Impedance Measurement With the D-Optimum Experimental Conditions

Marcin Witczak, Ryszard Rybski, and Janusz Kaczmarek

Abstract—A computer-based automatic alternating current (ac) bridge for impedance measurement is proposed. The problem of impedance measurement is formulated as a linear-in-parameter estimation one, and a suitable algorithm is proposed that ensures high measurement accuracy under a high convergence rate. In particular, the optimum experimental conditions regarding the reference resistance and the sampling time are developed. These conditions guarantee that the estimation is performed with possibly high accuracy. This paper also shows how to use the proposed approach for fault detection, which is very important from the point of view of modern control and fault diagnosis. The final part of this paper shows a number of illustrative experiments that justify the effectiveness of the proposed approach.

Index Terms—Experimental design, impedance measurement, measurement accuracy, model uncertainty, parameter estimation.

I. INTRODUCTION

VARIOUS physical quantities, e.g., force, pressure, flow, and displacement, can be transformed into impedance through various transducers. The measurements that are collected in such a way constitute the basic source of knowledge regarding any industrial system being controlled or diagnosed [8], [12]. Many different techniques and equipment have been developed over the past few years to meet demands on different operational ranges, accuracy, measurement rates, specific application targets, and costs. So far, there has been no doubt that high-accuracy measurements can be achieved by using ac bridges that are balanced either manually or iteratively. Unfortunately, such a balancing technique excludes fast measurement rates that are required by modern control and fault-diagnosis systems [8], [12]. To settle such a challenging problem, many different computer-based ac bridges have been developed over the last 15 years. Dutta et al. [5] introduced the idea of a “virtual ac bridge,” with a virtual arm implemented with the help of a microcomputer. Awad et al. [2] made an important contribution to this approach with respect to its convergence. Recently, Dutta et al. [6] have proposed another modification of this impedance measurement technique. In all above papers, the authors formulated the impedance-measurement problem as the nonlinear-in-parameter estimation one [11]. To solve such a problem, they employed the gradient descent algorithm [11], which made the required measurement rate difficult to attain. To settle this problem, Kaczmarek et al. [7] employed the so-called bounded-error parameter estimation technique [4], [11]. In spite of the high convergence rate, the technique does not make it possible to attain the accuracy that is comparable with that discussed in [2] and [6]. A slightly different approach was proposed by Angrisani et al. [1]. This technique is based on a bridge-balance loop comprising two internal loops. The first one is used for tuning the capacitive (inductive) part of the 52 impedance, whereas the second one is used to rearrange the resistive part. The main drawback of this approach is that the fast convergence seems very hard to attain. A fast balancing bridge 55 was proposed by Zhang et al. [13], and it seems suitable for a 56 wide range of industrial applications.

Unfortunately, the researchers have not considered the relation between the experimental conditions [10], [11] and the accuracy of the measurements. In this paper, the experimental conditions are related with the optimum selection of the reference resistance of the bridge and the sampling time. Angrisani et al. [1] and Zhang et al. [13] discovered, by numerous simulations, that the closer the selected reference resistance is to the measured impedance, the more precise measurement results can be obtained. However, they did not provide any explicit formulas determining the reference resistance.

In this paper, the answer to the challenging problem of the optimum experimental conditions is provided; that is, explicit formulas for the reference resistance and the sampling time are developed. This paper is organized as follows. Section II shows the formulation of the problem and the proposed measurement technique. In particular, the problem of impedance measurement is transformed into the linear-in-parameter estimation task. This makes it possible to achieve both high estimation quality and a fast convergence rate. In Section III, the results regarding the optimum experimental conditions are provided. Section IV shows a way of extending the proposed approach to the inductive impedance. Section V is devoted to the robustness 79 problems that are very important in practical applications. Subsequently, Section VI presents a comprehensive experimental 81 study, which confirms the high performance of the proposed approach. Finally, Section VII concludes this paper.

II. PROBLEM FORMULATION

The objective of this section is to propose a new impedance measurement method. This paper is motivated by the approach presented in [7]. The virtual bridge is composed of two arms, 87
samples, where \( f_p = 1/\tau \). Thus, it is clear that \( f_p \) should be 112 selected in such a way that \( n \) is an integer number. It should 113 also be pointed out that any incoherence of \( f_p \) causes \( n \) to be not 114 an integer number, and, hence, \( u_{2,k} \) cannot be determined with 115 the possibly highest precision. Undoubtedly, this uncertainty 116 in the input signal impairs the overall performance of the 117 proposed impedance-measurement method. Unfortunately, the 118 theoretical analysis regarding the influence of this unappealing 119 phenomenon on the final measurement quality seems impossi- 120 ble to realize.

Another important fact that can be observed while analyzing 122 (3) is that it can be perceived as a linear-in-parameter model 123 with respect to \( p_1 \) and \( p_2 \). Contrary to [2] and [5], where the 124 nonlinear parameter estimation techniques were employed for 125 obtaining \( R \) and \( C \), it is proposed to use the classical recursive 126 least squares (RLS) algorithm [11] for the estimation of \( p_1 \) and 127 \( p_2 \). Such an algorithm can be written as follows:

\[
\hat{p}_k = \hat{p}_{k-1} + k_\varepsilon \varepsilon_k \\
k_\varepsilon = P_k^{-1} R_k \varepsilon_k \\
\varepsilon_k = y_k - f(\hat{p}_{k-1}, u_k) \\
P_k = [I_{n_{p}} - k_r R_k] P_{k-1}
\]

where \( y_k \) is the \( k \)th measurement of \( v_k \) (cf. Fig. 1), 129 \( f(\hat{p}_k, u_{k+1}) = \hat{p}_{1,k} u_{1,k+1} + \hat{p}_{2,k} u_{2,k+1} \), \( R_k = [u_{1,k}, u_{2,k}]^T \), 130 \( \hat{p}_k = [\hat{p}_{1,k}, \hat{p}_{2,k}]^T \in \mathbb{R}^{n_{p}} \) denotes the \( k \)th estimate of \( p \), and 131 \( k = 1, \ldots, n_t \), where \( n_u \) stands for the number of 132 measurements of \( u \) and \( y \). Thus, knowing \( p \), it is possible to 133 obtain the estimates of \( R \) and \( C \) according to the following 134 equations:

\[
\hat{R} = - \frac{R_r (\hat{p}_1^2 + \hat{p}_2^2)}{\hat{p}_1^2 + \hat{p}_2^2 - p_1} \\
\hat{C} = - \frac{\hat{p}_2}{R_r \omega (\hat{p}_1^2 + \hat{p}_2^2)}
\]

which are obtained by solving (4) with respect to \( R \) and \( C \). It 136 should also be pointed out that when there is no need for online 137 estimation of the impedance, then the classical nonrecursive 138 least squares algorithm can be employed. The well-known 139 advantage of this algorithm, as compared with its recursive 140 counterpart, is that the highest estimation accuracy can be 141 attained with smaller \( n_t \). In this case, estimates of \( p_1 \) and 142 \( p_2 \) can be computed as follows:

\[
\hat{p}_1 = \frac{\gamma_2 \eta - \beta_2 \gamma_1}{\eta^2 - \beta_1 \gamma_2}, \quad \hat{p}_2 = \frac{\gamma_1 \eta - \beta_1 \gamma_2}{\eta^2 - \beta_1 \beta_2}
\]

where

\[
\gamma_i = \sum_{k=1}^{n_t} u_{i,k} y_k, \quad \eta = \sum_{k=1}^{n_t} u_{1,k} u_{2,k}, \quad \beta_i = \sum_{k=1}^{n_t} u_{i,k}^2.
\]

A. Initialization of the RLS Algorithm

As can be found in the literature [11] regarding the RLS algo- 146 rithm, the initial matrix \( P_k \), i.e., \( P_0 \), should be set as \( P_0 = \gamma I \).
where $\gamma$ stands for a sufficiently large positive constant (usually $10^3-10^{20}$). When some rough values of $R$ and $C$ are known, then $\hat{p}_0$ should be initialized according to (4). Otherwise, it can be observed from (10) that $\hat{p}_1^2+\hat{p}_2^2-\hat{p}_1<0$, and hence
\[
\frac{1}{2} - \frac{1}{2} \sqrt{1 - 4\hat{p}_2^2} < \hat{p}_1 < \frac{1}{2} + \frac{1}{2} \sqrt{1 - 4\hat{p}_2^2}.
\] (14)

Since $\hat{p}_2$ should satisfy $1 - 4\hat{p}_2^2 > 0$, while (10) and (11) indicate that $\hat{p}_2 < 0$ and $\hat{p}_1 > 0$, then it is clear that
\[
\frac{1}{2} < \hat{p}_2 < 0.
\] (15)

Thus, when no knowledge is available about $R$ and $C$, then $\hat{p}_0$ should be set to satisfy (14) and (15).

III. EXPERIMENTAL DESIGN

The one objective of this section is to provide rules for computing the accuracy of the measured impedance, i.e., a set of all $R$ and $C$ that are consistent with the measurements of $u$ and $v$. The main objective is to provide the optimum experimental conditions; that is, explicit formulas for the reference resistance and the sampling time are developed that make it possible to increase the measurement accuracy. The proposed solution is based on the following assumption:
\[
y_k = v_k + \epsilon_k
\] (16)

where $\epsilon_k$ stands for the zero-mean uncorrelated Gaussian noise sequence. In other words, $\epsilon_k$ represents the difference between the output of the model (3) and $y_k$ that represents the actual measurements of $v_k$ (cf. Fig. 1).

A. Confidence Region and Fault Detection

Since the estimates of $R$ and $C$ can be obtained according to (10) and (11), the next problem being considered is obtaining a set of all possible $R$ and $C$ that are consistent with the measurements of $u$ and $y$. Such a set can be obtained with the use of the $(1-\alpha)100\%$ confidence region [11] for $p$ and $q$, (4), where $\alpha$ stands for the significance level. As a result, the following inequality is given:
\[
d_k^TP^{-1}d_k \leq 2\hat{\sigma}_k^2F_{\alpha,2,k-2}
\] (17)

where
\[
d_k = \left[ \hat{p}_1 - \hat{p} \right] \left[ R(R + R_r), R_rC \omega R^2 \right]^T
\] (18)

$F_{\alpha,2,k-2}$ stands for the F-Snedecor distribution quantile with 2 and $k-2$ degrees of freedom, and $\hat{\sigma}$ is the estimate of the standard deviation of $\epsilon$. The inequality (17) is very important from the point of view of fault detection and control of industrial systems [8], [12]. Indeed, it can be used for checking whether the measured impedance satisfies the predefined bounds. However, the main drawback of (17) is that it does not take into account the inaccuracy of $R_r$. To tackle this problem, let us define the $(1-\alpha)100\%$ confidence interval for $p_1$ and $p_2$, i.e.,
\[
\hat{p}_1 - \Delta_1 < p_1,m < p_1 < p_1,M = \hat{p}_1 + \Delta_1
\]
\[
\hat{p}_2 - \Delta_2 < p_2,m < p_2 < p_2,M = \hat{p}_2 + \Delta_2
\] (19)

where $\Delta_1 = \hat{\sigma}t_{\alpha,k-3}\sqrt{P_{i,i,k}}$, where $P_{i,i,k}$ stands for the $i$th diagonal element of $P_k$, and $t_{\alpha,k-3}$ is the Student distribution quantile with $k - 3$ degrees of freedom. A similar interval can be defined for $R_r$, i.e.,
\[
R_{r,m} < R_r < R_{r,M}
\] (20)

which can be easily computed when the accuracy of $R_r$ is known. Thus, using (19) and (20) along with (10) and (11), it can be shown that
\[
\frac{R_{r,m}^2 \left( p_{1,m}^2 + p_{2,m}^2 \right)}{p_{1,m}^2 + p_{2,m}^2 - p_1,m} < R < \frac{R_{r,M}^2 \left( p_{1,M}^2 + p_{2,M}^2 \right)}{p_{1,M}^2 + p_{2,M}^2 - p_1,M}
\] (21)
\[
\frac{R_{r,M} \omega \left( p_{1,M}^2 + p_{2,M}^2 \right)}{p_{1,M}^2 + p_{2,M}^2 - p_1,M} < C < \frac{R_{r,m} \omega \left( p_{1,m}^2 + p_{2,m}^2 \right)}{p_{1,m}^2 + p_{2,m}^2 - p_1,m}
\] (22)

It should be pointed out that the frequency of the input signal is known precisely (or the associated error is negligibly small), but when this is not the case, then, by assuming that
\[
\omega_m < \omega < \omega_M
\] (23)

the inequalities in (22) should be written as follows:
\[
\frac{R_{r,m} \omega \left( p_{1,m}^2 + p_{2,m}^2 \right)}{p_{1,m}^2 + p_{2,m}^2 - p_1,m} < C < \frac{R_{r,m} \omega \left( p_{1,m}^2 + p_{2,m}^2 \right)}{p_{1,m}^2 + p_{2,m}^2 - p_1,m}
\] (24)

On the other hand, the problem of fault detection can be transformed into the task of testing the hypotheses. This means that, at the $\alpha$-level, the hypothesis
\[
H_0 : (R, C) = (R_0, C_0)
\]

versus
\[
H_1 : (R, C) \neq (R_0, C_0)
\] (25)

where $R_0$ and $C_0$ are the required (nominal) values of $R$ and $C$. If $H_0$ is rejected when inequalities (21) and (22) are violated. The acceptance of hypothesis $H_1$ denotes the faulty behavior of the impedance.

B. Optimum Experimental Conditions

As can be seen from (17) as well as (21) and (22), the size of the confidence region depends on the so-called Fisher information matrix (FIM) $P^{-1}$. On the other hand, FIM depends on the experimental conditions, e.g., $\xi = [u_1, \ldots, u_m]$. Thus, optimal experimental conditions can be obtained by optimizing some scalar function $\Phi(P^{-1})$. Such a function can be defined in several different ways [10], [11]. In this paper, the so-called 212
\[ P^{-1} = \sum_{k=1}^{n_t} r_k r_k^T, \quad r_k = \left[ \frac{\partial v_k}{\partial R} \right]_k^T. \] (26)

244 The purpose of further consideration is to obtain D-optimum values of \( R_\tau \) and \( \tau \), i.e., \( R_\tau \) and \( \tau \) that maximize \( \det(P^{-1}_k) \).

245 It can be observed that
\[ r_k = P_2 r_{2,k}, \quad P_1 = \sqrt{2 U R_e \rho^2} \text{diag}(1, \omega R^2) \]
\[ r_{1,k} = [a \sin(\omega \kappa + \tau_0) + b \cos(\omega \kappa + \tau_0) - a \cos(\omega \kappa + \tau_0)] \] (27)
\[ a = R^2 + 2 R_\tau R + R_\tau^2 (1 - \omega^2 R^2 C^2) \]
\[ b = -2 C \omega (R_\tau R^2 + R R_\tau^2). \] (28)

247 Bearing in mind that
\[ \sqrt{a^2 + b^2} \sin(\omega \kappa + \tau_0 + \arctan(a/b)) = a \sin(\omega \kappa + \tau_0) + b \cos(\omega \kappa + \tau_0) \] (29)
248 it is possible to write
\[ r_k = P_2 r_{2,k}, \quad P_2 = \sqrt{2 U R_e \rho^2} \text{diag}(1, \omega R^2) \]
\[ r_{2,k} = \sin(\omega \kappa + \tau_0 + \arctan(a/b)) \] (30)
249 Using (29), the FIM can be given as follows:
\[ P^{-1} = P_2 \sum_{k=1}^{n_t} r_{2,k} r_{2,k}^T P_2. \] (31)

250 The main difficulty that is associated with further deliberations is concerned with the selection of the number of measurements \( n_t \). Indeed, it is very difficult to give \( n_t \) a priori. Taking into account the nature of \( \sin(\omega \kappa + \tau_0) \), it is easy to see that the experimental conditions are repeated cyclically. When some experiments are repeated, then the number of experimental conditions is less than the total number of observations \( n_t \). The design resulting from this approach is called a continuous experimental design [10], [11]. The FIM can then be defined as
\[ P^{-1} = P_2 \sum_{k=1}^{n_t} \mu_k r_{2,k} r_{2,k}^T P_2. \] (31)

251 D-optimality criterion is used, i.e., \( \Phi(P^{-1}) = \det(P^{-1}) \) is maximized, which is equivalent to minimizing the volume of the confidence region. This means that an appropriate selection of the experimental conditions will make it possible to obtain a more reliable fault diagnosis system (through more accurate measurements of the impedance) than those designed without it [3]. It should also be pointed out that the experimental conditions are repeated cyclically. When some experiments are repeated, then the number of measurements that provide additional source of knowledge are exploited. First, let us define the FIM as follows:
\[ P^{-1} = \sum_{k=1}^{n_t} r_k r_k^T, \quad r_k = \left[ \frac{\partial v_k}{\partial R} \frac{\partial v_k}{\partial C} \right]_k^T. \] (26)

253 The FIM can then be defined as
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254 Using (31), now, the FIM can be given as follows:
\[ \det(P^{-1}) = \det(P_2)^2 \det \left( \sum_{k=1}^{n_t} \mu_k r_{2,k} r_{2,k}^T \right). \] (31)

255 After some relatively easy but lengthy calculations, it can be shown that
\[ \det \left( \sum_{k=1}^{n_t} \mu_k r_{2,k} r_{2,k}^T \right) = \sin(\omega \tau)^2 \left( 16 \mu_1 \mu_4 \cos(\omega \tau)^4 + 4 (\mu_1 \mu_3 + \mu_2 \mu_4 - 2 \mu_1 \mu_4) \right. \]
\[ \left. \times \cos(\omega \tau)^2 + \mu_1 \mu_4 + \mu_1 \mu_2 + \mu_2 \mu_3 + \mu_3 \mu_4 \right). \] (32)

256 It can be observed that (32) is independent of \( R_\tau \), \( C \), and \( R_e \). On 257 the other hand, \( P_2 \) does not depend on \( \tau \). This means that the 258 maximization of the determinant of the FIM with respect to \( \tau \) is equivalent to
\[ \tau^* = \arg \max_{\tau > 0, \mu_k, i = 1, \ldots, n_t} \det \left( \sum_{k=1}^{n_t} \mu_k r_{2,k} r_{2,k}^T \right). \] (33)

260 whereas the maximization of the determinant of the FIM with respect to \( R_\tau \) is equivalent to
\[ R_\tau^* = \arg \max_{R_\tau > 0} \det(P_2) = \arg \max_{R_\tau > 0} 2 \rho^2 U^2 R^2 R_\tau^2. \] (34)

262 The solution of (33) is given as follows:
\[ \tau^* = \frac{\pi}{2 \omega}. \] (35)

266 with \( \mu_k = 1/4, \quad k = 1, \ldots, n_t = 4 \). Finally, the D-optimum value of the reference resistance \( R_\tau^* \) is given by
\[ R_\tau^* = \frac{R}{\sqrt{1 + \omega^2 R^2 C^2}}. \] (36)

267 C. Other Properties

268 The objective of this section is to investigate the influence of experimental conditions (35) and (36) on the estimation 269
accuracy of \( p \). First, let us define the FIM for \( p \), i.e.,

\[
P^{-1} = \sum_{k=1}^{n_x} r_k^T r_k,
\]

\[
r_k = \left[ \frac{\partial v_k}{\partial p_1}, \frac{\partial v_k}{\partial p_2} \right]^T
\]

\[
= U \sqrt{2} [\sin(\omega \tau k + \tau_0), \cos(\omega \tau k + \tau_0)]^T.
\] (37)

Similarly as in Section III-B, the FIM for the continuous design 272 can be written as

\[
P^{-1} = \sum_{k=1}^{n_x=4} r_k^T r_k.
\] (38)

Substituting (35) into (38), and then \( \mu_k = 1/4, k = 1, \ldots, 274 n_x = 4 \), it can be shown that

\[
P^{-1} = 2U^2 \text{diag} \left( \frac{1}{2}, \frac{1}{2} \right).
\] (39)

From (39), it can be observed that the FIM is diagonal. A design 276 satisfying this property is called the orthogonal design. Its ap- 277 pealing property is that the covariance between the parameters 278 \( p_1 \) and \( p_2 \) is equal to zero, which means that they are estimated 279 independently. The remaining task is to check if experimental 280 conditions (35) and (36) are D-optimum for \( p \). To do that, the 281 following useful criterion can be used \([10],[11]\):

\[
r_k^T P r_k \leq n_p
\] (40)

when the equality holds for \( r_k \) satisfying experimental condi- 283 tions (35) and (36). Substituting \( n_p = 2 \) and then (39) into (40), 284 it can be shown that

\[
2 \sin \left( \frac{1}{2} \pi k + \tau_0 \right)^2 + 2 \cos \left( \frac{1}{2} \pi k + \tau_0 \right)^2 \leq 2.
\] (41)

This implies that experimental designs (35) and (36) are 286 D-optimum and orthogonal for \( p \).

IV. INDUCTIVE IMPEDANCE

The main objective of this section is to derive the D-optimum 289 experimental conditions for inductive impedance. This section 290 presents only the main results that are obtained according to 291 the derivation presented in the preceding sections. First, let 292 us observe that for inductive impedance, the following current 293 equality can be established:

\[
v(t) + \frac{1}{L} \int_0^t v(t) dt = \frac{u(t) - v(t)}{R_r}.
\] (42)

The discrete-time steady-state solution of (42) can be written as

\[
v_k = p_1 u_{1,k} + p_2 u_{2,k}
\] (43)

where

\[
p_1 = \rho L^2 \omega^2 (R_r + R)L, \quad p_2 = \rho L \omega R_r R^2
\] (44)

where \( \rho = ((R + R_r)^2 \omega^2 L^2 + R^2 R_r^2)^{-1} \). Knowing \( \hat{p} \), it is possible to obtain the estimates of \( \hat{R} \) and \( \hat{L} \) according to the 297 following equations:

\[
\hat{R} = -\frac{R_r (\hat{p}_1^2 + \hat{p}_2^2)}{\hat{p}_1^2 + \hat{p}_2^2 - \hat{p}_1}
\] (45)

\[
\hat{L} = \frac{R_r (\hat{p}_2^2 + \hat{p}_1^2)}{\hat{p}_2^2 - \hat{p}_1}.
\] (46)

which are obtained by solving (44) with respect to \( R \) and \( L \). 299

Finally, the D-optimum sampling time is given by (35), whereas 300 the D-optimum reference resistance is given by

\[
R_r = \frac{RL\omega}{\sqrt{R^2 + \omega^2 L^2}}.
\] (47)

Moreover, an equivalent form of (17), (21), and (22) can also 302 be derived relatively easily.

Similarly as in Section III-A, the initial values of parameters 304 \( p_1 \) and \( p_2 \) should satisfy

\[
\frac{1}{2} < \frac{1}{2} \sqrt{1 - 4 \hat{p}_2^2} < \frac{1}{2} < \frac{1}{2} \sqrt{1 - 4 \hat{p}_2^2} < \frac{1}{2}
\] (48)

\[
0 < \hat{p}_2 < \frac{1}{2}.
\] (49)

Another important property is that the optimality conditions 306 presented in Section III-C are valid for the inductive impedance 307 as well.

V. TOWARD ROBUSTNESS

It is clear from (36) [and (47)] that the D-optimum value 310 of the reference resistance \( R_r \) depends on the values of the 311 unknown parameters \( R \) and \( C(L) \). Indeed, it is well known 312 from the literature \([10],[11]\) that the dependence on the param- 313 eters that nonlinearly enter to the model is an unappealing 314 property of the nonlinear optimum experimental design. One 315 way out of this problem is to use the so-called sequential 316 design \([10],[11]\). When there are some rough estimates of \( R \) and \( C(L) \), then \( R_r \) can be calculated according to (36) [or 318 (47)], and then the impedance-measurement procedure can be 319 started. As a result, a more accurate impedance estimate can be 320 obtained, which can be employed to obtain new \( R_r \). This two- 321 step sequential procedure can be repeated several times until 322 satisfactory results are obtained, i.e., suitable measurement 323 accuracy is accomplished. On the other hand, when some prior 324 bounds for \( R \) and \( C(L) \) are given, i.e., \( R \in [R_{\min}, R_{\max}] \) and 325 \( C(L) \in [C(L)_{\min}, C(L)_{\max}] \), then it is possible to use (36) 326 [or (47)] to compute \( R_r \) for the average values of \( C(L) \) and \( R \), 327 which are defined as \( C(L) = 0.5(C(L)_{\min} + C(L)_{\max}) \) and 328 \( \bar{R} = 0.5(R_{\min} + R_{\max}) \), respectively (see \([10],[11]\)). 329

VI. EXPERIMENTAL RESULTS

The main objective of this section is to show the result 332 of experiments regarding the impedance measurement tech- 333 nique presented in the preceding sections. Owing to limited 334
space, only the results for capacitive impedance are presented.
All experiments were performed with the use of the measurement system that is carefully described in [7] and [9].
The excitation signal for a real arm of the “virtual bridge” is provided by HP33120A—a synthesized function/arbitrary generator controlled by an IEEE488 interface. The nominal value of the amplitude of the generated signal was adjusted on $\sqrt{2} \cdot 1.5$ [V], $f = 1$ [kHz]. The high-performance National Instruments AT2150C DAQ plug-in card was used for processing $u(t)$ and $y(t)$. Both voltages are simultaneously sampled and converted by two 16-bit sigma–delta analog-to-digital converters with sampling frequencies of 4 kHz (the basic sampling frequency in the experiments) and 32 kHz.

**A. Reference Resistance, Confidence Interval, and Fault Detection**

Let us consider an exemplary experiment for the following nominal parameters: $R = 1$ [kΩ] and $C = 168$ [nF]. Three different experiments were performed.

1. **Case 1**: optimal reference resistance and sampling frequency, i.e., $R_r = 700$ [Ω] [cf. (36)] and $f_p = 4$ [kHz] [cf. (35)];
2. **Case 2**: not optimal reference resistance and optimal sampling frequency, i.e., $R_r = 7$ [kΩ] and $f_p = 4$ [kHz];
3. **Case 3**: not optimal reference resistance and sampling frequency, i.e., $R_r = 7$ [kΩ] and $f_p = 32$ [kHz].

It should also be pointed out that the inaccuracy of $R_r$ is ± 0.05 [%]. Each of the above experiments was repeated 100 times and for which $n_t = 100$. Figs. 2–5 show the histograms (for Cases 1 and 3) of the relative measurement errors $\delta R$ and $\delta C$, which are defined as follows:

$$
\delta_{R,m} = \left| \frac{R_m - \hat{R}}{\hat{R}} \right| \times 100\% , \quad \delta_{R,M} = \left| \frac{R_M - \hat{R}}{\hat{R}} \right| \times 100\% 
$$

$$
\delta_{C,m} = \left| \frac{C_m - \hat{C}}{\hat{C}} \right| \times 100\% , \quad \delta_{C,M} = \left| \frac{C_M - \hat{C}}{\hat{C}} \right| \times 100\% 
$$

$$
\delta_R = \{ \delta_{R,m}, \delta_{R,M} \} , \quad \delta_C = \{ \delta_{C,m}, \delta_{C,M} \} 
$$

where $R_m(R_M)$ and $C_m(C_M)$ denote the maximum and minimum bounds of $R$ and $C$, which are defined by (21) and 366 (22), respectively. From these results, it can be seen that a considerable increase in the measurement accuracy can be achieved with the D-optimum experimental conditions, i.e., the

Fig. 2. Relative error $\delta_R$ [%] for Case 1.

Fig. 3. Relative error $\delta_R$ [%] for Case 3.

Fig. 4. Relative error $\delta_C$ [%] for Case 1.

Fig. 5. Relative error $\delta_C$ [%] for Case 3.
relative measurement error can be approximately reduced two times when the D-optimum setting is employed. This appealing phenomenon can also be observed while comparing bounds (21) and (22) (assuming that $\alpha = 0.01$, i.e., 99% confidence interval) and the estimated $R$ and $C$ in Figs. 6 and 7. Indeed, the maximum values of $\delta_R$ and $\delta_C$ are as follows:

376 Case 1: $\delta_R = 0.059\%$; $\delta_C = 0.058\%$.  
377 Case 2: $\delta_R = 0.103\%$; $\delta_C = 0.11\%$.  
378 Case 3: $\delta_R = 0.141\%$; $\delta_C = 0.135\%$.

The purpose of the subsequent example is to use the approach developed in Section III-A for fault detection of impedance. Let us assume that the nonfaulty $R$ and $C$ are $R = 993$ [Ω] and $C = 169.6$ [nF], respectively. Thus, the problem of fault detection boils down to the task of testing the following:

$$H_0 : (R, C) = (993 \, [\Omega], 169.6 \, [nF])$$

versus

$$H_1 : (R, C) \neq (993 \, [\Omega], 169.6 \, [nF]).$$

It can be observed from Figs. 6 and 7 and inequalities (21) and (22) that the hypothesis $H_0$ is rejected for Case 1, which means that a fault occurs. On the contrary, the hypothesis $H_0$ can either be accepted or rejected for Case 2, which means that the fault can be undetected. Finally, the hypothesis $H_0$ is accepted when Case 3 is being considered. These results clearly indicate that the application of the D-optimum experimental conditions increases the fault sensitivity, that is, it makes the proposed fault-diagnosis scheme more reliable.

### B. Accuracy Analysis

The main objective of this section is to estimate the measurement accuracy provided by the considered approach. For that purpose, a set of different impedances were selected for the experiment, for which the nominal values of $R$ and $C$ are as follows:

- Case 1: $C = 15$ [nF]; $R = 1$ [kΩ].  
- Case 2: $C = 15$ [nF]; $R = 10$ [kΩ].  
- Case 3: $C = 168$ [nF]; $R = 1$ [kΩ].

Each measurement was repeated 100 times, and then, the mean measured values $\bar{R}$ and $\bar{C}$ were calculated, as well as the coefficient of variation $\bar{\sigma}$ of the corresponding relative measurement errors ($50)$, i.e.,

$$\bar{\sigma}_R = \frac{\sigma_R}{\bar{R}} \times 100\% \quad \text{or} \quad \bar{\sigma}_C = \frac{\sigma_C}{\bar{C}} \times 100\%$$

where $\sigma_C$ (or $\sigma_R$) stands for the standard deviation of $\delta_C$ (or $\delta_R$), whereas $\delta_C$ (or $\delta_R$) denotes its arithmetic mean.

Owing to limited space, the $D$-optimum sampling frequency is used in all cases, whereas the main objective is to show the influence of reference resistance $R_r$. In particular, for each case, two scenarios were considered, i.e., $R_r$ that is near the optimal value (36) and $R_r$ that is approximately ten times larger than its optimal value. It should also be pointed out that the inaccuracy of $R_r$ is ± 0.05 [%]. This knowledge can easily be used to compute (20). Table I shows the achieved results. From these results, it is clear that the proposed approach provides high measurement accuracy. Indeed, the variability of $\delta_C$ and $\delta_R$ is considerably smaller for the D-optimum $R_r$. This 418 phenomenon can also be observed while analyzing the results presented in Figs. 6 and 7. For the sake of illustration, the impedance considered within Cases 1–3 was also measured with the HP4284A Precision LCR Meter. The achieved results are 422 shown in Table II. Some differences among the measurement results shown in Tables I and II are within the boundaries of 424
The main objective of this paper has been to propose a novel impedance-measurement method that can be used for a wide range of impedance. In particular, the problem of impedance measurement has been transformed into a parameter-estimation task. Contrary to the approaches presented in the literature, parameter estimation has been realized with the use of the 439 linear least-squares method (or the RLS method when an online measurement is required), which enables a fast convergence rate. Another important contribution of this paper has been the development of the D-optimum experimental conditions that make it possible to enhance measurement accuracy. In particular, explicit formulas for selecting reference resistance and sampling time have been provided. It has also been shown that the proposed approach can effectively be applied for fault detection, which is very important from the point of view of modern control and fault diagnosis.

Numerous experiments presented in this paper have shown that the proposed measurement method enables high accuracy under a relatively large convergence rate. We hope that the results presented in this paper will encourage engineers to apply the proposed measurement technique in practice.

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### REFERENCES


Marcin Witczak was born in Poland on December 19, 1973. He received the M.Sc. and D.Sc. degrees in electrical engineering from the University of Zielona Góra, Zielona Góra, Poland, in 1998 and 2007, respectively, and the Ph.D. degree in automatic control and robotics from the Wroclaw University of Technology, Wroclaw, Poland, in 2002.

Since 2002, he has been an Assistant Professor of automatic control and robotics with the Institute of Control and Computation Engineering, University of Zielona Góra. He has published more than 75 papers in international journals and conference proceedings. He has authored two monographs and eight book chapters. His current research interests include computational intelligence, fault detection and isolation, fault-tolerant control, and experimental design and control theory.

Ryszard Rybski was born in Poland in 1952. He received the M.Sc. degree in electrical engineering and the D.Sc. degree from the University of Zielona Góra, Zielona Góra, Poland, in 1979 and 2008, respectively, and the Ph.D. degree from the Wroclaw University of Technology, Wroclaw, Poland, in 1989.

Since 1979, he has been with the Institute of Electrical Metrology, University of Zielona Góra. His research interests include high-accuracy measurements of electrical quantities, particularly the precise impedance measurements.

Janusz Kaczmarek was born in Poland in 1964. He received the M.Sc. and Ph.D. degrees in electrical metrology from the University of Zielona Góra, Zielona Góra, Poland, in 1989 and 1996, respectively.

He is currently an Assistant Professor with the Institute of Electrical Metrology, University of Zielona Góra. He has also been a supervisor and the main executor of many research projects for industry. His research areas of interests include precise electrical measurements, instrumentation, and virtual instrumentation.
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Impedance Measurement With the D-Optimum
Experimental Conditions

Marcin Witczak, Ryszard Rybski, and Janusz Kaczmarek

Abstract—A computer-based automatic alternating current (ac) bridge for impedance measurement is proposed. The problem of impedance measurement is formulated as a linear-in-parameter estimation one, and a suitable algorithm is proposed that ensures high measurement accuracy under a high convergence rate. In particular, the optimum experimental conditions regarding the reference resistance and the sampling time are developed. These conditions guarantee that the estimation is performed with practically high accuracy. This paper also shows how to use the proposed approach for fault detection, which is very important from the point of view of modern control and fault diagnosis. The final part of this paper shows a number of illustrative experiments that justify the effectiveness of the proposed approach.

Index Terms—Experimental design, impedance measurement, measurement accuracy, model uncertainty, parameter estimation.

I. INTRODUCTION

VARIOUS physical quantities, e.g., force, pressure, flow, and displacement, can be transformed into impedance through various transducers. The measurements that are collected in such a way constitute the basic source of knowledge regarding any industrial system being controlled or diagnosed. Many different techniques and equipment have been developed over the past few years to meet demands on different operational ranges, accuracy, measurement rates, specific application targets, and costs. So far, there has been no doubt that high-accuracy measurements can be achieved by using ac bridges that are balanced either manually or iteratively. Unfortunately, such a balancing technique excludes fast measurement rates that are required by modern control and fault-diagnostics systems [8], [12]. To settle such a challenging problem, many different computer-based ac bridges have been developed over the last 15 years. Dutta et al. [5] introduced the idea of a "virtual ac bridge," with a virtual arm implemented with the help of a microcomputer. Awad et al. [2] made an important contribution to this approach with respect to its convergence. Recently, Dutta et al. [6] have proposed another modification of this impedance measurement technique. In all above papers, the authors formulated the impedance-measurement problem as the nonlinear-in-parameter estimation one [11]. To solve such a problem, they employed the gradient descent algorithm [11], [43] which made the required measurement rate difficult to attain.

To settle this problem, Kaczmarek et al. [7] employed the so-called bounded-error parameter estimation technique [4], [11]. In spite of the high convergence rate, the technique does not make it possible to attain the accuracy that is comparable with that discussed in [2] and [6]. A slightly different approach was proposed by Angrisani et al. [1]. This technique is based on a bridge-balance loop comprising two internal loops. The first one is used for tuning the capacitive (inductive) part of the 52 impedance, whereas the second one is used to rearrange the resistive part. The main drawback of this approach is that the fast convergence seems very hard to attain. A fast balancing bridge 55 was proposed by Zhang et al. [13], and it seems suitable for a 56 wide range of industrial applications.

Unfortunately, the researchers have not considered the relation between the experimental conditions [10], [11] and the accuracy of the measurements. In this paper, the experimental conditions are related with the optimum selection of the reference resistance of the bridge and the sampling time. 62 Angrisani et al. [1] and Zhang et al. [13] discovered, by numerous simulations, that the closer the selected reference resistance is to the measured impedance, the more precise measurement results can be obtained. However, they did not provide any explicit formulas determining the reference resistance.

In this paper, the answer to the challenging problem of the optimum experimental conditions is provided; that is, explicit formulas for the reference resistance and the sampling time are developed. This paper is organized as follows. Section II shows the formulation of the problem and the proposed measurement technique. In particular, the problem of impedance measurement is transformed into the linear-in-parameter estimation task. This makes it possible to achieve both high estimation quality and a fast convergence rate. In Section III, the results regarding the optimum experimental conditions are provided. Section IV shows a way of extending the proposed approach to the inductive impedance. Section V is devoted to the robustness problems that are very important in practical applications. Subsequently, Section VI presents a comprehensive experimental study, which confirms the high performance of the proposed approach. Finally, Section VII concludes this paper.

II. PROBLEM FORMULATION

The objective of this section is to propose a new impedance measurement method. This paper is motivated by the approach presented in [7]. The virtual bridge is composed of two arms, 87...
samples, where \( f_p = 1/\tau \). Thus, it is clear that \( f_p \) should be selected in such a way that \( n \) is an integer number. It should also be pointed out that any incoherence of \( f_p \) causes \( n \) to be not an integer number, and, hence, \( u_{2,k} \) cannot be determined with the highest precision. Undoubtedly, this uncertainty in the input signal impacts the overall performance of the proposed impedance-measurement method. Unfortunately, the theory analysis regarding the influence of this unappealing phenomenon on the final measurement quality seems impossible to realize.

Another important fact that can be observed while analyzing (3) is that it can be perceived as a linear-in-parameter model with respect to \( p_1 \) and \( p_2 \). Contrary to (2) and (5), where the nonlinear parameter estimation techniques were employed for obtaining \( R \) and \( C \), it is proposed to use the classical recursive least squares (RLS) algorithm [11] for the estimation of \( p_1 \) and \( p_2 \). Such an algorithm can be written as follows:

\[
\hat{p}_k = \hat{p}_{k-1} + k_k \varepsilon_k
\]

\[
k_k = P_k^{-1} P_k r_k \left( 1 + r_r^T P_k^{-1} r_r \right)^{-1}
\]

\[
\varepsilon_k = y_k - f(\hat{p}_{k-1}, u_k)
\]

\[
P_k = I_{n_p} - k_k r_k^T P_{k-1}
\]

where \( y_k \) is the \( k \)th measurement of \( v_k \) (cf. Fig. 1), 129 \( f(\hat{p}_k, u_{k+1}) = \hat{p}_{1,k} u_{1,k+1} + \hat{p}_{2,k} u_{2,k+1}, \quad r_k = [u_{1,k}, u_{2,k}]^T \).

\[
\hat{R} = -\frac{R_r (\hat{p}_1^2 + \hat{p}_2^2)}{\hat{p}_1^2 + \hat{p}_2^2 - p_1}
\]

\[
\hat{C} = -\frac{\hat{p}_2}{R_r \omega (\hat{p}_1^2 + \hat{p}_2^2)}
\]

which are obtained by solving (4) with respect to \( R \) and \( C \). It is also pointed out that when there is no need for online estimation of the impedance, then the classical nonrecursive least squares algorithm can be employed. The well-known advantage of this algorithm, as compared with its recursive counterpart, is that the highest estimation accuracy can be attained with smaller \( n_t \). In this case, estimates of \( p_1 \) and \( p_2 \) can be computed as follows:

\[
\hat{p}_1 = \frac{\gamma_1 \eta - \beta_1 \gamma_2}{\eta^2 - \beta_1 \beta_2}, \quad \hat{p}_2 = \frac{\gamma_1 \eta - \beta_1 \gamma_2}{\eta^2 - \beta_1 \beta_2}
\]

where

\[
\gamma_k = \sum_{k=1}^{n_t} u_{i,k} y_k, \quad \eta = \sum_{k=1}^{n_t} u_{1,k} u_{2,k}, \quad \beta_k = \sum_{k=1}^{n_t} u_{i,k}^2.
\]

### A. Initialization of the RLS Algorithm

As can be found in the literature [11], the matrix \( P_0 \), i.e., \( P_0 = \gamma I \), 147
where $\gamma$ stands for a sufficiently large positive constant (usually $10^3-10^{20}$). When some rough values of $R$ and $C$ are known, then $p_0$ should be initialized according to (4). Otherwise, it can be observed from (10) that $\hat{p}_1^2 + \hat{p}_2^2 - \hat{p}_1 < 0$, and hence
\begin{equation}
\frac{1}{2} - \frac{1}{2}\sqrt{1-4\hat{p}_2^2} < \hat{p}_1 < \frac{1}{2} + \frac{1}{2}\sqrt{1-4\hat{p}_2^2}.
\end{equation}

Since $\hat{p}_2$ should satisfy $1-4\hat{p}_2^2 > 0$, while (10) and (11) indicate that $\hat{p}_2 < 0$ and $\hat{p}_1 > 0$, then it is clear that
\begin{equation}
-\frac{1}{2} < \hat{p}_2 < 0.
\end{equation}

Thus, when no knowledge is available about $R$ and $C$, then $\hat{p}_0$ should be set to satisfy (14) and (15).

### III. Experimental Design

The one objective of this section is to provide rules for comparing the accuracy of the measured impedance, i.e., a set of all $R$ and $C$ that are consistent with the measurements of $u$ and $y$. The main objective is to provide the optimum experimental conditions; that is, explicit formulas for the reference resistance and the sampling time are developed that make it possible to increase the measurement accuracy. The proposed solution is based on the following assumption:
\begin{equation}
y_k = v_k + \epsilon_k
\end{equation}
where $\epsilon_k$ stands for the zero-mean uncorrelated Gaussian noise sequence. In other words, $\epsilon_k$ represents the difference between the output of the model (3) and $y_k$ that represents the actual measurements of $v_k$ (cf. Fig. 1).

#### A. Confidence Region and Fault Detection

Since the estimates of $R$ and $C$ can be obtained according to (10) and (11), the next problem being considered is obtaining a set of all possible $R$ and $C$ that are consistent with the $u$ and $y$. Such a set can be obtained with the use of the $(1-\alpha)100\%$ confidence region [11] for $p$ and $\gamma$ (4), where $\alpha$ stands for the significance level. As a result, the following inequality is given:
\begin{equation}
d_k^T P^{-1} d_k \leq 2\hat{\sigma}_k^2 F_{\alpha,2,k-2}
\end{equation}
where
\begin{equation}
d_k = \hat{p}_k - p \left[ R(R + R_r), R_r C \omega R^2 \right]^T
\end{equation}
$F_{\alpha,2,k-2}$ stands for the F-Snedecor distribution quantile with 2 and $k-2$ degrees of freedom, and $\hat{\sigma}$ is the estimate of the standard deviation of $\epsilon$. The inequality (17) is very important from the point of view of fault detection and control of industrial systems [8], [12]. Indeed, it can be used for checking whether the measured impedance satisfies the predefined bounds. However, the main drawback of (17) is that it does not take into account the inaccuracy of $R_r$. To tackle this problem, let us define the $(1-\alpha)100\%$ confidence interval for $p_1$ and $p_2$, i.e.,
\begin{equation}
\hat{p}_1 - \Delta_1 = p_{1,m} < p_1 < p_{1,M} = \hat{p}_1 + \Delta_1
\end{equation}
\begin{equation}
\hat{p}_2 - \Delta_2 = p_{2,m} < p_2 < p_{2,M} = \hat{p}_2 + \Delta_2
\end{equation}
where $\Delta_1 = \hat{\sigma} \bar{\tau}_{k-3} \sqrt{P_{i,k}}$, $P_{i,k}$ stands for the $i$th diagonal element of $P_k$, and $\bar{\tau}_{k-3}$ is the Student distribution quantile with $k-3$ degrees of freedom. A similar interval can be defined for $R_r$, i.e.,
\begin{equation}
R_{r,m} < R_r < R_{r,M}
\end{equation}
which can be easily computed when the accuracy of $R_r$ is known. Thus, using (19) and (20) along with (10) and (11), it can be shown that
\begin{equation}
-\frac{R_{r,M} \left( p_{1,m}^2 + p_{2,M} \right)}{p_{1,m} + p_{2,M} - p_{1,m}} < R < -\frac{R_{r,M} \left( p_{1,M}^2 + p_{2,m} \right)}{p_{1,M} + p_{2,m} - p_{1,M}}
\end{equation}
\begin{equation}
-\frac{R_{r,M} \omega \left( p_{1,M}^2 + p_{2,M} \right)}{p_{1,m}^2 + p_{2,M}^2} < C < -\frac{R_{r,m} \omega \left( p_{1,m}^2 + p_{2,m}^2 \right)}{p_{1,M}^2 + p_{2,m}^2}.
\end{equation}

It should be pointed out that the frequency of the input signal is known precisely (or the associated error is negligibly small), but when this is not the case, then, by assuming that $\omega_m < \omega < \omega_M$
the inequalities (22) should be written as follows:
\begin{equation}
-\frac{R_{r,M} \omega \left( p_{1,m}^2 + p_{2,M} \right)}{p_{1,m}^2 + p_{2,M}^2} < C < -\frac{R_{r,m} \omega \left( p_{1,m}^2 + p_{2,m}^2 \right)}{p_{1,M}^2 + p_{2,m}^2}.
\end{equation}

On the other hand, the problem of fault detection can be transformed into the task of testing the hypotheses. This means that, at the $\alpha$-level, the hypothesis
\begin{equation}
H_0 : (R, C) = (R_0, C_0)
\end{equation}
versus
\begin{equation}
H_1 : (R, C) \neq (R_0, C_0)
\end{equation}
where $R_0$ and $C_0$ are the required (nominal) values of $R$ and $C$, is rejected when inequalities (21) and (22) are violated. The acceptance of hypothesis $H_1$ denotes the faulty behavior of the 203 impedance.

#### B. Optimum Experimental Conditions

As can be seen from (17) as well as (21) and (22), the size of the confidence region depends on the so-called Fisher information matrix (FIM) $P^{-1}$. On the other hand, FIM depends on the experimental conditions, e.g., $\xi = [u_1, \ldots, u_m]$. Thus, optimal experimental conditions can be obtained by optimizing 210 some scalar function $\Phi(P^{-1})$. Such a function can be defined 211 in several different ways [10], [11]. In this paper, the so-called 212
**D-optimality criterion** is used, i.e., \( \Phi(P^{-1}) = \det(P^{-1}) \) is maximized, which is equivalent to minimizing the volume of the confidence region. This means that an appropriate selection of experimental conditions will make it possible to obtain a more reliable fault diagnosis system (through more accurate measurements of the impedance) than those designed without it [3]. It should also be pointed out that the experimental conditions are repeated cyclically. When it is concerned with the selection of the number of measurements, an appropriate selection of the number of measurements is used, i.e., \( \mu_k = 1/n_e \). Indeed, the measurement times \( t_k \) are parameterized by \( t_k = k \tau \), and hence, there is no justification to favor some of them. Unfortunately, the D-optimum design for \( n_e = 3 \) leads to \( \mu_k \), which are unequal. On the other hand, such a D-optimality criterion is possible to attain for \( n_e = 4 \), and hence, such a choice is to be employed in the sequel of this paper.

Using (31), it can be shown that

\[
\det(P^{-1}) = \det(P_2)^2 \det \left( \sum_{k=1}^{n_e} \mu_k r_{2,k} r_{2,k}^T \right).
\]

After some relatively easy but lengthy calculations, it can be shown that

\[
\det \left( \sum_{k=1}^{n_e} \mu_k r_{2,k} r_{2,k}^T \right) = \sin(\omega \tau)^2 (16\mu_1 \mu_4 \cos(\omega \tau)^4 + 4(\mu_1 \mu_3 + \mu_2 \mu_4 - 2\mu_1 \mu_4)) \times \cos(\omega \tau)^2 + \mu_1 \mu_4 + \mu_1 \mu_2 + \mu_2 \mu_3 + \mu_3 \mu_4.
\]

(32)

It can be observed that (32) is independent of \( R_r, C \), and \( R_e \). On the other hand, \( P_2 \) does not depend on \( \tau \). This means that the 258 maximization of the determinant of the FIM with respect to \( \tau \) is equivalent to

\[
\tau^* = \arg \max_{\tau > 0, \mu_k, k = 1, \ldots, n_e} \det \left( \sum_{k=1}^{n_e} \mu_k r_{2,k} r_{2,k}^T \right).
\]

(33)

whereas the maximization of the determinant of the FIM with 261 respect to \( R_r \) is equivalent to

\[
R_r^* = \arg \max_{R_r > 0} \det(P_2) = \arg \max_{R_r > 0} 2\rho^2 \omega U^2 R_r^2 R_r^2.
\]

(34)

The solution of (33) is given as follows:

\[
\tau^* = \frac{\pi}{2\omega}.
\]

(35)

with \( \mu_k = 1/4 \), \( k = 1, \ldots, n_e = 4 \). Finally, the D-optimum 264 value of the reference resistance \( R_r^* \) [being the solution of (34)] can be written according to

\[
R_r^* = \frac{R}{\sqrt{1 + \omega^2 R_r^2 C^2}}.
\]

(36)

C. Other Properties

The objective of this section is to investigate the influence of experimental conditions (35) and (36) on the estimation...
270 accuracy of $p$. First, let us define the FIM for $p$, i.e.,
\[ P^{-1} = \sum_{k=1}^{n_e} r_k r_k^T, \quad r_k = \left[ \frac{\partial v_k}{\partial p_1} \frac{\partial v_k}{\partial p_2} \right]^T \]
\[ = U \sqrt{2} [\sin(\omega \tau k + \tau_0), \cos(\omega \tau k + \tau_0)]^T. \] (37)

271 Similarly as in Section III-B, the FIM for the continuous design
272 can be written as
\[ P^{-1} = \sum_{k=1}^{n_e=4} r_k r_k^T. \] (38)

273 Substituting (35) into (38), and then $\mu_k = 1/4$, $k = 1, \ldots, 274 n_e = 4$, it can be shown that
\[ P^{-1} = 2U^2 \text{diag} \left( \frac{1}{2}, \frac{1}{2} \right). \] (39)

275 From (39), it can be observed that the FIM is diagonal. A design 276 satisfying this property is called the orthogonal design. Its ap- 277 pealing property is that the covariance between the parameters 278 $p_1$ and $p_2$ is equal to zero, which means that they are estimated 279 independently. The remaining task is to check if experimental 280 conditions (35) and (36) are D-optimum for $p$. To do that, the 281 following useful criterion can be used [10], [11]:
\[ r_k^T P r_k \leq n_p \] (40)

282 when the equality holds for $r_k$ satisfying experimental condi- 283 tions (35) and (36). Substituting $n_p = 2$ and then (39) into (40), 284 it can be shown that
\[ 2 \sin \left( \frac{1}{2} \pi k + \tau_0 \right)^2 + 2 \cos \left( \frac{1}{2} \pi k + \tau_0 \right)^2 \leq 2. \] (41)

285 This implies that experimental designs (35) and (36) are 286 D-optimum and orthogonal for $p$.

IV. INDUCTIVE IMPEDANCE

288 The main objective of this section is to derive the D-optimum 289 experimental conditions for inductive impedance. This section 290 presents only the main results that are obtained according to 291 the derivation presented in the preceding sections. First, let 292 us observe that for inductive impedance, the following current
293 equality can be established:
\[ v(t) + \frac{1}{L} \int_0^t v(t) dt = \frac{u(t) - v(t)}{R_r}. \] (42)

294 The discrete-time steady-state solution of (42) can be written as
\[ v_k = p_1 u_{1,k} + p_2 u_{2,k} \] (43)

295 where
\[ p_1 = \rho L^2 \omega^2 (R_r + R) R, \quad p_2 = \rho L \omega R, R^2 \] (44)
where $\rho = ((R + R_r)^2 \omega^2 L^2 + R^2 R_r^2)^{-1}$. Knowing $\hat{p}$, it is 296 possible to obtain the estimates of $\hat{R}$ and $L$ according to 297 the following equations:
\[ \hat{R} = -\frac{R_r (\hat{p}_1^2 + \hat{p}_2^2)}{\hat{p}_2^2 - \hat{p}_1}, \quad \hat{L} = \frac{R_r (\hat{p}_2^2 + \hat{p}_1^2)}{\hat{p}_2^2 - \hat{p}_1} \] (45)

298 which are obtained by solving (44) with respect to $R$ and $L$. 299 Finally, the D-optimum sampling time is given by (35), whereas 300 the D-optimum reference resistance is given by
\[ R_r = \frac{RL\omega}{\sqrt{R^2 + \omega^2 L^2}}. \] (47)

Moreover, an equivalent form of (17), (21), and (22) can also 302 be derived relatively easily.

Similarly as in Section III-A, the initial values of parameters 304 $p_1$ and $p_2$ should satisfy 305
\[ \frac{1}{2} - \frac{1}{2} \sqrt{1 - 4\hat{p}_2^2} < \hat{p}_1 < \frac{1}{2} + \frac{1}{2} \sqrt{1 - 4\hat{p}_2^2} \] (48)
\[ 0 < \hat{p}_2 < \frac{1}{2}. \] (49)

Another important property is that the optimality conditions 306 presented in Section III-C are valid for the inductive impedance 307 as well.

V. TOWARD ROBUSTNESS

309 It is clear from (36) [and (47)] that the D-optimum value 310 of the reference resistance $R_r$ depends on the values of the 311 unknown parameters $R$ and $C(L)$. Indeed, it is well known 312 from the literature [10], [11] that the dependence on the pa- 313 rameters that nonlinearly enter to the model is an unappealing 314 property of the nonlinear optimum experimental design. One 315 way out of this problem is to use the so-called sequential 316 design [10], [11]. When there are some rough estimates of $R$ 317 and $C(L)$, then $R_r$ can be calculated according to (36) [or 318 (47)], and then the impedance-measurement procedure can be 319 started. As a result, a more accurate impedance estimate can be 320 obtained, which can be employed to obtain new $R_r$. This two- 321 step sequential procedure can be repeated several times until 322 satisfactory results are obtained, i.e., suitable measurement 323 accuracy is accomplished. On the other hand, when some prior 324 bounds for $R$ and $C(L)$ are given, i.e., $R \in [R_{\text{min}}, R_{\text{max}}]$ and 325 $C(L) \in [C(L)_{\text{min}}, C(L)_{\text{max}}]$, then it is possible to use (36) 326 [or (47)] to compute $R_r$ for the average values of $C(L)$ and $R$, 327 which are defined as $C(L) = 0.5 (C(L)_{\text{min}} + C(L)_{\text{max}})$ and 328 $R_r = 0.5 (R_{\text{min}} + R_{\text{max}})$, respectively (see [10] and [11] for 329 further comments about average D-optimality).

VI. EXPERIMENTAL RESULTS

331 The main objective of this section is to show the result 332 of experiments regarding the impedance measurement tech- 333 nique presented in the preceding sections. Owing to limited 334
space, only the results for capacitive impedance are presented. All experiments were performed with the use of the measurement system that is carefully described in [7] and [9].

The excitation signal for a real arm of the “virtual bridge” is provided by HP33120A—a synthesized function/arbitrary generator controlled by an IEEE488 interface. The nominal value of the amplitude of the generated signal was adjusted on \( \sqrt{2} \cdot 1.5 \) [V], \( f = 1 \) [kHz]. The high-performance National Instruments AT2150C DAQ plug-in card was used for processing \( u(t) \) and \( y(t) \). Both voltages are simultaneously sampled and converted by two 16-bit sigma–delta analog-to-digital converters with sampling frequencies of 4 kHz (the basic sampling frequency in the experiments) and 32 kHz.

**A. Reference Resistance, Confidence Interval, and Fault Detection**

Let us consider an exemplary experiment for the following nominal parameters: \( R = 1 \) [kΩ] and \( C = 168 \) [nF]. Three different experiments were performed.

- **Case 1**: optimal reference resistance and sampling frequency, i.e., \( R_r = 700 \) [Ω] [cf. (36)] and \( f_p = 4 \) [kHz] [cf. (35)];
- **Case 2**: not optimal reference resistance and optimal sampling frequency, i.e., \( R_r = 7 \) [kΩ] and \( f_p = 4 \) [kHz];
- **Case 3**: not optimal reference resistance and sampling frequency, i.e., \( R_r = 7 \) [kΩ] and \( f_p = 32 \) [kHz].

It should also be pointed out that the inaccuracy of \( R_r \) is \( \pm 0.05 \) [%]. Each of the above experiments was repeated 100 times and for which \( n_t = 100 \). Figs. 2–5 show the histograms (for Cases 1 and 3) of the relative measurement errors \( \delta_R \) and \( \delta_C \), which are defined as follows:

\[
\delta_{R,m} = \frac{R_m - \hat{R}}{R} \cdot 100\% \quad \text{and} \quad \delta_{R,M} = \frac{R_M - \hat{R}}{R} \cdot 100\% \quad \text{[50]}
\]

\[
\delta_{C,m} = \frac{C_m - \hat{C}}{C} \cdot 100\% \quad \text{and} \quad \delta_{C,M} = \frac{C_M - \hat{C}}{C} \cdot 100\% \quad \text{[50]}
\]

\[
\delta_R = \{\delta_{R,m}, \delta_{R,M}\} \quad \text{and} \quad \delta_C = \{\delta_{C,m}, \delta_{C,M}\}
\]

where \( R_m(R_M) \) and \( C_m(C_M) \) denote the maximum and minimum bounds of \( R \) and \( C \), which are defined by (21) and (22), respectively. From these results, it can be seen that a considerable increase in the measurement accuracy can be achieved with the D-optimum experimental conditions, i.e., the
relative measurement error can be approximately reduced two

times when the D-optimum setting is employed. This appealing phenomenon can also be observed while comparing bounds (21) and (22) (assuming that $\alpha = 0.01$, i.e., 99% confidence interval) and the estimated $R$ and $C$ in Figs. 6 and 7. Indeed, the maximum values of $\delta_R$ and $\delta_C$ are as follows:

Case 1: $\delta_R = 0.059\%$; $\delta_C = 0.058\%$.
Case 2: $\delta_R = 0.103\%$; $\delta_C = 0.11\%$.
Case 3: $\delta_R = 0.141\%$; $\delta_C = 0.135\%$.

The purpose of the subsequent example is to use the approach developed in Section III-A for fault detection of impedance. Let us assume that the nonfaulty $R$ and $C$ are $R = 993 \, \Omega$ and $C = 169.6 \, nF$, respectively. Thus, the problem of fault detection boils down to the task of testing the following:

$$H_0 : (R, C) = (993 \, \Omega, 169.6 \, nF)$$

versus

$$H_1 : (R, C) \neq (993 \, \Omega, 169.6 \, nF).$$

It can be observed from Figs. 6 and 7 and inequalities (21) and (22) that the hypothesis $H_0$ is rejected for Case 1, which means that a fault occurs. On the contrary, the hypothesis $H_0$ is accepted when Case 3 is being considered. These results clearly indicate that the application of the D-optimum experimental conditions increases the fault sensitivity, that is, it makes the proposed fault-diagnosis scheme more reliable.

B. Accuracy Analysis

The main objective of this section is to estimate the measurement accuracy provided by the considered approach. For that purpose, a set of different impedances were selected for the experiment, for which the nominal values of $R$ and $C$ are as follows:

Case 1: $C = 15 \, nF$; $R = 1 \, k\Omega$.

Case 2: $C = 15 \, nF$; $R = 10 \, k\Omega$.

Case 3: $C = 168 \, nF$; $R = 1 \, k\Omega$.

Each measurement was repeated 100 times, and then, the mean measured values $\bar{R}$ and $\bar{C}$ were calculated, as well as the coefficient of variation $\bar{\sigma}$ of the corresponding relative measurement errors (50), i.e.,

$$\bar{\sigma}_R = \frac{\sigma_R}{\bar{R}} \times 100\% \quad \text{or} \quad \bar{\sigma}_C = \frac{\sigma_C}{\bar{C}} \times 100\% \quad (52)$$

where $\sigma_C$ (or $\sigma_R$) stands for the standard deviation of $\delta_C$ (or $\delta_R$), whereas $\delta_C$ (or $\delta_R$) denotes its arithmetic mean.

Owing to limited space, the D-optimum sampling frequency is used in all cases, whereas the main objective is to show the influence of reference resistance $R_r$. In particular, for each case, two scenarios were considered, i.e., $R_r$ that is near the optimal value (36) and $R_r$ that is approximately ten times larger than its optimal value. It should also be pointed out that the inaccuracy of $R_r$ is $\pm 0.05 \%$. This knowledge can easily be used to compute (20). Table I shows the achieved results. From these results, it is clear that the proposed approach provides high measurement accuracy. Indeed, the variability of $\delta_C$ and $\delta_R$ is considerably smaller for the D-optimum $R_r$. This phenomenon can also be observed while analyzing the results presented in Figs. 6 and 7. For the sake of illustration, the impedance considered within Cases 1–3 was also measured with the HP4284A Precision LCR Meter. The achieved results are shown in Table II. Some differences among the measurement results shown in Tables I and II are within the boundaries of

<table>
<thead>
<tr>
<th>Case</th>
<th>$R_1$ [Ω]</th>
<th>$R_2$ [Ω]</th>
<th>$C$ [nF]</th>
<th>$\bar{\sigma}_R$ [%]</th>
<th>$\bar{\sigma}_C$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1004.67</td>
<td>999.76</td>
<td>13.766</td>
<td>0.8</td>
<td>2.4</td>
</tr>
<tr>
<td>2</td>
<td>7119.53</td>
<td>10157.3</td>
<td>13.785</td>
<td>2.2</td>
<td>1.8</td>
</tr>
<tr>
<td>3</td>
<td>709.814</td>
<td>994.12</td>
<td>169.371</td>
<td>1.5</td>
<td>1.4</td>
</tr>
</tbody>
</table>
The main objective of this paper has been to propose a novel impedance-measurement method that can be used for a wide range of impedance. In particular, the problem of impedance measurement has been transformed into a parameter-estimation task. Contrary to the approaches presented in the literature, the parameter estimation has been realized with the use of the 439 linear least-squares method (or the RLS method when an online measurement is required), which enables a fast convergence rate. Another important contribution of this paper has been the development of the D-optimum experimental conditions that make it possible to enhance measurement accuracy. In particular, explicit formulas for selecting reference resistance and sampling time have been provided. It has also been shown that the proposed approach can effectively be applied for fault detection, which is very important from the point of view of modern control and fault diagnosis.

Numerous experiments presented in this paper have shown that the proposed measurement method enables high accuracy under a relatively large convergence rate. We hope that the results presented in this paper will encourage engineers to apply the proposed measurement technique in practice.

TABLE II
MEASUREMENT RESULTS WITH THE LCR METER

<table>
<thead>
<tr>
<th>Case</th>
<th>$R$ [Ω]</th>
<th>$C$ [nF]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>998.41</td>
<td>13.742</td>
</tr>
<tr>
<td>2</td>
<td>10121.5</td>
<td>13.756</td>
</tr>
<tr>
<td>3</td>
<td>892.65</td>
<td>109.36</td>
</tr>
</tbody>
</table>

Fig. 8. Evolution of $\delta_R$ [%].

Fig. 9. Evolution of $\delta_C$ [%].

Finally, Figs. 8 and 9 show the evolution of the relative errors (50) for Case 1, $R_e = 709.814$ Ω in the consecutive iterations of the proposed algorithm. From these results, it is clear that high measurement accuracy can be achieved after a dozen or so iterations only.

VII. CONCLUSION

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REFERENCES

Marcin Witczak was born in Poland on December 19, 1973. He received the M.Sc. and D.Sc. degrees in electrical engineering from the University of Zielona Góra, Zielona Góra, Poland, in 1998 and 2007, respectively, and the Ph.D. degree in automatic control and robotics from the Wroclaw University of Technology, Wroclaw, Poland, in 2002. Since 2002, he has been an Assistant Professor of automatic control and robotics with the Institute of Control and Computation Engineering, University of Zielona Góra. He has published more than 75 papers in international journals and conference proceedings. He has authored two monographs and eight book chapters. His current research interests include computational intelligence, fault detection and isolation, fault-tolerant control, and experimental design and control theory.

Ryszard Rybski was born in Poland in 1952. He received the M.Sc. degree in electrical engineering and the D.Sc. degree from the University of Zielona Góra, Zielona Góra, Poland, in 1979 and 2008, respectively, and the Ph.D. degree from the Wroclaw University of Technology, Wroclaw, Poland, in 1989. Since 1979, he has been with the Institute of Electrical Metrology, University of Zielona Góra. His research interests include high-accuracy measurements of electrical quantities, particularly the precise impedance measurements.

Janusz Kaczmarek was born in Poland in 1964. He received the M.Sc. and Ph.D. degrees in electrical metrology from the University of Zielona Góra, Zielona Góra, Poland, in 1989 and 1996, respectively. He is currently an Assistant Professor with the Institute of Electrical Metrology, University of Zielona Góra. He has also been a supervisor and the main executor of many research projects for industry. His research areas of interests include precise electrical measurements, instrumentation, and virtual instrumentation.
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AQ3 = Please provide issue number and month of publication in Ref. [5].

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