of several lines in a logic order by length, width, height and colors.

Usually, the graphical modeling applied on economics until today, it is based on the application of 2D Cartesian coordinate plane. It is based on the observation of eight hundred different papers were published in twenty-one reputable economics journals\footnote{JSTOR and SCOPUS, 2009}, between 1909 and 2009, it can be observed that common types of graphical representations applied in economics were the 2-Dimensional Cartesian coordinate plane with 99%. Only 1% of them applied the 3-Dimensional coordinate spaces on the uses of surfaces.

According to this paper the 2D Cartesian coordinate plane show certain limitations in the process to visualize multi-variable economic data into the same graphical space. The limitations
of the 2D Cartesian coordinate plane encourage propose an alternative set of multi-dimensional coordinate spaces that can facilitate the visualization multi-variable economic data in the same graphical space. Hence, the transition in the uses from 2D Cartesian coordinate plane to MD coordinate space depends on the learning and teaching process to plot and draw MD graphs. The main idea to use MD graphs is to observe possible changes of multivariable economic data behavior simultaneously. To understand better the uses and application of MD coordinate spaces, this paper suggests the review of Econographicology (Ruiz Estrada, 2007) in order to learn and understand much better MD graphs. The construction and analysis of multi-dimensional graphs suggest three basic steps follow by:

1. Basic knowledge about how to plot and draw 2-Dimensional graphs on the 2D Cartesian coordinate plane.
2. Basic knowledge of the Euler’s geometry ($\mathbb{R}^n$-space)
3. The construction of solid prototype(s) (Ruiz Estrada, 2009) by using basic materials such as wood, papers, plastics and others to represent different MD coordinates spaces

Additionally, the uses of MD graphs request the application of the Omnia Mobilis assumption by Ruiz Estrada, Yap and Shyamala (2008) to generate an environment of freedom in different variables that interacting in the same MD graph. Hence, the use of Ceteris Paribus assumption application on MD graphs can be considered minimum. In view of this, a new, multi-dimensional graphical approach is introduced here. It is based on the uses of different multi-dimensional coordinate spaces: the pyramid coordinate space; the diamond coordinate space; the 4-dimensional coordinate space; the 5-dimensional coordinate space; the infinity-dimensional coordinate space; the inter-linkage coordinate space; the cube-wrap coordinate space; the mega-surface coordinate space.

**Figure 1:**
The 2-Dimensional Cartesian Coordinate Plane and 3-Dimensional Coordinate Space
3. Classification of Multi-Dimensional Coordinate Spaces

3.1. The Pyramidal Coordinate Space

The pyramidal coordinate space consists in four independent axes \((X_1, X_2, X_3, X_4)\) and one dependent axis \((Y^*)\). The \(Y^*\) axis is positioned in the center part of this coordinate space among of the other four axes: \(X_1, X_2, X_3, X_4\). The function used by the pyramidal coordinate is fixed by \(Y^* = f(X_1, X_2, X_3, X_4)\). The pyramidal coordinate space show clearly into the same graphical space any possible change(s) of any or all values plotted on each or all \(X_1, X_2, X_3, X_4\) axes that can affect directly on the behavior of \(Y^*\) axis value.

In order to plot different values in each axis into the pyramidal coordinate space, we need to plot each value directly on its axis line. At the same time, all values were plotted on each axis line need to be joined together by straight lines until we can visualize a pyramid-shaped figure with five faces (see Figure 2). Therefore, we have two possible graphical scenarios: first graphical scenario, if all or any \(X_1, X_2, X_3, X_4\) axes values move from outside to inside, then \(Y^*\) axis value move down. Second graphical scenario, if all or any \(X_1, X_2, X_3, X_4\) axes values move from inside to outside, then \(Y^*\) axis value move up. Basically, the pyramidal coordinate system is represented by: 

\[
(1.) \quad ([X_1, X_2, X_3, X_4], Y^*)
\]

*Figure 2: The Pyramidal Coordinate Space*

3.2. The Diamond Coordinate Space

The diamond coordinate space has two levels of analysis and ten axes. Each level of analysis is represented by \((X_{L:i}, Y_{L:i})\), where “L” represents the level of analysis, in this case either level one (L1) or level two (L2); “i” represents the quadrant level of analysis (in this case, quadrant 1, 2, 3 or 4). In order to plot different values in each axis into the diamond coordinate space with ten axes, we need to plot each value directly on its axis line respectively. At the same time, all values were plotted on each axis line need to be joined together by straight lines until we can build a diamond-shaped figure with eight faces (see Figure 3). It is important to mention at this juncture that the first level (L1) has five axes represented by \(X_{1:1}, X_{1:2}, X_{1:3}, X_{1:4}, Y_1\). Four independent axes represented by \(X_{1:1}, X_{1:2}, X_{1:3}, X_{1:4}\) and one dependent axis fixed by \(Y_1\) respectively. The second level (L2) has five axes represented by \(X_{2:1}, X_{2:2}, X_{2:3}, X_{2:4}, Y_2\). We assume that does not exist any relationship between level one (L1) and level two (L2) of
analysis. The common issue between these two levels of analysis is that both levels use the same $X_{L,1}$ axes in the diamond coordinate space.

However, level one (L1) of the analysis cannot affect level two (L2) of the analysis, and vice versa. If we draw different levels of analysis in the diamond coordinate space, we can visualize and compare two different scenarios in the same diamond coordinate space at the same time (see Figure 5). It is crucial to mention at this point that the fifth and tenth axes ($Y_1$ and $Y_2$) is positioned in the center part of the diamond coordinate space among the other eight axes: $X_{1:1}$, $X_{1:2}$, $X_{1:3}$, $X_{1:4}$, $X_{2:1}$, $X_{2:2}$, $X_{2:3}$, $X_{2:4}$. We assume that both $Y_{L}$ ($Y_1$, $Y_2$) use only real positive numbers $\mathbb{R}$. Therefore, the diamond coordinate space all $X_{1:1}$, $X_{1:2}$, $X_{1:3}$, $X_{1:4}$, $Y_1$, $X_{2:1}$, $X_{2:2}$, $X_{2:3}$, $X_{2:4}$, $Y_2$ axes are either on the positive side of respective axes together. The only uses of positive axes in the diamond coordinate space request the uses of absolute values. The uses of absolute values in each axis are based on the application of the non-negative properties. Hence, all axes $/X_{1:1}$, $/X_{1:2}$, $/X_{1:3}$, $/X_{1:4}$, $/Y_1$, $/X_{2:1}$, $/X_{2:2}$, $/X_{2:3}$, $/X_{2:4}$, $/Y_2$ always use values large or equal than zero. The final result, if the two levels of analysis are joined, is possible to visualize a diamond-shaped figure. The diamond coordinate system is represented by: (2.) $([X_{1:1}, X_{1:2}, X_{1:3}, X_{1:4}], Y_1)$ & ($[X_{2:1}, X_{2:2}, X_{2:3}, X_{2:4}], Y_2$)

**Figure 3:** The Diamond Coordinate Space

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### 3.3. The 4-Dimensional coordinate Space

The 4-dimensional coordinate space in vertical position offer four axes: $X_1$, $X_2$, $X_3$, $Y^\ast$. All these four axes are distributed by three independent axes: $X_1$, $X_2$, $X_3$ and one dependent axis: $Y^\ast$. The $X_1$, $X_2$, $X_3$, $Y^\ast$ axes are fixing positive and negative real numbers $\mathbb{R}_{\ast\ast}$. In order to plot different values in each axis into the 4-dimensional coordinate space, we need to plot each value directly on its axis line. All values were plotted on each axis line need to be joined together by straight lines until we can build pyramid-shaped figure with four faces in vertical position (see Figure 4). Additionally, the $Y^\ast$ axis is positioned in the center part of the 4-dimensional coordinate space (among the other three axes). It is the convergent point of all the other three axes: $X_1$, $X_2$, $X_3$. In other words, all $X_1$, $X_2$, $X_3$ axes converge always in the $Y^\ast$ axis. The 4-dimensional coordinate system is represented by: (3.) $([X_1, X_2, X_3], Y^\ast)$
3.4. The 5-Dimensional coordinate Space

The 5-dimensional coordinate space in vertical position consists of five axes: $X_1, X_2, X_3, X_4, Y^*$. All these five axes are distributed by four independent axes: $X_1, X_2, X_3, X_4$ and one dependent axis $Y^*$. The $X_1, X_2, X_3, X_4, Y^*$ are fixing positive and negative real numbers $\mathbb{R}/\mathbb{R}$. In order to plot different values in each axis into the 5-dimensional coordinate space, we need to plot each value directly on its axis line. All values were plotted on each axis line need to be joined together by straight lines until we can build pyramid-shaped figure with five faces in vertical position (see Figure 5). Therefore, the $Y^*$ axis is positioned in the center of the 5-dimensional coordinate space (among the other four axes). The $Y^*$ axis is the convergent axis of all the other four axes: $X_1, X_2, X_3, X_4$. The 5-dimensional coordinate system is represented by: (4.) $((X_1, X_2, X_3, X_4), Y^*)$
3.5. The Infinity Coordinate Space

Basically, the infinity coordinates space under the specific approach offer a new coordinate system according to expression 5. The basic coordinate space system is formed by three levels of analysis: general-space (i); sub-space (j); micro-space (k). In the case of plotting into this coordinate space start with define our specific general-space (i), sub-space (j), micro-space (k), alpha-space (α) and beta-space (β) respectively.

\[
(\alpha_{<i:j:k>}, \beta_{<i:j:k>})
\]

The infinity coordinate space under the specific approach is available to show different dimensions that is not possible to be observed in the classic 2-dimensional Cartesian coordinate plane and 3-dimensional coordinate space. Hence, the 2-dimensional Cartesian coordinate plane and 3-dimensional coordinate space can be considered as sub-axes systems into the infinity coordinate space under specific approach. The structure of the infinity coordinate space under specific approach is formed by infinite general-spaces (i), sub-spaces (j) and micro-spaces (k). There are distributed into different places along the general cylinder (see Figure 6). Therefore, the infinity coordinate space under the specific approach starts from the general space zero (\(i_0\)) until the general space infinity (\(i_\infty\)). And each sub-space starts from sub-space zero (\(j_0\)) until the sub-space infinity (\(j_\infty\)).

Finally, the micro-space starts from micro-space zero (\(K_0\)) until the micro-space infinity (\(K_\infty\)) (see Expression 6). The infinity coordinate space under the specific approach is available to connect a large number of micro-spaces (k) distributed into the same sub-space (j) and general space (i) by the application of the inter-linkage connectivity of micro-spaces (\(\overline{\varphi}\)). At the same time, the infinity coordinate space under the specific approach is also available to connect a large number of general spaces in the same coordinate space. It is based on the application of the inter-linkage connectivity of general-spaces (\(\overline{\varphi}\)).

\[
\begin{align*}
(\alpha_{<0:0:0>}, \beta_{<0:0:0>}) & \overline{\varphi} (\alpha_{<0:0:1>}, \beta_{<0:0:1>}) \overline{\varphi} \ldots \overline{\varphi} (\alpha_{<0:0:\infty>}, \beta_{<0:0:\infty>}) \\
(\alpha_{<0:1:0>}, \beta_{<0:1:0>}) & \overline{\varphi} (\alpha_{<0:1:1>}, \beta_{<0:1:1>}) \overline{\varphi} \ldots \overline{\varphi} (\alpha_{<0:1:\infty>}, \beta_{<0:1:\infty>}) \\
(\alpha_{<0:2:0>}, \beta_{<0:2:0>}) & \overline{\varphi} (\alpha_{<0:2:1>}, \beta_{<0:2:1>}) \overline{\varphi} \ldots \overline{\varphi} (\alpha_{<0:2:\infty>}, \beta_{<0:2:\infty>}) \\
(\alpha_{<0:0:0>}, \beta_{<0:0:0>}) & \overline{\varphi} (\alpha_{<0:0:1>}, \beta_{<0:0:1>}) \overline{\varphi} \ldots \overline{\varphi} (\alpha_{<0:0:\infty>}, \beta_{<0:0:\infty>}) \\
(\alpha_{<0:1:0>}, \beta_{<0:1:0>}) & \overline{\varphi} (\alpha_{<0:1:1>}, \beta_{<0:1:1>}) \overline{\varphi} \ldots \overline{\varphi} (\alpha_{<0:1:\infty>}, \beta_{<0:1:\infty>}) \\
(\alpha_{<0:2:0>}, \beta_{<0:2:0>}) & \overline{\varphi} (\alpha_{<0:2:1>}, \beta_{<0:2:1>}) \overline{\varphi} \ldots \overline{\varphi} (\alpha_{<0:2:\infty>}, \beta_{<0:2:\infty>}) \\
(\alpha_{<0:0:0>}, \beta_{<0:0:0>}) & \overline{\varphi} (\alpha_{<0:0:1>}, \beta_{<0:0:1>}) \overline{\varphi} \ldots \overline{\varphi} (\alpha_{<0:0:\infty>}, \beta_{<0:0:\infty>}) \\
(\alpha_{<0:1:0>}, \beta_{<0:1:0>}) & \overline{\varphi} (\alpha_{<0:1:1>}, \beta_{<0:1:1>}) \overline{\varphi} \ldots \overline{\varphi} (\alpha_{<0:1:\infty>}, \beta_{<0:1:\infty>}) \\
(\alpha_{<0:2:0>}, \beta_{<0:2:0>}) & \overline{\varphi} (\alpha_{<0:2:1>}, \beta_{<0:2:1>}) \overline{\varphi} \ldots \overline{\varphi} (\alpha_{<0:2:\infty>}, \beta_{<0:2:\infty>}) \\
(\alpha_{<0:0:0>}, \beta_{<0:0:0>}) & \overline{\varphi} (\alpha_{<0:0:1>}, \beta_{<0:0:1>}) \overline{\varphi} \ldots \overline{\varphi} (\alpha_{<0:0:\infty>}, \beta_{<0:0:\infty>})
\end{align*}
\]
3.6. The Inter-Linkage Coordinate Space

The inter-linkage coordinate space is formed by infinite number of general axes \((A_0, A_1, \ldots, A_n \ldots)\), perimeter levels \((L_0, L_1, \ldots, L_n \ldots)\) and windows refraction \((W_0, W_1, \ldots, W_n \ldots)\) (see Figure 7). Each window refraction is based on join its sub-x axis \((X_{A-L})\) with its sub-y axis \((Y_{A-L})\) respectively. Therefore, the window refraction \((W_0, W_1, \ldots, W_n \ldots)\) is follow by the coordinate Space \((X_{A-L}, Y_{A-L})\).

All windows refraction on the same general axis \((A_0, A_1, \ldots, A_n \ldots)\) will be joined together under the application of the inter-linkage connectivity of windows refraction represented by “®”. The inter-linkage connectivity of windows refraction is represented by the symbol “®”. The inter-linkage connectivity of windows refraction “®” will inter-connect all windows refraction \((W_0, W_1, \ldots, W_n \ldots)\) on the same general axis \((A_0, A_1, \ldots, A_n \ldots)\) but in different perimeter levels \((L_0, L_1, \ldots, L_n \ldots)\). Moreover, the inter-linkage coordinate system is represented by (see Expression 7):

\[
\text{Perimeter level } P_0 \circ \text{Perimeter level } P_1 \circ \ldots \circ \text{Perimeter level } P_n
\]

\[\text{General Axis } 0 \ (A_0): \quad W_{0-0} = (x_{0-0}, y_{0-0}) \circ \quad \ldots \circ \quad \ldots \circ \quad W_{0-\infty} = (x_{0-\infty}, y_{0-\infty})\]

\[\text{General Axis } 1 \ (A_1): \quad W_{1-0} = (x_{1-0}, y_{1-0}) \circ \quad \ldots \circ \quad \ldots \circ \quad W_{1-\infty} = (x_{1-\infty}, y_{1-\infty})\]

\[\text{General Axis } 2 \ (A_2): \quad W_{2-0} = (x_{2-0}, y_{2-0}) \circ \quad \ldots \circ \quad \ldots \circ \quad W_{2-\infty} = (x_{2-\infty}, y_{2-\infty})\]

\[\text{General Axis } 3 \ (A_3): \quad W_{3-0} = (x_{3-0}, y_{3-0}) \circ \quad \ldots \circ \quad \ldots \circ \quad W_{3-\infty} = (x_{3-\infty}, y_{3-\infty})\]

\[\text{General Axis } 4 \ (A_4): \quad W_{4-0} = (x_{4-0}, y_{4-0}) \circ \quad \ldots \circ \quad \ldots \circ \quad W_{4-\infty} = (x_{4-\infty}, y_{4-\infty})\]

\[\text{General Axis } 5 \ (A_5): \quad W_{5-0} = (x_{5-0}, y_{5-0}) \circ \quad \ldots \circ \quad \ldots \circ \quad W_{5-\infty} = (x_{5-\infty}, y_{5-\infty})\]

\[\ldots\]

\[\text{General Axis } n \ (A_n): \quad W_{n-0} = (x_{n-0}, y_{n-0}) \circ \quad \ldots \quad \ldots \circ \quad W_{n-\infty} = (x_{n-\infty}, y_{n-\infty})\]
Finally, the inter-linkage coordinate space is available to fix a large number of different functions located in different windows refraction (W_0, W_1, ..., W_n, ...), perimeter levels (L_1, L_2, ..., L_n, ...) and general axes (A_1, A_2, ..., A_n, ...) (see Expression 8):

(8.) \( \text{Perimeter level } P_0 \circ \text{Perimeter level } P_1 \circ \ldots \circ \text{Perimeter level } P_n \)

**General Axis 0** \( (A_0) \): \( y_{0,0} = f(x_{0,0}) \circ y_{0,1} = f(x_{0,1}) \circ \ldots \circ y_{0,\infty} = f(x_{0,\infty}) \)

**General Axis 1** \( (A_1) \): \( y_{1,0} = f(x_{1,0}) \circ y_{1,1} = f(x_{1,1}) \circ \ldots \circ y_{1,\infty} = f(x_{1,\infty}) \)

**General Axis 2** \( (A_2) \): \( y_{2,0} = f(x_{2,0}) \circ y_{2,1} = f(x_{2,1}) \circ \ldots \circ y_{2,\infty} = f(x_{2,\infty}) \)

**General Axis 3** \( (A_3) \): \( y_{3,0} = f(x_{3,0}) \circ y_{3,1} = f(x_{3,1}) \circ \ldots \circ y_{3,\infty} = f(x_{3,\infty}) \)

**General Axis 4** \( (A_4) \): \( y_{4,0} = f(x_{4,0}) \circ y_{4,1} = f(x_{4,1}) \circ \ldots \circ y_{4,\infty} = f(x_{4,\infty}) \)

**General Axis 5** \( (A_5) \): \( y_{5,0} = f(x_{5,0}) \circ y_{5,1} = f(x_{5,1}) \circ \ldots \circ y_{5,\infty} = f(x_{5,\infty}) \)

\[ \ldots \]

**General Axis n** \( (A_n) \): \( y_{n,0} = f(x_{n,0}) \circ \ldots \circ y_{n,\infty} = f(x_{n,\infty}) \)

**Figure 7**: The Inter-Linkage Coordinate Space

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3.7. The Cube-Wrap Coordinate Space

The cube-wrap coordinate space is willing to offer an alternative coordinate space. The main objective of the cube-wrap coordinate space is to show unknown dimensions that cannot be visualized by the 2-dimensional Cartesian plane and 3-Dimensional coordinate space. Initially, the cube-wrap coordinate space is divided by two quadrants. The first quadrant is located on the top of the cube-wrap coordinate space that represents all \( X_i \)-axes. The second quadrant is located under the button of the cube-wrap coordinate space that represents all \( Y_j \)-axes (see Figure 8).
In the process to plot on this coordinate space start by plot each value in its axis line respectively. The space that exists between /X_/i/ -axes and /Y_/j/ -axes will be called “Quadratic-Space Refraction”. It mean that each /X_/i/-axis has its /Y_/j/-axis respectively. The construction of the quadratic space refraction is based on two basic steps: the first step is to plot each value on the /X_/i/-axis line and /Y_/j/-axis line, we suggest to apply the inter-linkage connectivity of micro-spaces (QString) (see Expression 9). Second step is to join the values located on /X_/i/-axis and /Y_/j/-axis by a single straight vertical line.

\( (9. \ ((/X_/i/ \neq \ -/Y_/j/) \) \)

We assume that between \(X_/i/-\)axes and \(Y_/j/-\)axes exists a common single straight vertical line that joint both set of axes. This common single straight line is called the zero space. Hence, the cube-wrap coordinate space starts from the quadratic space refraction zero \( (L_/0/) \) until the quadratic space refraction infinity \( (L_/\infty/) \).

According to the cube wrap Cartesian coordinate space requests the application of absolute values \( |R_/x/-/| \) because the cube-wrap Cartesian coordinate space works only with positive real numbers \( R_/+/-/ \). The final coordinate system to build the cube-wrap space is represented by expression 10.

\( (10. \ CW = ([S_/0/] = (/X_/00/ \ -/Y_/00/) \ \neq \ - ([S_/1/] = (/X_/01/ \ -/Y_/01/) \ \neq \ \\ ... \ \neq \ ([S_/\infty/] = (/X_/\infty/ \ -/Y_/\infty/) \) \)

In the final stage of analysis in the cube-wrap space, it is based on its size. We can have three possible stages that the cube-wrap space can experience anytime:

(11.) If all values are growing constantly in /X_/i/ and /Y_/j/ then the cube-wrap experience an Expansion-stage

(12.) If all values are decreasing constantly in /X_/i/ and /Y_/j/ then the cube-wrap experience a Contraction-stage

(13.) If all values are keeping constant in /X_/i/ and /Y_/j/ then the cube-wrap experience a Static-stage

**Figure 8:** The Cube-Wrap Coordinate Space and the Cube-Wrap Space
3.8. The Mega-Surface Coordinate Space

The mega-surface coordinate space is formed by infinite number of axes in vertical position. Each vertical axis \((X_{ij})\) show positive integer numbers on the top and negative integer numbers on the bottom in the same vertical axis. At the same time, all the vertical axes can be located by its row number \((i)\) and column number \((j)\) in the mega-surface coordinate space (see Expression 14). The idea to apply the mega-surface coordinate space is to build the mega-surface. The mega-surface can show how all vertical axes values behave together in the same graphical space. Initially, the construction of the mega-surface start by join each vertical axis value by straight lines with its neighbor vertical axis: front side; left side; right side; back side (see Figure 9). To join all axes are necessary to apply the inter-linkage connectivity condition \((\nabla)\) on all vertical axes simultaneously.

\[(14.) \quad \nabla X_{ij} \Rightarrow \begin{align*}
X_{11} & \quad X_{12} \\
X_{21} & \quad X_{22} \\
X_{31} & \quad X_{32} \\
\vdots & \quad \vdots \\
X_{\infty 1} & \quad X_{\infty 2} \\
\end{align*} \]

The final analysis of the mega-surface is based on the location of the mega-surface in the mega-surface coordinate space. The possible stages that the mega-surface can experience are:

- (15.) If all \(X_{ij}\) values > 0 then the mega-surface shows an expansion stage
- (16.) If all \(X_{ij}\) values = 0 then the mega-surface shows a stagnation stage
- (17.) If \(X_{ij}\) values < 0 then the mega-surface shows a contraction stage
- (18.) If some \(X_{ij}\) is sharing positive, negative or zero values in different vertical axes then the mega-surface shows an unstable performance stage.

Figure 9: The Mega-Surface Coordinate Space and the Mega-Surface

3. Comments

This paper offers a new set of multi-dimensional coordinate spaces for economists, academics and policy makers that can facilitate the visualization of multi-variable economic data in the same graphical space and time. Hence, these new multi-dimensional coordinate spaces open the possibility to visualize all possible changes variable by variable according to this new multidimensional graphical approach. The idea to propose a new set of multi-dimensional
coordinate spaces, it is originated by the limitations that the 2-dimensional Cartesian coordinate plane and the 3-dimensional coordinate space show in the moment to visualize multi-variable economic data behavior in the same graphical space and time.

References


Endnotes