Joint Design of Tx-Rx Beamformers in MIMO Downlink Channel

M. Codreanu, A. Tölli, M. Juntti and M. Latva-aho

Centre for Wireless Communications, University of Oulu, P.O. Box 4500, FIN-90014, Oulu, Finland,
{marian.codreanu, antti.tolli, markku.juntti, matti.latva-aho}@ee.oulu.fi

Abstract—We consider a single-cell multiple-input multiple-output (MIMO) downlink channel where linear transmission and reception strategy is employed. The base station (BS) transmitter is equipped with a scheduler using a simple opportunistic beamforming strategy, which associates an intended user for each of the transmitted data streams. For the case when the channel of the scheduled user is available at the BS, we propose a general method for joint design of the transmit and the receive beamformers according to different optimization criteria. The proposed method can handle multiple antennas at the BS and mobile user with single and/or multiple data streams per scheduled user. By exploiting the uplink-downlink SINR duality, we decompose the original optimization problem as a series of simpler optimization problems which can be efficiently solved by using standard convex optimization tools. The simulations show that the algorithms converge fast to a solution, which can be a local optimum, but is still efficient. Only one iteration of the proposed method is enough to substantially outperform the zero forcing based solution.

Index Terms—Broadcast channels, multiple-input multiple-output (MIMO) systems, beamforming, geometric programming, signomial programming.

I. INTRODUCTION

The knowledge of channel state information (CSI) at the transmitter side can dramatically improve the performance (data rate, quality of service, etc.) of multiple-input multiple-output (MIMO) communications systems. However, especially in case of a multiuser system, it is difficult to achieve full CSI knowledge of every user at the transmitter side.

An efficient method to exploit the multiuser diversity gain with very limited CSI feedback is the opportunistic beamforming technique [1]. It has been shown that in case of systems with very large numbers of users, the opportunistic beamforming performs very close to the optimum beamforming [1]. However, the performance deteriorates rapidly for such systems with moderate or small numbers of users [2]. For such cases, the use of a supplementary CSI feedback has been proposed in [2] to allow beamforming, and, thus, to reduce the detrimental effect at small to moderate numbers of users.

For the case when the channel of the scheduled users is available at the BS we propose a general method for joint design of the linear transmit and the receive beamformers according to different optimization criteria. The proposed method can handle multiple antennas at the BS and multiple user with single and/or multiple data streams per scheduled user. We focus in this paper to rate maximization problem, and we provide a general algorithm that maximizes the weighted sum of the individual data streams’ rates under a sum power constraint. The proposed method has the advantage that it can be easily modified to accommodate different optimality criteria and supplementary constraints.

The optimization problems encountered in the beamformer design are not convex in general. Therefore, the problem of finding the global optimum is intrinsically non-tractable. However, by exploiting the uplink-downlink SINR duality [3], we decompose the original optimization problem as a series of simpler optimization problems which can be efficiently solved by using standard convex optimization tools. Even though each subproblem is optimally solved, there is no guarantee that the global optimum has been found due to the nonconvexity of the problem. However, the simulations show that the algorithms converge fast to a solution, which can be a local optimum, but is still efficient.

II. SYSTEM MODEL

We consider a single cell MIMO multi-user system with $T$ antennas at BS and $K$ decentralized users, user $k$ equipped with $R_k$ antennas, operating in frequency-flat fading channel. The downlink (DL) channel is mathematically described as

$$y_k = H_k x + n_k, \quad k = 1, \ldots, K$$

(1)

where $H_k \in \mathbb{C}^{R_k \times T}$ is the channel matrix between BS and the user $k$, $x \in \mathbb{C}^T$ is the transmitted signal vector, $y_k \in \mathbb{C}^{R_k}$ is the signal vector received by the $k$'th user, and $n_k \sim \mathcal{CN}(0, \mathbf{R}_k)$ models the noise and inter-cell interference at the $k$'th terminal. Without loss of generality, we assume $\mathbf{R}_k = \mathbf{I}$. Notice that this assumption is not a restriction since colored additive noise can be viewed as white after an appropriate whitening transform on the channel matrix [3]. Thus, it is assumed in (1) that the whitening filter is contained in $H_k$.

We consider a linear transmission and reception strategy. The base station transmits $S \leq T$ independent data streams, i.e., $x = V \text{diag}(p)^{1/2} d$, where $d \in \mathbb{C}^{S \times 1}$, $\text{E} \{dd^H\} = \mathbf{I}$, contains the current transmitted data symbols, $V = [v_1, \ldots, v_S] \in \mathbb{C}^{T \times S}$, $\|v_s\|_2 = 1$, is the normalized beamforming matrix and the vector $p = [p_1, \ldots, p_S]^T$ controls the power allocated to each stream. For each data stream $s$, $s = 1, \ldots, S$, the base station’s scheduler unit associates
an intended user $k_s$. Notice that more than one stream can be associated to one user, therefore, the cardinality of the set of scheduled users, $S = \{k_s|s = 1, \ldots, S\}$, is less or equal to $S$. Let $u_s, \|u_s\|_2 = 1$ be the normalized receive beamformer (or antenna combiner vector) used by the $k_s$'th user to generate the decision variables for $s$'th data stream, $d_s = u_s^H y_{k_s}$. By collecting all the decision variables into the vector $d = [d_1, \ldots, d_S]^T$, the downlink linear processing model is given by:

$$d = U^H V \text{diag}(p)^{1/2} d + n$$

(2)

where

$$U^H \triangleq \begin{bmatrix} u_{1H} & 0 & \ldots & 0 \\ 0 & u_{2H} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & u_{S,H} \end{bmatrix}, \quad H \triangleq \begin{bmatrix} H_{k_1} \\ \vdots \\ H_{k_S} \end{bmatrix}, \quad n \triangleq \begin{bmatrix} n_{1} \\ \vdots \\ n_{S} \end{bmatrix}$$

(3)

Notice that $E([n_s]^2) = \|u_s\|_2 = 1$. Therefore, the SINR of the $s$'th stream, $\gamma_s$, is given by

$$\gamma_s = \frac{p_s \|u_s^H H_{k_s} v_s\|^2}{1 + \sum_{i=1,i \neq s}^S p_i \|u_i^H H_{k_s} v_s\|^2}$$

(4)

Following the approach of [3, Sect. III] the dual (or reciprocal) uplink linear processing model is defined as

$$d' = V^H H^H U \text{diag}(q)^{1/2} d' + n$$

(5)

where the vector $q = [q_1, \ldots, q_S]^T$ controls the uplink power allocated to each stream. $u_s$ is the transmit beamformer used by the $k_s$'th user, $v_s$ is receive beamformer used at the BS to generate the decision variable for the $s$'th stream, $d_s$. The SINR of the $s$'th stream, $\gamma_s$, is given by

$$\gamma_s = \frac{q_s \|v_s^H H_{k_s} u_s\|^2}{1 + \sum_{i=1,i \neq s}^S q_i \|v_i^H H_{k_s} u_s\|^2}$$

(6)

Our proposed iterative algorithm is based on the following uplink-downlink SINR duality theorem proved in [3].

Theorem 1: For any given (fixed) set of normalized beamformers $\{u_1, \ldots, u_S\}$ and $\{v_1, \ldots, v_S\}$, the set of SINR values, $\gamma = [\gamma_1, \ldots, \gamma_S]^T$, is achievable in downlink channel (2) under the sum-power constraint $1^T p \leq P_{\text{max}}$ if and only if the same set of SINR values is achievable in uplink channel (5) under the same power constraint, $1^T q \leq P_{\text{max}}$.

The following definitions are useful in description of our proposed algorithm.

Definition 1: The normalized maximum SINR receiver for the downlink channel (2) is given by the set of beamformers, $\{u_1^*, \ldots, u_S^*\}$, that solves the following multi-objective optimization problem [4, Sect. 4.7.5] (the beamformers are normalized such that their Euclidian norms are equal to unity)

$$\begin{align*}
\text{maximize} & \quad (\text{w.r.t. } \mathbb{R}_+^S) \gamma_{\text{dl}} \triangleq [\gamma_1^*, \ldots, \gamma_S^*]^T \\
\text{subject to} & \quad \|u_s\|_2 = 1, \quad s = 1, \ldots, S
\end{align*}$$

(7)

where the maximization is with respect to the nonnegative orthant, $\mathbb{R}_+^S$ [4, Sect. 4.7.5]. The optimum set $\{u_1^*, \ldots, u_S^*\}$ is obtained by normalizing the combiner vectors of the linear minimum mean square error (LMMSE) receiver [5], i.e.,

$$u_s^* = \frac{\hat{u}_s}{\|\hat{u}_s\|_2}, \quad \hat{u}_s^H = p_s v_s H_{k_s}^H \left( \sum_{i=1}^S p_i H_{k_s} v_i v_i^H H_{k_s} + I \right)^{-1}$$

(8)

Definition 2: The normalized maximum SINR receiver for the uplink channel (5) is given by the set of beamformers, $\{v_1^*, \ldots, v_S^*\}$, that solves the following optimization problem

$$\begin{align*}
\text{maximize} & \quad (\text{w.r.t. } \mathbb{R}_+^S) \gamma_{\text{ul}} \triangleq [\gamma_1^*, \ldots, \gamma_S^*]^T \\
\text{subject to} & \quad \|v_s\|_2 = 1, \quad s = 1, \ldots, S.
\end{align*}$$

(9)

The optimum set $\{v_1^*, \ldots, v_S^*\}$ is obtained by normalizing the LMMSE receiver’s combiner vectors [5], i.e.,

$$v_s^* = \frac{\hat{v}_s}{\|\hat{v}_s\|_2}, \quad \hat{v}_s^H = q_s u_s^H H_{k_s} \left( \sum_{i=1}^S q_i H_{k_i}^H u_i u_i^H H_{k_i} + I \right)^{-1}$$

(10)

III. GENERAL ITERATIVE OPTIMIZATION ALGORITHM

The uplink-downlink SINR duality theorem suggests that (8) and (10) can be used to iteratively adjust the sets of normalized beamformers, $\{u_1, \ldots, u_S\}$ and $\{v_1, \ldots, v_S\}$, by sequentially switching between the downlink and uplink channels and by updating the power allocation vectors, $p$ and $q$, according to a certain optimization criterion before each channel switching. A simple heuristic procedure that increases the SINR vector at each iteration is described in the following.

A. A Simple Example: Heuristic SINR Vector Maximization

Consider a downlink channel specified by $\{v_1, \ldots, v_S\}$ and $p(0), 1^T p(0) = P_{\text{max}}$. The optimum set of receive beamformers, $\{u_1(0), \ldots, u_S(0)\}$, that maximizes the SINR vector, $\gamma_{\text{dl}}$ (w.r.t. $\mathbb{R}_+^S$) is given by (8) and the optimum SINR vector, $\gamma_{\text{dl}}(0) \triangleq \gamma_{\text{dl}}(\{u_1(0), v_1(0), p(0)\}_{s=1,\ldots,S})$, is given by (4). Now we turn to the reciprocal uplink channel (5), where the users’ transmit beamformers are $\{u_1(0), \ldots, u_S(0)\}$ and the BS’s combining vectors are $\{v_1(0), \ldots, v_S(0)\}$. Theorem 1 ensures that there exists a power allocation $q(1), 1^T q(1) = P_{\text{max}}$ such that $\gamma_{\text{ul}}(1) = \gamma_{\text{dl}}(\{u_1(0), v_1(0), q(1)\}_{s=1,\ldots,S}) = \gamma_{\text{dl}}(0)$. Now the key point is to observe that for this new power allocation the BS’s combiners $\{v_1(1), \ldots, v_S(1)\}$, are not necessarily optimal, and, therefore, the SINRs can be increased by updating them to a new value $\{v_1(1), \ldots, v_S(1)\}$ given by (10). The definition (9) ensures that $\gamma_{\text{ul}}(1) = \gamma_{\text{ul}}(0)$, where $\gamma_{\text{ul}}(1)$ is the new SINR vector given by (6), i.e., $\gamma_{\text{ul}}(1) \triangleq \gamma_{\text{ul}}(\{u_1(1), v_1(1), q(1)\}_{s=1,\ldots,S})$. Now we can turn again to the downlink channel (2), which uses $\{v_1(1), \ldots, v_S(1)\}$ as transmit beamformers, and find a power allocation vector $p(1), 1^T p(1) = P_{\text{max}}$ such that $\gamma_{\text{dl}}(\{u_1(1), v_1(1), p(1)\}_{s=1,\ldots,S}) = \gamma_{\text{dl}}(1)$, then update the users’ receive beamformers to $\{u_1(1), \ldots, u_S(1)\}$ given by (8). The definition (7) ensures that $\gamma_{\text{dl}}(1) = \gamma_{\text{dl}}(\{u_1(1), v_1(1), p(1)\}_{s=1,\ldots,S}) \geq \gamma_{\text{dl}}(1)$. The sequential switching between the uplink and downlink channel continues until the SINR value converges or a satisfactory solution is found.
Notice that at each iteration the sum power is kept fixed, i.e., $1^T q^{(i)} = 1^T p^{(i)} = P_{\text{max}}$, and the achieved SINR is monotonically increased, i.e., $\gamma_{dl}^{(i+1)} \geq \gamma_{dl}^{(i)}$ and $\gamma_{ul}^{(i)} \geq \gamma_{ul}^{(i+1)}$, where $\gamma_{dl}^{(i)} = \gamma_{dl}^{(i)}\{u_i, v_i, p_i\}_{s=1, \ldots, S}$ and $\gamma_{ul}^{(i+1)} = \gamma_{ul}^{(i+1)}\{u_i, v_i, p_i\}_{s=1, \ldots, S}$ for $i \geq 0$.

B. General Algorithm

By modifying the power update policies before each channel switching the previous method can be further generalized to include different optimality criteria. The general iterative optimization algorithm, which includes at the first step the considered scheduling algorithm based on the opportunistic beamforming strategy [1], is described in the following.

Algorithm 1: Joint Beamformer and Power Optimization

Step 1 Let $S = \min\{T, \sum_{k=1}^{K} R_k\}$, $p^{(0)} = 1P_{\text{max}}/S$ and form $S$ orthogonal beams by generating randomly the orthogonal set, $\{v_1^{(0)}, \ldots, v_S^{(0)}\}$ with $\{v_i^{(0)}\}^H v_j^{(0)} = 0$ for $1 \leq i \neq j \leq S$. Each user measures the SINR achieved in each beam by using the LMMSE receiver and sends it back to the BS. Based on this information the BS’s scheduler unit makes the beam-to-users association described by the following set of active users

$$S = \{k_s | k_s = \arg \max_{k=1, \ldots, K} \gamma_{k,s}, s = 1, \ldots, S\}$$

where $k_s$ is the user associated to $s$’th beam (or data stream) and $\gamma_{k,s}$ denotes the SINR of the $k$’th user in the $s$’th beam. Let us denote by $\{u_1^{(0)}, \ldots, u_S^{(0)}\}$ the optimal set of receive beamformers given by (8) and by $\gamma_{dl}^{(0)}$ the SINRs of the scheduled users, i.e., $\gamma_{dl}^{(0)} = [\gamma_{k1,1}, \ldots, \gamma_{kS,s}]^T$. In the following, we assume that the selected users feed back their instantaneous CSI to the BS. Let $i = 0$ and go to Step 2.

Step 2 Consider the uplink channel (5) specified by $\{u_1^{(i)}, \ldots, u_S^{(i)}\}$ and $\{v_1^{(i)}, \ldots, v_S^{(i)}\}$. Update the power allocation vector $q^{(i+1)}$ by solving a certain optimization problem which will be specified in detail in Sect. IV, i.e.,

$$q^{(i+1)} = f_{ul}\left(\{u_i^{(i)}, v_i^{(i)}\}_{s=1, \ldots, S}\right).$$

Update the base station’s beamformers to $\{v_1^{(i+1)}, \ldots, v_S^{(i+1)}\}$ given by (10). Compute the achieved SINR vector $\gamma_{ul}^{(i+1)}$ given by (6) and go to Step 3.

Step 3 Consider the downlink channel (2) specified by $\{u_1^{(i)}, \ldots, u_S^{(i)}\}$ and $\{v_1^{(i+1)}, \ldots, v_S^{(i+1)}\}$. Update the power allocation vector $p^{(i+1)}$ by solving a certain optimization problem which will be specified in Sect. IV, i.e.,

$$p^{(i+1)} = f_{dl}\left(\{u_i^{(i)}, v_i^{(i+1)}\}_{s=1, \ldots, S}\right).$$

Update the users’ beamformers to $\{u_1^{(i+1)}, \ldots, u_S^{(i+1)}\}$ given by (8). Compute the achieved SINR vector $\gamma_{dl}^{(i+1)}$ given by (4) and test a stopping criterion. If it is not satisfied let $i = i + 1$ and go to Step 2, otherwise STOP.

To clarify the main idea of Algorithm 1, let us give an example. Consider the problem of maximizing a system performance criterion under a sum power constraint, and assume that the performance criterion can be expressed as a nondecreasing function of the SINR values of the data streams, i.e., $f(\gamma) \text{ such that } \gamma_1 \geq \gamma_2 \Rightarrow f(\gamma_1) \geq f(\gamma_2)$, where $\gamma$ contains the SINR values of the data streams. For such a problem, (12) and (13) compute an optimum uplink and downlink power allocation that maximize $f(\gamma_{ul}^{(i)})$ and $f(\gamma_{dl}^{(i)})$, respectively. Following the reasoning of Sect. III-A, it is easy to observe that Theorem 1 ensures that $f(\gamma)$ is monotonically increased at each iteration of Algorithm 1, i.e., $f(\gamma_{dl}^{(i+1)}) \geq f(\gamma_{dl}^{(i)}) \geq f(\gamma_{ul}^{(i)}) \geq f(\gamma_{ul}^{(i+1)})$.

IV. COMPUTING THE POWER ALLOCATION VECTORS

In this section, we derive the detailed optimization problems to obtain the power allocation vectors $q^{(i+1)}$ and $p^{(i+1)}$ in (12) and (13) in Algorithm 1. Since the objective functions and the constraints depend only on the streams’ SINRs and the sum power, the uplink-downlink SINR duality theorem allows us to formulate similar optimization problems for computing the power allocation vectors, $p^{(i+1)}$ for the downlink channel and $q^{(i+1)}$ for the uplink channel.

A. Weighted Sum Rate Maximization

Suppose we want to maximize the weighted sum of the rates of the individual data streams under a sum power constraint, $P_{\text{max}}$. The weight vector, $\mathbf{w} = [w_1, \ldots, w_S]^T$, $\mathbf{w} \succeq 0$, is used to prioritize differentially the data streams and it can be chosen based on different criteria, e.g., the states of queues or buffers in case of cross-layer optimization schemes. The proposed weighted sum rate maximization algorithm can also be used to compute the achievable rate region under the linear processing constraint. Obviously, $\mathbf{w} = 1$ corresponds to the usual sum rate maximization or best effort. Assuming optimal channel coding (i.e., Gaussian codebook [6]) for each data stream, the weighted sum rate can be expressed as

$$R_{\mathbf{w}} = \sum_{s=1}^{S} w_s r_s = \sum_{s=1}^{S} w_s \log(1 + \gamma_s) = -\log \prod_{s=1}^{S} (1 + \gamma_s)^{-w_s}$$

where $r_s$ and $\gamma_s$ are the rate and the SINR of the $s$’th data stream. Since $R_{\mathbf{w}}$ increases with respect to each $\gamma_s$ and $-\log(\cdot)$ is a decreasing function, the weighted sum rate maximization problem can be formulated as follows

minimize $\prod_{s=1}^{S} (1 + \gamma_s)^{-w_s}$

subject to $\gamma_s \leq \frac{x_{s} g_{s,s}}{1 + \sum_{k=1, k \neq s}^{S} x_{k} g_{s,k}}$, $s = 1, \ldots, S$ (14)

$\sum_{s=1}^{S} x_{s} \leq P_{\text{max}}$; $x_{s} \geq 0$, $s = 1, \ldots, S$

where the variables are $x_1, \ldots, x_S, \gamma_1, \ldots, \gamma_S$ and the problem data, $g_{s,k}, k = 1, \ldots, S$, are defined according to which (uplink or downlink) power allocation is solved, i.e.,

$$g_{s,k} = \begin{cases} \left(\left|v_i^{(i)}\right|^H H_{k, s} u_i^{(i)}\right)^2 \text{ for solving (12)} \quad \text{(15)} \end{cases}$$

The optimization problem (14) is a signomial optimization problem [7]. Thus, it is not convex in general. In high SINR
region \( \gamma_s \gg 1 \), it can be accurately approximated by a geometric program [4]. This approach means that the objective is replaced by the monomial function \( \prod_{s=1}^{S} \gamma_s^{a_s - w_s} \). Clearly, the condition \( \gamma_s \gg 1 \) can not be controlled (e.g., imposed as a constraint) without affecting the solution since \( \gamma_s \) is a problem variable. Therefore, a solving method applicable to the whole SINR range is needed. Such a procedure consists of searching for a close local minima by solving a sequence of geometric programs which locally approximate the original problem. This procedure is presented in the following.

Lemma 1: Let \( m(\gamma_1, \ldots, \gamma_S) = c \prod_{s=1}^{S} \gamma_s^{a_s} \) be a monomial function [7] used to approximate the objective of (14), \( f_0(\gamma_1, \ldots, \gamma_S) = \prod_{s=1}^{S} (1 + \gamma_s)^{-w_s} \), near the point \( \hat{\gamma} = (\hat{\gamma}_1, \ldots, \hat{\gamma}_S) \). The parameters \( c \) and \( a_s \) of the best monomial local approximation are given by

\[
a_s = w_s \hat{\gamma}_s (1 + \hat{\gamma}_s)^{-1}, \quad c = f_0(\hat{\gamma}_1, \ldots, \hat{\gamma}_S) \prod_{s=1}^{S} \hat{\gamma}_s^{-a_s}
\]

Proof: The monomial function \( m \) is the best local approximation of \( f_0 \) near the point \( \hat{\gamma} = (\hat{\gamma}_1, \ldots, \hat{\gamma}_S) \) if [7]

\[
\begin{align*}
& m(\hat{\gamma}_1, \ldots, \hat{\gamma}_S) = f_0(\hat{\gamma}_1, \ldots, \hat{\gamma}_S) \\
& \sqrt{m(\hat{\gamma}_1, \ldots, \hat{\gamma}_S)} = \sqrt{f_0(\hat{\gamma}_1, \ldots, \hat{\gamma}_S)}
\end{align*}
\]

and by replacing the expressions of \( m \) and \( f_0 \) in (17) we obtain the following system of equations

\[
\begin{align*}
& c \prod_{s=1}^{S} \hat{\gamma}_s^{a_s} = f_0(\hat{\gamma}_1, \ldots, \hat{\gamma}_S) \\
& c \prod_{s=1}^{S} \hat{\gamma}_s^{a_s} \hat{\gamma}_s^{-1} = \frac{-w_s f_0(\hat{\gamma}_1, \ldots, \hat{\gamma}_S)}{(1 + \hat{\gamma}_s)}, \ s = 1, \ldots, S
\end{align*}
\]

which have the solution given by (16).

By using the local approximation given by Lemma 1 in the objective function of problem (14), and ignoring the multiplicative constant \( c \) which does not affect the problem solution, we obtain the following iterative algorithm.

Algorithm 2: Weighted Sum Rate Maximization

Step 1. Let the initial SINR guess, \( \hat{\gamma} = (\hat{\gamma}_1, \ldots, \hat{\gamma}_S) \), be

\[
\hat{\gamma} = \begin{cases} 
\{\gamma_{dl}^{(i)}\} & \text{for solving (12)} \\
\{\gamma_{ul}^{(i+1)}\} & \text{for solving (13)}
\end{cases}
\]

Step 2. Solve the following geometric program,

\[
\text{minimize} \quad \prod_{s=1}^{S} (1 + \gamma_s)^{-w_s} \sqrt{\gamma_s}
\]

subject to \( (1 - \alpha) \hat{\gamma}_s \leq \gamma_s \leq (1 + \alpha) \hat{\gamma}_s \), \( s = 1, \ldots, S \)

\[
\frac{\gamma_s}{g_s x_s} \leq \frac{\sum_{k=1, k \neq s}^{S} g_{s,k} x_{k}}{g_s x_s} \leq 1, \quad s = 1, \ldots, S
\]

\[
\sum_{s=1}^{S} k x_s \leq P_{\text{max}}
\]

with positive variables \( x_1, \ldots, x_S, \gamma_1, \ldots, \gamma_S \). Denote the solution by \( x_1^{*}, \ldots, x_S^{*}, \gamma_1^{*}, \ldots, \gamma_S^{*} \). If \( \text{max}_{s} \{\gamma_s^{*} - \hat{\gamma}_s\} \geq \epsilon \) set \( \gamma = (\gamma_1^{*}, \ldots, \gamma_S^{*}) \) and go to Step 2, otherwise STOP.

The optimal points \( x_1^{*}, \ldots, x_S^{*} \) are the power allocation vectors required at Step 2 and Step 3 of Algorithm 1, i.e., \( q^{(i+1)} = [x_1^{*}, \ldots, x_S^{*}]^{T} \) when solving (12) or \( p^{(i+1)} = [x_1^{*}, \ldots, x_S^{*}]^{T} \) when solving (13). The problem (19) is the geometric program which approximates the original sigmoidal problem (14) around the point \( \hat{\gamma} = (\hat{\gamma}_1, \ldots, \hat{\gamma}_S) \). The first set of inequality constraints of problem (19) are called trust region constraints [7] and they limit the domain of variables \( \gamma_s \) in a region where the monomial approximation is accurate enough. The small constant \( \alpha \ll 1 \) controls the desired approximation accuracy and a typical value is \( \alpha = 0.1 \) [7].

B. Maximization of the minimum SINR

Suppose that we want to maximize the sum rate under a sum power limit, \( P_{\text{max}} \), and all the individual data streams are constrained to have equal SINR. Since the rate is an increasing function of SINR, the problem we want to solve is equivalent to maximization of the minimum SINR among the data streams under the sum power constraint, \( P_{\text{max}} \), i.e.,

\[
\text{maximize} \quad \min_{s} \frac{x_s g_{s,s}}{1 + \sum_{k=1, k \neq s}^{S} x_k g_{s,k}}
\]

subject to \( \sum_{s=1}^{S} x_s \leq P_{\text{max}} \) \( x_s \geq 0 \), \( s = 1, \ldots, S \)

(20)

where the variables are \( x_1, \ldots, x_S \) and the problem data, \( g_{s,k} \), \( k = 1, \ldots, S \), are given by (15). The objective function is quasiconcave since it is the minimum of a family of linear fractional [4]. Thus, it can be solved by using the bisection method. We will show that it can be also casted as a geometric program [4], and, therefore, it can be solved more efficiently. We first transform the problem (20) by inverting its objective

\[
\text{minimize} \quad \max_{s} \frac{g_{s,s}^{-1} x_s^{-1}}{1 + \sum_{k=1, k \neq s}^{S} x_k g_{s,k}}
\]

subject to \( \sum_{s=1}^{S} x_s \leq P_{\text{max}} \) \( x_s \geq 0 \), \( s = 1, \ldots, S \)

(21)

By expressing the problem (21) in the epigraph form [4], we obtain the following equivalent geometric program

\[
\text{minimize} \quad t
\]

subject to \( \frac{1}{g_{s,s} x_s} + \sum_{k=1, k \neq s}^{S} g_{s,k} x_k \leq t \), \( s = 1, \ldots, S \)

\[
\sum_{s=1}^{S} x_s \leq P_{\text{max}}
\]

with positive variables \( t, x_1, \ldots, x_S \). The optimal value \( t^{*} \) have the significance of the inverse of the streams’ SINRs and the optimal values \( x_1^{*}, \ldots, x_S^{*} \) are the optimum power allocation vectors required at Step 2 and Step 3 of Algorithm 1.

C. Other Power Update Criteria

Other power update methods can be developed. An algorithm that minimizes the weighted sum of the mean square errors (MSE) of the data streams under the sum power constraints is described in [8]. One can also minimize the sum power required to satisfy same minimum QoS requirements for the data streams [8]. Other supplementary constraints (e.g., lower and/or upper bounds of streams rates, etc.) can be also easily accommodated by all the previous algorithms [8].

V. Numerical Results

For the numerical results we have considered a frequency-flat MIMO downlink channel with \( T = 4 \) antennas at the BS and \( R_k = 2 \) antennas at each user terminal. We consider uncorrelated Rayleigh fading antennas, i.e., vec(H_k) \( \sim CN(0, I) \).
users, \(256\) antennas and SNR = \(1\) dB; right: SNR = \(10\)dB, antennas and \(6\) antennas.

Algorithm 2 is designed to improve the sum rate performance by iteratively updating the power allocation vectors \(\mathbf{q}^{(i+1)}\) and \(\mathbf{p}^{(i+1)}\) at Step 1 respectively Step 2 have been updated according to different optimization criteria, as, sum rate maximization algorithm, sum MSE minimization \([8]\), minimum SINR maximization and heuristic SINR vector maximization. The minimum SINR method is within 0.5 – 1.5 bits/second/Hz close to the sum capacity, which represents more than 90% of the sum capacity. The sum rate improvement relative to the simple opportunistic beamforming increases dramatically with SNR (Fig. 1(a)) and it remains large even in the case of \(K = 256\) users (Fig. 1(b)). The proposed method converges relatively fast, i.e., depending on the SNR point and \(K\), it requires about 30–70 iterations to arrive to a sum rate stationary point. Notice that most of the gain is achieved in the first few iterations. Therefore, the proposed algorithm is suitable for practical implementation.

In Fig. 2 we compare the sum rate provided by the true sum rate maximization algorithm, Algorithm 2, to the resulted sum rate when the power allocation vectors \(\mathbf{p}^{(i+1)}\) and \(\mathbf{q}^{(i+1)}\) are updated based on different optimization criteria, as, sum MSE minimization \([8]\), minimum SINR maximization and heuristic SINR vector maximization. The minimum SINR
maximization criterion is equivalent to imposing the supplementary constraint of equal SINR for all data streams, but, as discussed in Sect. IV-B, it can be reformulated as a geometric program. Therefore, it has reduced complexity. The results show that for \( K > 8 \) users, all the considered algorithms achieve a sum rate within 0.5 bits/sec/Hz close to the sum rate maximization algorithm. In other words, the resulted sum rate is just slightly decreased if all the data streams are constrained to have equal SINR values, e.g., due to some fairness requirements. Moreover, the heuristic SINR vector maximization algorithm provides essentially the same sum rate as the minimum SINR maximization. From a practical perspective this is an important remark, since the heuristic SINR vector maximization has the least complexity. The sum MSE minimization algorithm appears to be the closest to the true sum rate maximization algorithm. This confirms the intuition that, under linear processing constraint, a rate efficient design can be obtained by minimizing the sum MSE.

Fig. 2 shows also the sum rate provided by the zero forcing solutions proposed by Spencer et al. [9]. Specifically, for identical stream-to-user allocation as in case of the proposed algorithms, we have used the coordinated transmit-receive channel block diagonalization method combined with the water-filling power allocation, as described in [9, Sect. V]. The results show that the proposed sum rate maximization method provides more than 4 dB SNR gain (Fig. 2(a)) and up to 3.5 bits/sec/Hz better spectral efficiency (Fig. 2(b)) as compared to zero forcing solution. By comparing Figs. 1 and 2, it is also interesting to remark that only one iteration of the proposed method is enough to substantially outperform the zero forcing based solution. However, it must be noted that the considered opportunistic beamforming based scheduling algorithm is not particularly well suited for the zero forcing applications. Zero forcing based schemes may work substantially better when full CSI knowledge of every user is available at the transmitter, and, consequently, the most orthogonal users can be scheduled at the same time. However, this kind of CSI level is often difficult to achieve and falls outside the scope of this paper.

Fig. 3 shows the weighted sum rate performance of the algorithms considered in Fig. 2(b), for a different weight vector, \( \mathbf{w} = [4/3, 4/3, 2/3, 2/3] \), i.e., the first two streams have double weights compared to the last two streams. The results show that the gap between performance of different algorithms increases as the spread of the weights increases. Therefore, to accurately compute the entire rate region achievable with linear transmission strategy the weighted sum rate maximization algorithm is required.

**VI. CONCLUSIONS**

We provided a general framework for joint design of linear transmit and receive beamformers in a MIMO multiuser transmission according to different optimality criteria. The proposed method can handle multiple antennas at the BS and mobile users with arbitrary numbers of data streams per a scheduled user. In contrast to other previously proposed solutions, our method has the advantage that it can be easily modified to accommodate different optimality criteria and supplementary constraints. By exploiting the uplink-downlink SINR duality, we decomposed the original optimization problem as a series of simpler optimization problems which can be efficiently solved by using standard convex optimization tools.

The simulations shown that the algorithms converge fast to a solution, which can be a local optimum, but is still efficient. The iterative solution, where most of the improvement is achieved in the first few iterations, makes the method attractive for practical implementation. Only one iteration of the proposed method is enough to substantially outperform the zero forcing based solution. The proposed weighted sum rate maximization algorithm can be also a useful tool to find the gap between the achievable rate region with only linear processing and the rate region achievable with DPC.

**REFERENCES**