Improved PSO Algorithms for Electromagnetic Optimization

L. Matekovits(1), M. Mussetta(2), P. Pirinoli(1), S. Selleri(3), R.E. Zich(2)
(1) DEE, Politecnico di Torino, c. Duca degli Abruzzi 24, 10129 Torino, Italy
(2) DEE, Politecnico di Milano p. Leonardo da Vinci 32 - 20133 Milano, Italy
(3) DET, University of Florence Via C. Lombroso 6/17 50134 Florence, Italy

Some variations over the basic Particle Swarm algorithm are here proposed, aimed at a
more efficient search over the solution space and exhibiting a negligible overhead in
complexity and speed. The proposed algorithms are then applied to the test case of a
microwave filter to show their superior capabilities with respect to the conventional
algorithm.

Introduction

Particle Swarm Optimization (PSO) is a relatively novel approach for global stochastic
optimization. It simulates a swarm, or flock, of beings wandering in the parameter space
and is based on a “social interaction” metaphor in which the path of the beings in the
parameter space is controlled according to a swarm- or flock-like set of rules. The
position of each particle is used to compute the value of the function to be optimized.
Individual particles are then attracted, with a stochastic-varying strength, by both the
position of their best past performance and the position of the global best performance of
the whole swarm [1-2].

PSO is akin to Genetic Algorithms (GA) and Simulated Annealing (SA) inasmuch it is a
stochastic method performing a global search in the parameter space without getting
trapped in local minima. The basic PSO technique has recently been applied to
electromagnetic problems [3-6].

In this contribution some new PSO based techniques, aimed to improve the performances
of the PSO and based on multiple swarm interactions, are presented. The implementation
is similar but simpler than those of Cooperative Particle Swarm (Co-PS) methods
presented, for example, in [7,8], yet their performances are quite interesting, as numerical
results presented here will show. As an example of application, the new techniques have
been used for the optimization of a microstrip filter [6].

Conventional PSO and variations

In a conventional PSO algorithm a set of \( i = 1, \ldots, N_p \) particles, or agents, are
categorized by their position \( \overline{X}_i \) and velocity \( \overline{V}_i \) in the \( M \)-dimensional space domain
of the function \( f(\cdot) \) to be optimized. At the beginning positions and velocities have
random values \( \overline{X}_i^{(0)} \) and \( \overline{V}_i^{(0)} \), then they are updated iteratively according to the rules:

\[
\begin{align*}
\overline{V}_i^{(k+1)} &= \omega^{(k)} \overline{V}_i^{(k)} + \phi_1 \eta_1 \left( \overline{P}_i - \overline{X}_i^{(k)} \right) + \phi_2 \eta_2 \left( \overline{G} - \overline{X}_i^{(k)} \right) \\
\overline{X}_i^{(k+1)} &= \overline{X}_i^{(k)} + \overline{V}_i^{(k+1)}
\end{align*}
\]  

(1)
being $\overrightarrow{P}_i = X_i^{(k)}$ the best position ever attained by particle $i$ at step $k$, named agent “personal knowledge” and $\overrightarrow{G}$ the best position ever attained by the particle swarm, i.e. the “social knowledge”, shared by all the agents; $w_i$ is a friction factor, slowing down the agents, $\eta_i$ and $\eta_2$ the weights tuning the pulls towards $\overrightarrow{P}_i$ and $\overrightarrow{G}$ and $\phi$ a stochastic number varying in the $[0,1]$ range.

The first variation to the standard PSO that we propose will be named in the following Meta-PSO (MPSO) and essentially consists in the use of more than a single swarm; as a consequence, a further term has to be added in the first eq. in (1), that takes into account the inter-swarm or “racial” knowledge. Particles are now characterized by two indices: an index $j = 1,\ldots,N_s$ defining the swarm they belong to and an index $i = 1,\ldots,N_{pj}$, that numbers the particle inside the swarm $j$. The update rules become therefore:

$$V_j^{(k+1)} = \omega \cdot V_j^{(k)} + \phi \eta_1 (\overrightarrow{P}_{i,j} - X_j^{(k)}) + \phi \eta_2 (\overrightarrow{S}_{j} - X_j^{(k)}) + \phi \eta_3 (\overrightarrow{G} - X_j^{(k)})$$

$$X_j^{(k+1)} = X_j^{(k)} + V_j^{(k+1)} \quad (2)$$

where $\overrightarrow{P}_{i,j}$ is still the agent personal best position, $\overrightarrow{P}_j$ is the global best position of swarm $j$ (swarm social knowledge) and $\overrightarrow{G}$ is the global best position of all swarms (racial knowledge), while the other symbols have the same meaning than in (1).

MPSO can be further enhanced by keeping swarms apart from each other, and hence widening the global search. This can be done by introducing an inter-swarm repulsion:

$$\sum_{s\neq j} \phi \xi \frac{B_s^{(k)} - X_{i,j}^{(k)}}{|B_s^{(k)} - X_{i,j}^{(k)}|^\alpha} \quad (3)$$

as a subtractive term in (2). In (3) $B_s^{(k)}$ are the barycenters of the other swarms. The so obtained scheme will be called Modified MPSO (M²PSO) in the following.

As a further upgrade of the M²PSO, the rules governing the repulsion among the swarm can be modified, so that the swarm which is performing best, is no longer repelled by the other swarms, i.e. it stabilizes its position (and for this the scheme is named Stabilized M²PSO, SM²PSO). This allows for the best swarm to keep exploring the surroundings of the current best position, refining it, whereas other swarms are repelled and obliged to extend the search in other points of the space, hence enhancing the possibility of escaping a local minimum.

**Numerical Results**

The performances of the introduced techniques has been investigated and compared with those of the PSO first considering their speed of convergence. Since all the schemes (apart from the PSO) are multi-swarms, the analysis has been done considering also the effect due to changing the number of swarms $N_s$ and correspondently the number of individuals of each swarm $N_{pj}$, chose equal for all the $N_s$ swarms, so that the total population $N_p = N_s N_{pj}$ remains constant. The comparison among the four schemes has been done applying them to the optimization of several test functions that are usually used for this purpose.
In particular, the two plots in Fig.1 show the curves of convergence, i.e. the average cost function value computed over 50 trials vs. the number of iterations, in the case of a $N$-dimensional sinc function, with $N = 10$. The population is always of $N_p = 72$ particles, but in the plot on the left $N_s = 2$ and $N_{pj} = 36$, for all $j$, while in that on the right $N_s = 8$ and $N_{pj} = 9$, for all $j$.

From Fig.1 it appears that the PSO, whose performances obviously depend only on the total number of particles $N_p$, never reaches the function minimum. For what concerns the other schemes, their behavior varies with the number of swarms: they act almost in the same way when $N_s = 2$, but as this number increases the SM$^2$PSO becomes worse, while the performances of the MPSO improve.

In view of these results, we have applied the MPSO scheme to the design of a symmetric micro strip band-pass filter (see inset in Fig.2), consisting in a sequence of $(2N+1)$ lines, each with length $\lambda_g/2$ at the central frequency ($\lambda_g$ is the guided wavelength), but different width, in order to have different characteristic impedance [6].

The cost function takes into account the bandwidth, the absolute value of the transmission coefficient $|S_{21}|$ in the rejection band and the ripple in the pass band. Taking $N_p = 72$, we have varied the values of $N_s$ and $N_{pj}$: in Table 1 the most significant filter parameters obtained with five different combination of $N_s$ and $N_{pj}$ are summarized. All these values have been obtained after 3000 iterations, and one trial.

The results in Table 1 show that the ripple decreases when $N_s$ increases. The optimal configuration, at least for what concerns the $|S_{21}|$ behavior in the pass band, is that consisting in 6 swarms, each one of 12 particles.
The filter transfer function obtained in this case is shown in Fig.2, where a blow out of the behavior of |S21| in the pass band is also reported, showing that the ripple is very low in the entire band.

Also the configuration with \( N_s = 8 \) and \( N_pj = 9 \) gives very good results for what concerns the ripple, and it is better than the previous one for what concerns the value of |S21| in the rejection band, but the –10 dB bandwidth is slightly narrower.

<table>
<thead>
<tr>
<th>( N_s ) = 2, ( N_pj = 36 )</th>
<th>( N_s ) = 3, ( N_pj = 24 )</th>
<th>( N_s ) = 4, ( N_pj = 18 )</th>
<th>( N_s ) = 6, ( N_pj = 12 )</th>
<th>( N_s ) = 8, ( N_pj = 9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10 dB bandwidth (%)</td>
<td>63.750</td>
<td>61.875</td>
<td>62.500</td>
<td>63.125</td>
</tr>
<tr>
<td></td>
<td>(</td>
<td>S21</td>
<td>) in rejection band (dB)</td>
<td>-54.6759</td>
</tr>
<tr>
<td></td>
<td>maximum ripple (dB)</td>
<td>-0.4198</td>
<td>-0.4568</td>
<td>-0.4949</td>
</tr>
<tr>
<td></td>
<td>mean ripple in pass band (dB)</td>
<td>-0.0421</td>
<td>-0.0909</td>
<td>-0.1236</td>
</tr>
</tbody>
</table>

Table 1. Principal filter parameters obtained with the MPSO scheme for several values of the number of swarms and the population of each of them.

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![Figure 2. Filter transfer function, obtained with \( N_s = 6 \) and \( N_pj = 12 \). Inset: scheme of the filter.](image)

**References**


